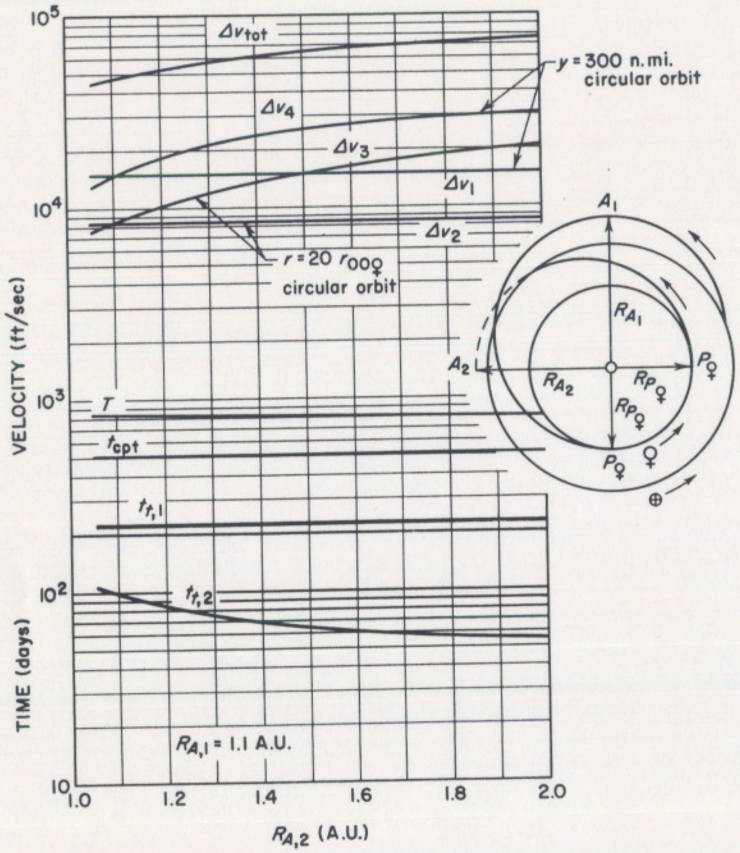

FIG. 9-65 Mission Profile 21 for Earth-Venus Operation (Planar; circular planet orbits).

well as human factors are astronautically interwoven in selecting the mission profile which best suits the spacecraft and the over-all mission purpose.

9-9 INTERPLANETARY FLIGHTS INVOLVING SEVERAL PLANETS

Instead of attempting to shorten the round trip to *one* planet in the manner described in the preceding paragraph, one may design the return path in such a manner that one or two other planets are contacted en route. This approach not only avoids the lengthy capture periods but in addition permits exploratory activity near other planets and shortens the over-all mission period.

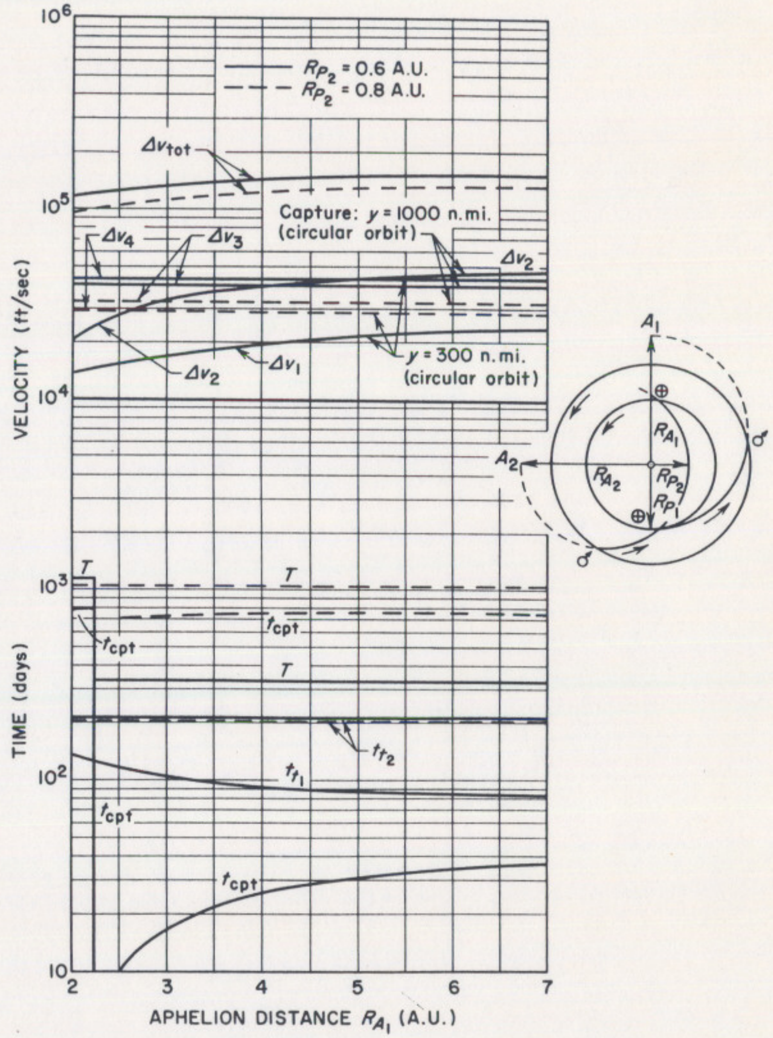

FIG. 9-66 Mission Profile 15 for Earth-Mars Operation (Planar; circular planet orbits).

The concept of a multi-planet round-trip already intrigued Hohmann, who in 1928 briefly investigated an Earth-Mars-Venus-Earth flight and an Earth-Mars-Venus-Mercury-Earth round trip. Later the Italian aviation pioneer and scientist G. A. Crocco presented a more detailed analysis of an Earth-Mars-Venus-Earth mission during the seventh International Astronautical Congress in Rome, 1956.

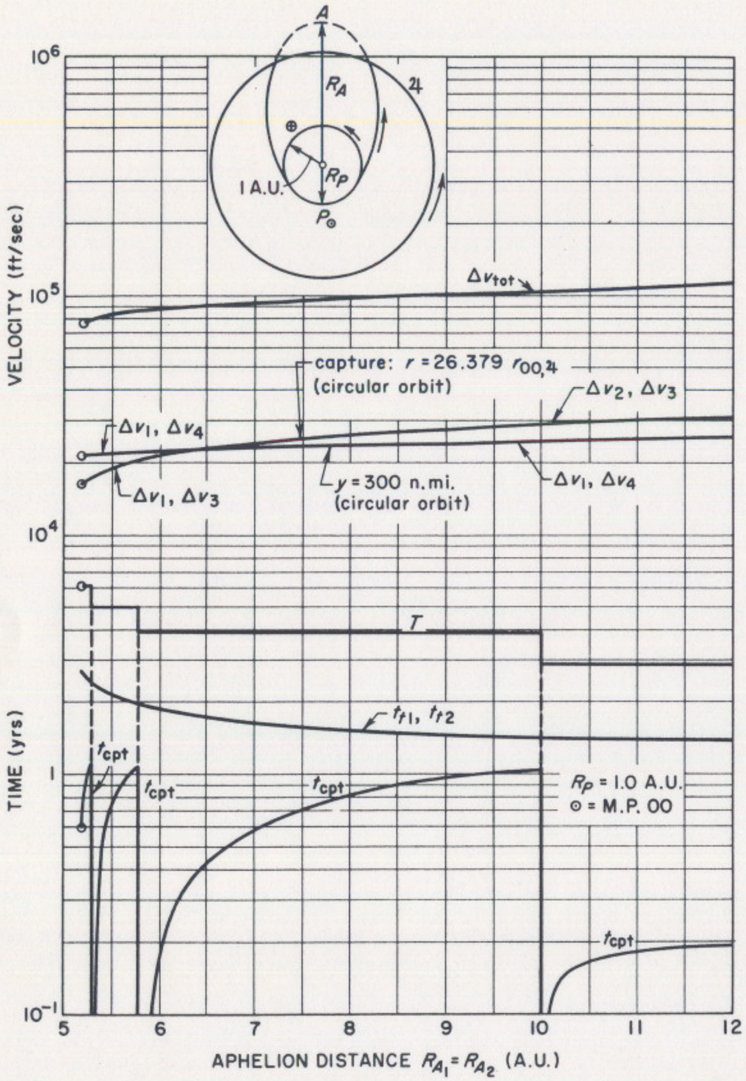

FIG. 9-67a Mission Profile 11 for Earth-Jupiter Operations (Planar; circular planet orbits).

Considering the multitude of transfer orbits, it is apparent that the basic single target planet mission can be extended in many ways to cover two, if not all three possible target planets in the inner solar system. Among these, three mission profiles should be singled out, namely, the *mono-elliptic*, the *tri-elliptic* and the *bi-elliptic mission profile*.

The *mono-elliptic* mission profile consists of a single elliptic orbit

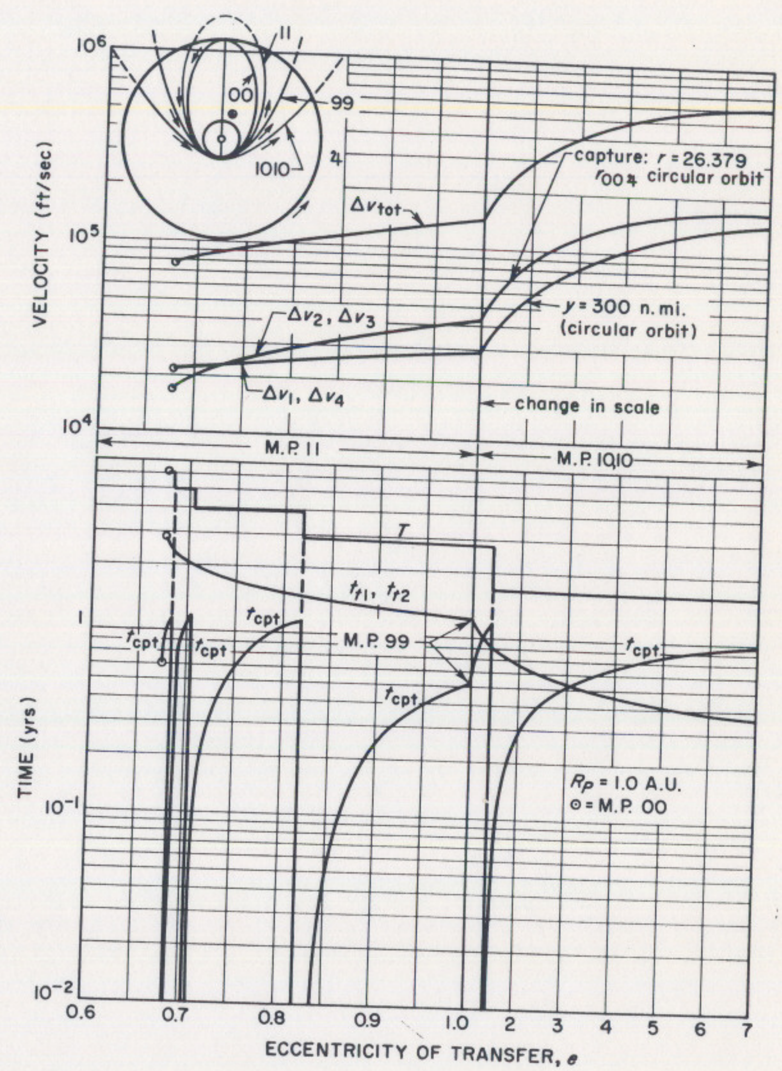

FIG. 9-67b Mission Profiles 00, 11, 99 and 1010 for Earth-Jupiter Operations (Planar; circular planet orbits).

which connects the Earth with the target planets visited. Because of the inclination of the planet orbits, orthogonal impulses will have to be applied at strategic points, but these do not, of course, change the eccentricity and major axis of the orbit. A mono-elliptic mission profile is moreover based on the stipulation that no planetary capture is involved. Observations and measurements can be made only to the extent possible during a hyperbolic encounter during which the vehicle remains within a few radii distance only for a few hours. Perturbations by the planetary encounter are assumed to be corrected, preferably while nearest to the planet so that a heliocentric ellipse closely resembling the original ellipse is resumed by the time the vehicle is sufficiently removed from the planet. Mono-elliptic mission profiles must have periods equal to or a multiple of the Earth's period to ascertain encounter with the Earth at the termination of the round trip. If the period of the orbit, T', is measured in sidereal years and the semi-major axis in astronomical units it is

$$K_{\odot} = 4\pi^2 = 39.478418 \text{ A.U.}^3/yr^2 \tag{9-75}$$

and

$$T' = a^{3/2} \tag{9-76}$$

For a mission period $T = T'$ of one or two years, the semi-major axis of the elliptic orbit must be either 1 A.U. or 2.8284 A.U. If the orbit is to contact tangentially the orbit of Mars ($a_{♂} = 1.52$), the perihelion must therefore be a distance of

$$R_P = 2a - R_A = 2a - a_{♂}$$

i.e., at $R_P = 0.48$ A.U. or at 4.1368 A.U. The latter case obviously is not feasible. The 1-year round trip which contacts the Mars orbit leads, therefore, deep inside the Venus orbit. The 2-year round trip requires an aphelion distance of

$$R_A = 2a - R_P = 5.6568 - 0.72$$

i.e., 4.93 A.U. if the perihelion touches the Venus orbit or 5.27 A.U. if the perihelion lies in the orbit of Mercury. Since the 2-year orbit would lead necessarily far into trans-Martian space, the 1-year orbit remains the only practical mono-elliptic mission profile in the inner solar system. When touching the orbit of Mercury ($a_{☿} = 0.38$) the aphelion is at the distance $R_A = 1.62$ which is within the aphelion-perihelion tolerance of the Martian orbit. It is this seen that the 1-year mono-elliptic mission profile permits near-tangential encounter with Mars and Mercury but only intersecting encounter with Venus. Three 1-year mission profiles are depicted in Fig. 9-68. These orbits could intersect or contact the Martian orbit while being tangential to the orbit of Mercury, because of the ellipticity of both planet orbits.

The *tri-elliptic* mission profile provides greater elasticity in that it can accommodate *any* mission period if at least one encounter with the Earth is intersecting. If all encounters are to be tangential or nearly so the mission periods must be 0.5, 1.5, 2.5, etc., years. Of these, the 1.5 year mission period is of greatest practical significance for flights in the inner solar system. Assuming circular planet orbits at their respective mean

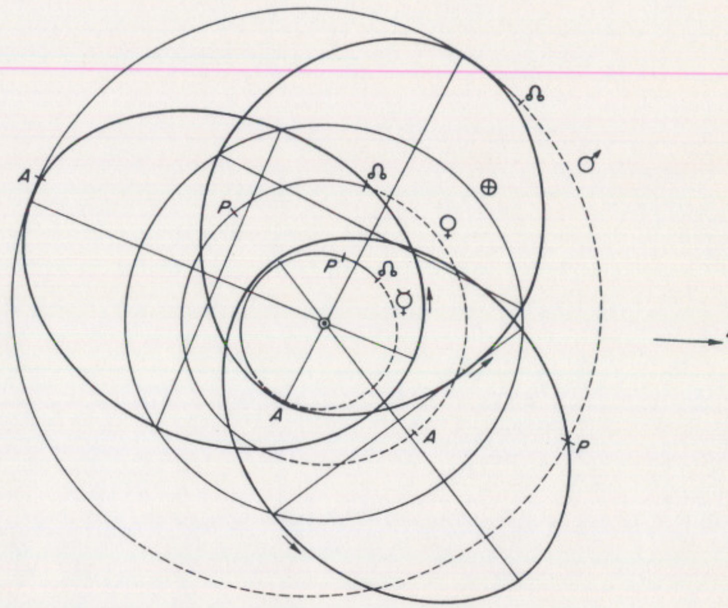

FIG. 9-68 1-Year Mono-Elliptic Round Trip Orbits Involving Venus and Mars or Mercury, Venus and Mars.

distances (but not necessarily co-planar), this mission consists of three exact half-ellipses. Their semi-major axes have the value (in astronomical units),

$$a_1 = \frac{1+R_A}{2}$$

$$a_2 = \frac{R_A+R_P}{2} \tag{9-77}$$

$$a_3 = \frac{1+R_P}{2} = a_2 + \frac{1-R_A}{2}$$

The mission period T (in sidereal years) is given by

$$a_1^{3/2} + a_2^{3/2} + a_3^{3/2} = 2T$$

One half ellipse can assumed to be known, since one must specify which planet shall be contacted first and whether the encounter is to be tangential or intersecting. In the latter case the respective associated apsis (either aphelion or perihelion) will be known also. Suppose semi-major axis a_1 is known; the mission period T is specified; then it follows for a_2

and a_3, substituting for a_3 the second expression in the last of Eq. (9-77),

$$a_2^{3/2}+\left(a_2+\frac{1-R_A}{2}\right)^{3/2} = 2T-a_1^{3/2}. \tag{9-78}$$

In order to facilitate the solution, Fig. 9-38b has been prepared, which shows the correlation between a and $a^{3/2}$, the period of the corresponding orbit. Suppose, for example, the orbit of Mars ($a_{♂} \simeq 1.52$) is to be contacted and the mission period is to be $T=1.5$ years. It follows,

$$a_1 = \frac{1+1.52}{2} = 1.26; \qquad a_1^{3/2} = \sqrt{2} = 1.414$$

$$2T_1-a_1^{3/2} = 3-1.414 = 1.584$$

$$R_A = a_{♂} = 1.52$$

$$\frac{1-R_A}{2} = -0.26$$

$$a_2^{3/2}+(a_2-0.26)^{3/2} = 1.584$$

With the aid of Fig. 9-38b,

$$\begin{aligned} a_2 = 1.00 \text{ yields}&: 1.0+0.625 = 1.625 \\ 0.96 &: 0.93+0.58 = 1.51 \\ 0.98 &: 0.96+0.6 = 1.56 \\ 0.99 &: 0.98+0.61 = 1.59 \end{aligned}$$

finally

$$a_2 = 0.987$$
$$R_P = 2a_2-R_A = 0.454$$
$$a_3 = 0.727$$

The transfer times along the half ellipses are

$$\tfrac{1}{2}a_1^{3/2} = t_{t,1} = \frac{1.414}{2} = 0.707$$

$$\tfrac{1}{2}a_2^{3/2} = t_{t,2} = 0.487$$

$$\tfrac{1}{2}a_3^{3/2} = t_{t,3} = 0.306$$

All three transfer times add up to $T=1.5$ years. The resulting trielliptic mission profile is depicted in Fig. 9-69. As in the case of the 1-year mono-elliptic mission profile, the Venus orbit is intersected rather than contacted. By raising the perihelion to the mean Venus distance ($a_{♂} \simeq 0.72$) the aphelion is necessarily reduced and the Martian orbit can no longer be contacted. It is in this case,

$$a_1 = \frac{1+R_P}{2} = \frac{1+0.72}{2} = 0.86; \qquad a_1^{3/2} = 0.79$$

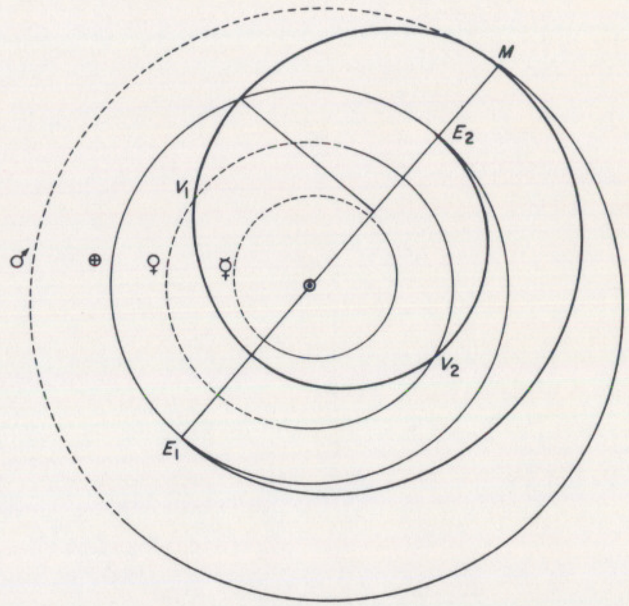

FIG. 9-69 1.5-Year Tri-Elliptic Round Trip Orbit Involving Venus and Mars.

$$2T_1 = 3-0.79 = 2.21$$

$$\frac{1-R_P}{2} = 0.14$$

$$a_2^{3/2}+(a_2+0.14)^{3/2} = 2.21$$

This equation is satisfied by $a_2 \approx 1.0$ whence $R_A=2a-R_P=1.28$, which is well below even the Martian perihelion distance. On the other hand, by stipulating tangential contact with the mean Mercury orbit, the resulting aphelion distance for $T=1.5$ years is so close to the mean distance of Mars that tangential contact mean Mars distance is feasible, provided a capture period of about seven weeks can be interspersed at Mars. It is, indeed,

$$\begin{aligned} &\text{Earth-Mars}: a_1 = 1.26; \quad \tfrac{1}{2}a_1^{3/2} = t_{t,1} = 0.7070 \text{ years} \\ &\text{Mars-Mercury}: a_2 = 0.85; \quad \tfrac{1}{2}a_2^{3/2} = t_{t,2} = 0.3918 \text{ years} \\ &\text{Mercury-Earth}: a_3 = 0.69; \quad \tfrac{1}{2}a_3^{3/2} = t_{t,3} = 0.2866 \text{ years} \\ &\hline \\ &T = 1.3854 \text{ years} \end{aligned}$$

yielding a required capture period near Mars (or near Mercury, which probably would be technically considerably more difficult) of $\Delta T=t_{\text{cpt}}=0.1146$ years. The resulting mission profile is depicted in Fig. 9-70.

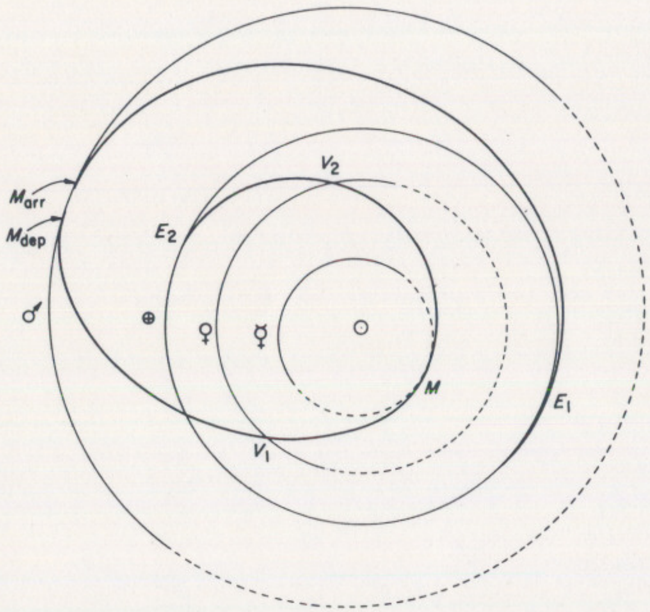

FIG. 9-70 1.5-Year Tri-Elliptic Round Trip Orbit Involving Mercury, Venus and Mars.

The *bi-elliptic* mission profile involves necessarily an intersecting encounter in the Earth orbit, either at departure or at arrival. Figure 9-71 shows such a mission profile. Only one flight path change is made,

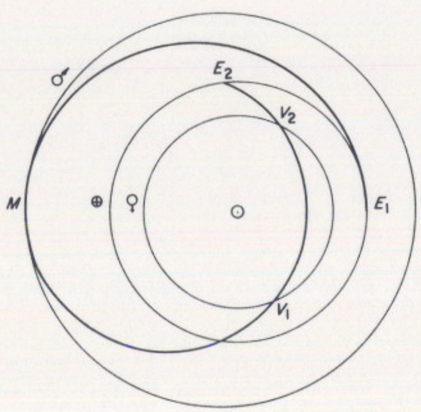

FIG. 9-71 Bi-Elliptic Round Trip Orbit Involving Venus and Mars.

namely near Mars. The periods of the two ellipses are (a in astronomical units, T in sidereal years),

$$T_1 = a_1^{3/2}; \qquad T_2 = a_2^{3/2}$$

The mission period T is

$$T = \tfrac{1}{2}T_1 + \tfrac{1}{2}T_2 + t_\eta \tag{9-79}$$

where $\frac{1}{2}T_1$ is the half period of the Earth-Mars transfer ellipse, $\frac{1}{2}T_2$ the flight time from Mars to the perihelion and t_η the flight time from the perihelion to the intersection with the Earth orbit which takes place at a point $R = R_\oplus = 1$ and

$$\eta = \cos^{-1}\frac{\dfrac{p}{R}-1}{e} = \cos^{-1}\frac{\dfrac{R_P}{R}(1+e)-1}{e} \tag{9-80a}$$

or, in this particular case

$$\eta = \cos^{-1}\frac{R_P(1+e)-1}{e}$$

$$e = \frac{n-1}{n+1} \tag{9-80b}$$

$$n = R_A/R_P$$

The flight time from $\eta = 0$ to η follows from the Kepler equation

$$\begin{aligned} t_\eta &= a_2^{3/2}(E - e\sin E) \\ &= a_2^{3/2}\left[\cos^{-1}\frac{a_2-R}{a_2 e} - e\sin\left(\cos^{-1}\frac{a_2-R}{a_2 e}\right)\right] \\ &= a_2^{3/2}\left[\cos^{-1}\frac{a_2-R}{a_2 e} - e\sqrt{1-\left(\frac{a_2-R}{a_2 e}\right)^2}\right] \end{aligned} \tag{9-81}$$

where, as before, a_2 is taken in astronomical units, t_η in sidereal years. For a given mission period T and a given semi-major axis a_1, the equation for the determination of a_2 is thus

$$\tfrac{1}{2}a_2^{3/2} + t_\eta = T - \tfrac{1}{2}a_1^{3/2} \tag{9-82a}$$

or substituting Eq. (9-81),

$$a_2^{3/2}\left[\tfrac{1}{2} + \cos^{-1}\frac{a_2-R}{a_2 e} - e\sqrt{1-\left(\frac{a_2-R}{a_2 e}\right)^2}\right] = T - \tfrac{1}{2}a_1^{3/2} \tag{9-82b}$$

This equation must also be solved by trial and error, but it is much more time consuming to solve than Eq. (9-78). In order to obtain rapidly a first approximation it is therefore preferable to compute η from Eq. (9-80) and, with this value, to enter Eq. (3-56a) which gives the fraction of the period, τ, for elliptic arcs of true anomaly η. Therewith one obtains immediately

$$t_\eta = \tau a_2^{3/2} \tag{9-83}$$

for a given eccentricity and true anomaly. The numerical value of a_2 is thus obtained in the following manner: Assume an estimated value of R_P, compute $a_2=(R_A+R_P)/2$, then find τ, thereafter compute $a_2^{3/2}$ (Fig. 9-38b) and t_η and compare the sum of these two values with the value of $T-\frac{1}{2}a_1^{3/2}$. This process is repeated until a reasonably accurate value of a_2 is found. If still greater accuracy is desired, it is necessary to switch to Eq. (9-82b), starting with the last and most accurate value of a_2 found in the preceding trial and error procedure.

The results of the preceding discussion of three mission profiles can be summarized as follows:

(a) Mono-elliptic mission profiles are restricted to an orbital period of one year, offering the advantage of a relatively short mission period, but only limited flexibility in arranging the orbit to suit specific mission purposes. Earth departure and arrival energies are high.
(b) Tri-elliptic mission profiles are practical only for a mission period of 1.5 years if all arcs flown are half-ellipses. If this requirement does not exist, considerably greater flexibility in varying the mission period is possible by returning to the Earth along an intersecting flight path.
(c) Bi-elliptic mission profiles resemble in their characteristics the tri-elliptic profiles which do not consist entirely of half-elliptic arcs.
(d) Neither the mono-elliptic nor the tri-elliptic (3 half ellipses) profile permit a tangential contact with the orbit of Venus, but only one with Mars and Mercury. The Venus orbit is intersected in all of these cases. A tangential encounter with Venus and a tangential or intersecting encounter with Mars leads to a mission period between 1.5 and 2 years and for this reason requires intersecting return encounter with Earth.

The energy requirement for these round trip missions can easily be computed on the simplified basis of co-planar circular planet orbits. However, if the actual conditions, including perturbations in the course of a close hyperbolic encounter are taken into account, the picture may vary considerably, requiring assessment of the energy requirement of individual missions. One rule, however, remains generally valid (cf. Par. 9-5): *If at all possible, maneuvers for changing the heliocentric orbital elements should be carried out during the hyperbolic encounter with a planet, rather than in heliocentric space.* The greater the planet's mass, the greater the energy saving.

Multiplanet round-trips of this kind face two fundamental difficulties not contained in missions to and from one target planet. Firstly, the intervals between the correct position of all planets involved are much longer, hence the opportunities for such a mission are considerably more rare, than for missions to one planet. The period between two equal positions of three planets $T_{3,\,syn}$ is given, within the accuracy limits of first order differences by

$$T_{3,\,syn} = \frac{T_{2,\,syn}}{\left|1-\dfrac{T_{2,\,syn}}{T'_{2,\,syn}}\right|} = \frac{T'_{2,\,syn}}{\left|1-\dfrac{T_{2,\,syn}}{T'_{2,\,syn}}\right|} \tag{9-84}$$

assuming again circular, co-planar planet orbits. $T_{2,\,syn}$ and $T'_{2,\,syn}$ designate the synodic periods of Earth and, e.g., Venus and Mars, respectively. Thus, with $T_{syn} = 1.6$ years for Venus and $T'_{2,\,syn}=2.13$ yr for Mars, the value of $T_{3,\,syn}$ in the first case is $4\cdot 1.6=6.4$ yr, in the second case $3\cdot 2.13=6.39$ yr. This means that if at any epoch $T=0$ years all three planets are in the correct constellation, then this constellation will be repeated after 6.4 yr with Venus being in the correct position and Mars 0.01 yr = 3.6 days off, or after 6.39 yr with Mars in the correct position and Venus 0.01 yr off. This second order difference of 0.01 yr is small enough to be unimportant for a number of periods following $T=0$. Eventually, however, the "off-position" of the one or the other planet will become so large that a three-planet round-trip becomes impractical until this second order period nears completion. In the present example with the second order difference being about 0.01 yr, the related period is 100 yr. There will, therefore, be long periods where the one target planet is not in a favorable position when Earth and the other target planet form a transfer constellation. A more or less large fraction of this period can be covered by including intersecting encounters and fast orbits, elliptic and hyperbolic transfers, in the flight plans. However, the practicality of this approach depends exclusively on the propulsion energy available. As in the case of one-planet missions, the width of the launch window and the frequency of launch window recurrence increases with the capability of flying short transfer orbits.

Secondly, the nonplanarity of the planet orbit planes requires additional plane correction en route. Since the plane change is the more expensive the closer the departure point from a given planet is at 90 degrees with respect to the nodal line of this planet's plane with respect to the target planet, some of the three-body constellations will be more expensive than others. As a result, the transfer orbits from one planet to the other will vary and, consequently, the interval between two possible 3-planet round-trip missions of approximately equal mission energy, if practical at all, will be sometimes less than 6.4 yr, sometimes more, depending on the position of the several planet contact points with respect to the nodal lines of the orbits of Venus and Mars.

In general, therefore, a 3-planet round trip involving Earth, Venus and Mars is very much more involved than a 2-planet round trip from the flight mechanical point of view. To this other complications are added by the considerably greater differences in environmental conditions to which the vehicle is exposed, ranging from Martian or trans-Martian to Venusian or intra-Venusian space environmental conditions. The variations in corpuscular radiation conditions of solar origin are so far only very approximately known and so are the variations in micrometeoritic density. Furthermore, the corpuscular radiation belt conditions near Mars can be expected to be considerably different from those near Venus. All these factors will complicate the vehicle design and increase its weight. It is therefore unlikely that initial manned expeditions will have a mission assignment involving two target planets. For instrumented space probes such mission appears not practical because of the enormous accuracy requirements involved. One argument in favor of such a mission is the reduced mission period (1 or 1.5 yr) compared to much longer minimum-energy periods to Venus and Mars. This argument overlooks the fact that with the energy needed for a 3-planet round trip, a 2-planet trip can likewise be shortened considerably.

In conclusion, it should be pointed out that 1-year or 1.5-year round trips, involving *one* target planet, can be flown with an instrumented probe. In this case the findings of the probe do not have to be transmitted to Earth over distances of the order of an astronomical unit (distance Sun-Earth), but can be stored and transmitted during the subsequent close passage of the Earth. Again, because of the great error sensitivity of a hyperbolic encounter, such a mission becomes increasingly delicate from the viewpoint of guidance and navigation the closer the desired hyperbolic encounter. However, even if the probe passes the target planet at greater distance (say, 50 to 100 planet radii), the problem of long-term storage of photographic pictures and measuring data remains. A 1-way mission to the target planet, involving a close encounter without stringent specifications of the post-encounter orbit has many simplifying features in its favor, except for the greater power requirement and directional accuracy of the transmitter antenna. These appear less serious problems in the light of early capabilities in the 1962–66 period. However, on the long run the 1-year to 1.5-year round trip involving one target planet offers interesting and attractive possibilities.

9-10 LAUNCHING OF INTERPLANETARY VEHICLES

Following the treatment of the heliocentric portion of interplanetary transfer orbits this paragraph considers the geocentric escape process. This process consists of the following steps:

Step 1: Ascent from surface into a low-altitude circular orbit of proper inclination with respect to the equator.

Step 2: Geocentric departure by co-planar orbit change to the proper escape hyperbola. Step 2 may either follow immediately the attainment of circular velocity (***uninterrupted departure***) or following a certain coasting period in a parking orbit (***interrupted departure***).

Step 3: Hyperbolic escape which is completed when the desired ***heliocentric*** departure condition is reached at a sufficiently large distance from the Earth where terrestrial attractions can be neglected compared to the heliocentric force field.

In the analysis of a launch process, these steps are reversed. First the following parameters must be determined (Fig. 9-72a):

(a) Year and day of ***heliocentric departure*** (heliocentric departure date), assuming that the variation in hyperbolic velocity excess during 24 hours can be neglected.

(b) Heliocentric departure vector $\vec{V}_1$ of the space vehicle.

Both parameters follow from the mission specification. For example, from transfer calculations as described in Par. 9-18 the following information is obtained for a specific transfer case:

Heliocentric departure date
Magnitude V_1 of departure vector
Initial heliocentric planar path angle θ_1
Inclination i_t of transfer orbit relative to ecliptic plane

The second through fourth items determine the magnitude and direction of the heliocentric departure vector $\vec{V}_1$. The departure date fixes the origin of $\vec{V}_1$ in the heliocentric ecliptic coordinate system (longitude l, latitude b, distance R, taken as unit),

$$\vec{V}_1 = V_1 k_V \tag{9-85}$$

where k_V is the unit vector of $\vec{V}_1$. The magnitude of the vector is

$$V_1^2 = \dot{R}_1^2 + R_1^2 \dot{l}_1^2 + V_{w,1}^2 \tag{9-86}$$

where R_1, l_1, $V_{w,1}$ are the heliocentric distance, the heliocentric longitude

PRINCIPLES OF
GUIDED MISSILE DESIGN

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SPACE FLIGHT

II. DYNAMICS

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