THE MISSION FOR A MANNED EXPEDITION TO MARS

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ABSTRACT

It is the objective of this thesis to define the mission for a manned expedition to Mars with the major portion of the work being devoted to the selection of the best interplanetary transfer. The approach used is to develop simple approximate models in order to gain an understanding of the complicated orbital mechanics, make conclusions based on a study of the simple models, and verify the conclusions by accurate analysis with the electronic computer. For round-trip missions of about 400 days duration velocity savings over single-elliptical transfer of about 3000 feet per second are possible by making one leg of the trip a bi-elliptical transfer. During every third opposition period it is possible to increase the saving to about 7000 feet per second by making the bi-elliptical transfer during a close approach to Venus. The opportunity to perform this mission will exist for several months during late 1970. Conditions will not be as attractive again until after the turn of the century. A method for analyzing all practical free-fall interplanetary mission combinations is presented. One specific mission for a manned expedition to Mars is recommended together with alternatives.

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CHAPTER 1

INTRODUCTION

The first attempts to place man into space outside the protecting cover of the Earth's atmosphere have met with such success that many manned space missions are now being planned. The current United States Apollo program has the objective of placing a man on the moon prior to 1970. The next logical mission in space after lunar exploration is the exploration of the planet Mars. The study of the mission for man on Mars is the subject of this thesis. The term "mission" includes the objectives of the expedition, the operations to be conducted by the crew during the flight and on Mars, the things to be brought back to Earth, the dates of departure and arrival and the flight paths to be followed.

1.1 The Reason for Going

The primary reasons for man to go to Mars initially are scientific research and exploration. The possibility that some form of life can or did at some time exist on Mars makes it a particularly noteworthy target. Of all the planets in the solar system, Mars is the only one suspected capable of supporting life save Earth and possibly Venus. The exploration of Mars may be the only way open for man to establish the existence of extra-terrestrial life.

Mars and Venus are the two planets most easily reached from Earth. Of the two, Mars offers the more attractive setting for man's next mission in space after lunar exploration. The surface of Venus is covered by a dense yellow atmosphere which is opaque to visible light. Astronomers have never had a clear view of the planet's surface and consequently important parameters such as the rotational period are in doubt. Recent measurements taken aboard the Mariner spacecraft indicate that the surface temperature is several hundred degrees Fahrenheit. In contrast, the atmosphere of Mars is transparent to visible light and the surface has been studied by astronomers. An excellent summary of the history of this study has been made by de Vaucouleurs.[2]

Prior to the launch of a manned interplanetary expedition man will have placed optical telescopes in balloons, rockets, satellites or on the surface of the moon that will allow greater resolution than possible with present Earth-mounted telescopes that are limited by the distorting properties of the Earth's atmosphere. These viewing positions will allow the telescope to give more and better information about the Martian surface. The atmospheric pressure and density at the surface of Mars is similar to that of Earth at an altitude of 56,000 feet. Surface conditions in general appear to make Mars quite adaptable to a manned mission.

The sciences having the most direct interest in the results of a manned expedition to Mars are biology, geophysics, astronomy and meteorology. The biologist is interested in the possibility of extra-terrestrial life on Mars. The geophysicist is interested in all the physical aspects of the planet. Having another planet besides Earth to study firsthand can help uncover the secrets of the origin of the solar system. The astronomer has studied Mars using telescope, spectrometer and theory for thousands of years. Firsthand information to verify or disprove the theories would be invaluable to the science of astronomy. Since the moon has no atmosphere Mars is the easiest place to study extra-terrestrial meteorology. Mars offers a model for the study of weather and climate that is expected to be less complex than for Earth because of the smaller size of the planet and the lesser amount of water in the atmosphere. An understanding of the meteorology of Mars would help in the understanding of our own more complex meteorological system. The volume of scientific information available from a manned expedition to Mars is immense.

Taylor and Blockley[3] have considered in detail the reasons for man to go on space missions instead of sending equipment alone. Even with the added difficulty of protecting the man from the space environment, man is the best piece of equipment that can be found to do the job. Despite the arguments pro and con whether man should go, history has proven that if he has the capability, he will go.

There are secondary reasons for going to Mars. The foremost of these is the fact that the results of scientific research have always lead to useful applications in society. In many cases the results could never have been predicted at the onset of the research. It is the research itself that uncovers the useful application. Just the possibility of what might be found on Mars offers a reason for going. It is speculative but not impossible that some millions of years ago Mars had a more abundant atmosphere and higher forms of life existed on the surface. If so, then evidence of their culture might be discovered and offer lessons for our own. If Mars is capable of supporting life then it is possible that Mars in time could be cultivated and even colonized by man. The important question initially is to what extent life can or did exist there, and the best way to establish the answer is for man to go.

In order that a manned expedition to Mars accomplish its objectives the crew must get to Mars, perform the planned scientific research and exploration, and return safely to Earth. The work to be done includes running the ship. This requires that the crew perform the functions of navigation, power plant control, maintenance, communication and survival. The scientific research and exploration work involves making measurements, gathering samples, taking pictures and mapping. The details of this work will be determined by the "state of the art" at the time of the expedition in those sciences already mentioned as having the most interest in Mars' exploration. (It is expected that once a telescope is placed above most of the Earth's atmosphere there will be a major increase in the knowledge concerning Mars.) What will actually be accomplished on the surface of Mars will also depend upon what the crew finds there. Like any scientific research, the course of their work cannot be completely predicted before it is started. Probably the major question to be answered is whether Mars was, is, or could be, capable of supporting life at some time in the past, present or future.

1.2 The Work of Others

A number of proposals have been made suggesting specific mission profiles for a manned expedition to Mars. Notable is the early work of von Braun[7][8] who has shown with substantiating calculations that the voyage is feasible with present day propellants and technology. In Reference (7) von Braun describes in detail an expedition of ten vehicles manned by a crew of seventy, a very large undertaking. In Reference (8) Ley and von Braun describe in detail a more modest expedition of two vehicles manned by a crew of twelve. Both calculations are based on coplanar circular orbits for Earth and Mars and both voyages go via a Hohman trajectory taking two hundred and sixty days each way with a wait time at Mars of four hundred and fifty days. Himmel et al[9] have considered the controlling effect of the radiation shielding requirement in the mission planning and have recommended a four hundred day expedition using a nuclear rocket. The weight of the passive shielding alone takes up over seventy tons of the allowed payload weight.

Recently a large volume of work has been done on interplanetary missions under the EMPIRE (Early Manned Planetary-Interplanetary Roundtrip Expedition) program, initiated by the Future Projects Office of the National Aeronautics and Space Administration at the Marshall Space Flight Center, Huntsville, Alabama. NASA has contracted several companies to conduct studies of specific interplanetary missions. The Aeronutronic Division of Ford Motor Company[1] has studied dual planet flyby missions of the Crocco[5] and symmetric types[1]. These missions launch from an Earth orbit and free-fall past Venus and Mars returning to Earth in a direct atmospheric entry. The Missiles and Space Division of Lockheed Aircraft Corporation[4] has made an extensive orbital parameter study of orbits to both Venus and Mars as well as single planet and dual planet flyby missions. The Astronautics Division of General Dynamics Corporation[6] has studied initial landing missions and assumes the use of thrust for velocity changes at each of the terminals of the interplanetary phase. Each of the Empire reports represents work by a complete study group consisting of several individuals.

Because of the large volume of work on different aspects of a mission to Mars it would be impossible to make reference to all of the literature on the subject. It should be noted, however, that the author has found no mention in the literature of the specific missions suggested in this work, namely trips to Mars via bi-elliptical transfer or via a Venus encounter that includes a significant velocity change near Venus.

1.3 Approach to the Problem

A manned expedition to Mars will be a large undertaking. The United States is presently planning to send a man to the surface of the Moon around 1967. The total cost of the lunar expedition has been estimated at approximately $40 billion over roughly a five-year period. This averages a cost of about $40 per year for every inhabitant of the country. The technological advances resulting from the lunar expedition can be applied directly to the Mars expedition, but it will still probably cost at least as much to go to Mars as it will to the Moon. If cost were the only factor a mission could be chosen by selecting that trip which minimized the cost. In the case of the lunar expedition the United States is choosing the mission profile with the motivation of getting there as soon as possible. It is not realistic to assume that the mission will be based on optimization of cost or any other factor alone. It will depend on the unpredictable actions and decisions of many different people up to and including the heads of state of the major powers of the world. Looking at the United States Apollo program, similar decisions were based not on a scientific optimization but rather on the informed opinions of responsible persons who still publically disagreed after the decision was made[10]. In the case of the Mars trip it does seem universally agreed that the interplanetary phase of the trip, through one hundred million miles of space, is the predominant consideration in establishing the mission. Because of the long time of flight of the interplanetary transfer in comparison with the other phases, the major aspects of the mission will be determined by the interplanetary requirement. The problem is still complex due to the many engineering trade-offs involved in the determination of the interplanetary transfer paths. In order to obtain an understanding of the total problem the author has attempted to make gross simplifications of those aspects which are complex so that the major contributing factors are shown in clear perspective. An example of the application of this approach is the simplified approximate model described in Chapter 4. There the complexities of the orbital mechanics are reduced to a crude linear model. Based on an understanding of the crude model conclusions may be drawn and subsequently checked by more rigorous analysis with the aid of the electronic computer. As verification of the value of this approach the savings associated with the bi-elliptical transfer including Venus encounter missions (described in Chapters 7, 11, 12) were predicted solely by use of the simple model and prior to the use of the electronic computer. The basic approach is to use simple models to gain an understanding of the problem, then make conclusions based on the simple model, and finally check the conclusions by more rigorous analysis using the electronic computer.

CHAPTER 2

THE ORBITAL GEOMETRY OF EARTH, VENUS, AND MARS

2.1 Units

The unit for interplanetary distance will be the astronomical unit (a.u.) and for planetary distance the planet radius. Dates will be specified in Julian date. Time intervals specified in days refer to the terrestrial mean solar day. When time is expressed symbolically in an equation the unit is the terrestrial year (365.25 mean solar days). The unit for velocity is the Earth's Mean Orbital Speed (EMOS) which is equivalent to 2π a.u. per year or about 100,000 feet per second or 30 kilometers per second or 18.6 miles per second.

2.2 Major Aspects

The major aspects of the orbital geometry for Earth, Venus, and Mars are shown in Figures 1a and 1b. The opposition positions of Earth and Mars are given through 1978 and the conjunction positions of Earth and Venus are given for the period 1970 through 1977 in a sun-centered inertial frame. The orbit of Venus is quite circular (ϵ = .007), that of Earth is approximately circular (ϵ = .017), while the eccentricity of Mars' orbit (ϵ = .093) is sufficient to cause its radial distance from the Sun to vary as much as a quarter of an astronomical unit. Aphelion, α, and perihelion, π, for Earth, ⊕, Venus, ♀, and Mars, ♂, are shown together with the first point of Aires, ♈. The ascending node, ☊, of either Venus or Mars is shown as the point on the orbit where the planet crosses the ecliptic plane from south to north. The descending node, ☋, is displaced by 180 degrees from the ascending node. The orbital inclinations of $1^{o}51'$ for Mars and $3^{o}24'$ for Venus while small are important considerations in an orbital transfer. The polar obliquity of Mars (24^{o} - 25^{o}) is about the same as that of Earth giving rise to a similar cycle of seasons. To a first approximation the northern hemisphere of Mars celebrates summer in the part of the sky where Earth celebrates winter. An opposition of Mars which occurs near perihelion is a favorable opposition since at that time Mars is not only closest to the Sun but also closest to the Earth. The next favorable opposition can be seen to occur in 1971. The favorable oppositions due in 1986 and 1988 are not shown. If they were shown in Figure 1 they would straddle the 1971 opposition. The most favorable oppositions occur when Mars is near its maximum distance below the plane of the ecliptic. Similarly the least favorable oppositions occur when Mars is near its position of maximum distance above the plane of the ecliptic.

2.2 Synodic Periods

The relative positions of Earth, Venus, and Mars will be important in the work to follow. The easiest way to visualize these relative positions is to show the relative times of Mars' oppositions and Venus' conjunctions. The orbital period of Mars is 687 days (the term "day" will refer to the "terrestrial mean solar day"). The orbital period of Earth is 365 days. This causes the synodic period, the time between successive oppositions, to be about 780 days. The orbital period of Venus is 224 days causing the synodic period between successive conjunctions of Venus to be about 580 days. Considering the paths of the planets around the sun as a racetrack, Earth overtakes Mars at each opposition, and Venus overtakes Earth at each conjunction. These events are plotted on a time scale in Figure 2 for the years 1964-1980. Since Venus is the fastest of the three planets it overtakes both Earth and Mars. Since Mars is the slowest of the three, Mars is overtaken by Venus more often than Earth is overtaken by Venus. The period between successive alignments of Venus and Mars is about 340 days. These events are also plotted in Figure 2 and will be referred to later.

CHAPTER 3

THE ORBITAL PARAMETERS OF ALL POSSIBLE TRANSFER ORBITS

3.1 The Planet-Centered Coordinate Frame

Define a planet-centered reference frame with x-axis pointing radially outward from the Sun through the planet in the plane of the planet's motion. The y-axis is in the circumferential direction also in the planet's orbital plane. The z-axis is perpendicular to the planet's orbital plane and completes a right-hand orthogonal set. For a planet in circular orbit the Sun is fixed on the negative x-axis and the velocity vector is along the positive y-axis. When velocity components are specified at a planet they will almost always be in this reference frame.

3.2 Loci of Constant Orbital Parameters

If a spaceship is to follow a free-fall trajectory between two planets then specification of the time of launch and the time of arrival is sufficient to uniquely determine the trajectory. The time of launch fixes the position of the launch planet at launch and hence determines the four coordinates in space and time of the launch event. Similarly, the arrival date fixes the four coordinates of the arrival event. The difference between the times of arrival and launch determines the time of flight and this is sufficient to select a unique orbit. In a similar manner, the specification of any two independent parameters of the transfer orbit will determine a trajectory between two given planets. By choosing two of the initial velocity components of a spaceship as independent variables it is possible to express all the parameters of all possible orbits between two planets as a function of these two components of the initial launch velocity. Loci of constant orbital parameters can then be shown graphically.

The method described here is similar to a method suggested by Vertregt[11] and carried out by Dugan[12]. Vertregt's suggestion was to plot all the possible orbits on an eccentricity versus semi-latus rectum plane. Dugan constructed these plots assuming circular coplanar orbits. He then used the plots as "working charts" to develop curves for round-trip mission planning. The method described here is considered an improvement because it displays the information in a manner more useful for mission planning and can also take into account the orbital inclination and eccentricity in a simple manner. Furthermore, the present method can be directly applied to the problem of pairing two interplanetary orbits which have a common terminus. This will be discussed in Chapter 11. In order to fix ideas the method will be applied to free-fall transfers between Earth and Mars.

3.3 Free-fall Orbits from Earth to Mars

This example assumes a circular orbit for Earth. The ellipticity and orbital inclination of Mars are handled by looking at trajectories that terminate at a few representative points on the true Martian orbit. All the orbital parameters except the out-of-plane velocity increment will be determined assuming coplanar orbits. The out-of-plane velocity increment will be added vectorially to the component in the Earth's orbital plane to obtain the total velocity increment. The V_x, V_y plane will be the x-y plane described in Section 3.1 with axes calibrated in velocity units. Plot the projection of the velocity of an interplanetary space vehicle relative to the Earth after escape as a point in the V_x, V_y plane. Any possible free-fall orbit is represented by one point on the plane. If the radial distance of Mars at arrival is specified, all orbits which do not reach Mars lie in one region of the plane. This region can be separated from the region containing all orbits which do reach Mars by the locus of those orbits which are just tangent to the orbit of Mars. See Figure 3a. Of the orbits which do reach Mars, the many orbital characteristics can be displayed by plotting the locus of orbits having the same value of the orbital parameter of interest.

A few of the more important parameters are now discussed.

3.4 Constant Launch Velocity, V

The launch velocity relative to Earth is indicative of the fuel required for the trip to Mars. These curves plot as circles about the origin. V is the projection of the total velocity increment on the Earth's orbital plane and does not yet include the out-of-plane component.

$$V_x^2 + V_y^2 = V^2 \tag{3.1}$$

3.5 Constant Orbital Period, T, or Semi-major Axis, a

These orbital parameters depend on the velocity relative to the Sun and plot as circles about the point $V_x = 0$, $V_y = -1$.

$$V_x^2 + (V_y + 1)^2 = \frac{2a - 1}{a} \tag{3.2}$$

$$T = (a)^{3/2} \tag{3.3}$$

3.6 Constant Angular Momentum, h, or Semi-latus Rectum, p

These orbital parameters depend only on the tangential velocity and plot as vertical straight lines.

$$1 + V_y = h \tag{3.4}$$

$$(1 + V_y)^2 = p = h^2 \tag{3.5}$$

3.7 Constant Eccentricity, ϵ

The eccentricity is given by Equation (3.6)

$$\epsilon^2 = 1 + [V_x^2 + (1 + V_y)^2 - 2] [1 + V_y]^2 \tag{3.6}$$

For V_x and V_y small the locus is a two by one ellipse about the origin. The $\epsilon = 1$ locus is a circle about the $V_x = 0$, $V_y = -1$ point. The $\epsilon = 1$ locus separates the elliptical and hyperbolic orbits. If $V_y = 0$ then $\epsilon = V_x$.

3.8 Orbits which reach Mars

The criterion for reaching Mars is that the aphelion radial distance from the Sun be greater than the radius of the Martian orbit, $r_♂$, at arrival. The locus of orbits which just reach Mars can be plotted from the ϵ and p loci that satisfy

$$r_♂ = \frac{p}{1 - \epsilon} \tag{3.7}$$

Because the radial distance of Mars varies considerably throughout its orbit the loci will be graphed for several values of $r_♂$. The minimum value at perihelion, the average value near the line of nodes, and the maximum value at aphelion are sufficient to show the effect of eccentricity in a simple model. Each value of $r_♂$ corresponds to an arrival at Mars at a specific point in the Martian orbit.

3.9 Constant Time of Flight, t_f

The time of flight to reach Mars is one of the more important parameters. For elliptical orbits two loci will go through each point since there are two possible crossings of the Martian orbit. The expression for the time of flight depends in part on the type of orbit. These expressions are given in Reference (12). For the hyberbolic orbits, for example

$$t_f = \frac{a^{3/2}}{2\pi} [\, \gamma_1 - \gamma_2 - (\sinh \gamma_1 - \sinh \gamma_2)] \qquad (3.8)$$

where

$$\gamma_1 = \text{arc cosh } (\frac{a+1}{a})$$

$$\gamma_2 = \text{arc cosh } (\frac{a+r_♂}{a})$$

3.10 Constant Heliocentric Trip Angle, Θ

For elliptical orbits there will be two Θ loci through each point since there are two possible crossings of the Martian orbit. For a hyperbolic orbit the locus is given by Equation (3.9). For the elliptical orbits there are sign changes to make Equation (3.9) applicable.

$$\Theta = \left| \text{arc cos } (\frac{p-r_♂}{r_♂ \epsilon}) - \text{arc cos } (\frac{p-1}{\epsilon}) \right| \qquad (3.9)$$

3.11 Constant Configuration at Launch, ϕ_L

The launch configuration angle is the heliocentric angle between Mars' radius vector and the Earth's radius vector at launch. See Figure 3b. This parameter sets the launch date relative to the current opposition date. It is important for determining the date of a launch opportunity or the duration of a launch window.

$$\phi_L = \Theta_M - \Theta \qquad (3.10)$$

where Θ_M = the heliocentric angle measured from Mars' radius vector at launch to Mars' radius vector at arrival.

3.12 Constant Configuration at Arrival, ϕ_A

This parameter gives the angular orientation of Earth and Mars at the completion of the trip to Mars. It is the heliocentric angle between Mars' radius vector and the Earth's radius vector at arrival. This parameter sets the arrival date relative to the current opposition. It is probably the most important parameter for round trip considerations because it represents the end conditions with which one must start to plan the return trip. It also gives the orientation of Earth and Mars at the start of the exploration phase which is important for communication requirements.

$$\phi_A = 2\pi t_f - \theta \qquad (3.11)$$

3.13 The Out-of-Plane Velocity Increment, V_z

The out-of-plane velocity increment is given by

$$V_z = \frac{z(1 + V_y)}{r_{♂} \sin \theta} \qquad (3.12)$$

The distance, z, of Mars above the ecliptic plane at arrival is determined by the corresponding value of $r_{♂}$. The relationship can be seen in Figure 1. The maximum positive value of z occurs near Martian aphelion and the maximum negative value of z occurs near perihelion. V_z should be added vectorially to V and a total velocity locus plotted to show the net effect of the orbital inclination on the total velocity requirement.

3.14 Velocity Increment at Mars

Because of the possibility of using atmospheric braking at Mars the expression for the required velocity increment could be complex. A simple bound is given by the case using only thrust braking and the case using only atmospheric braking. The velocity increment at Mars can also be visualized by plotting all of the contours on the arrival planet's V_x, V_y plane.

There may be other important parameters to be considered but the point is that they can all be represented as loci on some V_x, V_y plane. To consider the round trip the same curves are constructed for orbits starting in the vicinity of Mars and proceeding to Earth. It is possible using a large scale with light colored lines to present all or several loci on one chart. The equations presented are based on a simplified two-body representation but the device described is adaptable to the presentation of the same information deduced from more accurate models. The advantage of the device is that all possible orbits are displayed and at a glance all properties of any proposed orbit may be determined. Further, the direction and magnitude of the velocity increment to accomplish the particular orbit is apparent. All combinations of flyby and stopover missions may be analyzed by pairing inbound and outbound velocity vectors at the destination planet.

CHAPTER 4

THE SIMPLIFIED APPROXIMATE MODEL

4.1 The Need for Simplicity

Because the closed form analytic solutions of many problems in celestial mechanics are difficult or impossible to develop, recourse is made to a digital computer. Once the computer is used the basic understanding of what is going on becomes confused in graphs and tables. As an aid to mission planning it appears desirable to have a model that permits a simple visualization of complex orbital characteristics and gives at least approximate numerical results without requiring a computer solution. In Reference (13) the author developed such a model in connection with the problem of orbital rendezvous. The model expresses the position and velocity of a freely-falling spaceship as a function of time and initial conditions or as a function of time and the orbital parameters. The model shows the first order effects of ellipticity and orbital plane inclination. Transfer between sun-centered and planet-centered coordinate frames both inertial and rotating is simple and the effects of orbital maneuvering can be visualized in the several frames even though the numerical results are approximate. The model is too crude for accurate trajectory selection but it is extremely useful for the demonstration of complicated orbital characteristics and optimizations under limiting conditions. Optimizations too complicated to be developed rigorously(14) can be done relatively easily using the model. One such example is given in Appendix C. In those cases where optimizations have been done rigorously (by Lawden(14) or Long(15) for example) the conclusions using the model have been substantiated.

4.2 The Model

Define an Earth-centered frame (x, y, z) and a Sun-centered frame (ρ, Ø, z) that rotate with respect to inertial space at the rotational velocity of the Earth around the Sun. The x, y, ρ and Ø measurements are made in the Earth's orbital plane. See Figure 4. The z coordinate measures the distance above or below the Earth's orbital plane. For units use the astronomical unit (a.u.) for distance, the radian for angle, the year for time, and the Earth's Mean Orbital Speed for velocity.* At time t = 0 a vehicle leaves the Earth at E with a velocity relative to the Earth given by the components V_x, V_y and V_z. At a time t years later the vehicle is freely falling in the Sun's gravitational field at point P with coordinates x, y, and z or ρ, Ø and z. If P is close to E then the

*Note that one EMOS = 2π a.u. per year. The rationalized velocity unit is chosen to avoid the 2π factor.

results derived in Appendix A give the coordinates of P approximately by Equations (4.1).

$$x = sV_x + rV_y$$

$$y = -rV_x + qV_y$$

$$z = sV_z \qquad (4.1)$$

where: $q = q(t) = 4\sin 2\pi t - 6\pi t$

$r = r(t) = 2 - 2\cos 2\pi t$

$s = s(t) = \sin 2\pi t$

Equivalently, if $\rho \cong 1$ a.u.

$$\rho = 1 + sV_x + rV_y$$

$$\phi = -rV_x + qV_y \qquad (4.2)$$

$$z = sV_z$$

Because the distance from E to P is not always small compared with the distance from the Earth to the Sun the values of the coordinates are crude. The accuracy is discussed in Appendix B. The equations are, however, sufficiently representative of the orbital mechanics to be of great value in mission planning. The charts outlined in Chapter 3, for instance, can be sketched using only slide rule calculations. The numerical results are approximate but the orbital characteristics can be easily visualized.

4.3 Visualizations

The results of velocity components in the three directions are visualized simply. Referring to Figure 5, V_x alone gives rise to a 2 x 1 elliptical path. V_y alone gives rise to a cycloid-like motion when viewed in this rotating frame. A V_y velocity component is four times as effective as a V_x component in producing an x-displacement, but it carries with it a stronger y-displacement, and the times to realize the maximum displacements are different. V_z alone gives a simple harmonic motion above and below the plane of the ecliptic. In general the motion will consist of all three components. Since q, r, and s are functions of time only, the equations are a simple linear set when t is specified.

It should be noted that positive V_x and V_y velocity components cause the vehicle to fall behind Earth while negative V_x and V_y components cause the vehicle to run ahead of Earth. Since a round trip will require that the vehicle return to Earth, the total y displacement must sum to zero (or a multiple of 2π). Consequently trajectories cannot be chosen without concern for the y motion.

In this model a planet in a coplanar, circular orbit will move along the line:

$$x = \text{constant}; \quad z = 0 \tag{4.3}$$

and will have a velocity given by:

$$\dot{x} = 0$$
$$\dot{y} = -\tfrac{3}{2}x \tag{4.4}$$

This shows that Earth overtakes the outer planets such as Mars and in turn is overtaken by the inner planets such as Venus. The orbits of Venus and Mars are shown in Figure 4 as they would appear in this frame assuming circular orbits.

4.4 Relative Velocity at Arrival, V_a

To determine the velocity of the spaceship at arrival relative to a planet in circular orbit, differentiate* Equations (4.1) and subtract (4.4)

$$\begin{bmatrix} V_{a_x} \\ V_{a_y} \end{bmatrix} = \begin{bmatrix} c & 2s \\ -\frac{s}{2} & c \end{bmatrix} \begin{bmatrix} V_x \\ V_y \end{bmatrix} \tag{4.5}$$

where $c = c(t) = \cos\ 2\pi t$

A simple matrix transforms the launch velocity into the arrival velocity.

In order to study the orbits which return to Earth from space let V_x and V_y be the velocity components of

*Since Equations (4.1) were obtained in Appendix A by a direct integration there is no additional approximation here.

the vehicle relative to Earth at arrival instead of launch and let time run backwards, i.e. replace t by minus t in all equations. Equation (4.5) will then give $\overline{V}_a$ as the velocity of the vehicle relative to the launch planet (in circular orbit) at launch. To visualize time running backwards in Figure 5 merely proceed in the direction opposite to the arrows.

4.5 Orbital Inclination and Eccentricity

Orbital inclination and eccentricity in the case of Mars cause the path of the destination planet to have a slight slope in this frame relative to the path of a planet in coplanar, circular orbit. Because of the orbital inclination the z of Mars can vary from -.044 to +.054 a.u. during its two-year orbit. Because of the orbital eccentricity the x of Mars can vary from .38 to .67 a.u. The actual values of x and z at arrival will be determined by the date of arrival. Because of round-trip considerations the arrival date is almost always within a few months of the current opposition date. This means that it is only necessary to substitute the x and z of Mars near the current opposition date to compare the effects of different opposition dates on the launch velocity.

4.6 Superposition

As a consequence of the linearity of the simple model, the result of two independent velocity increments applied to the vehicle at different times may be deduced by adding the results of each increment applied separately. If velocity V_1 is applied at time t_1 and velocity, V_2 is applied at time t_2, then the resulting motion for $t > t_2$ is given by

$$
\begin{aligned}
x &= s(t - t_1)V_{1_x} + r(t - t_1)V_{1_y} + s(t - t_2)V_{2_x} + r(t - t_2)V_{2_y} \\
y &= -r(t - t_1)V_{1_x} + q(t - t_1)V_{1_y} - r(t - t_2)V_{2_x} + q(t - t_2)V_{2_y} \\
z &= s(t - t_1)V_{1_z} + s(t - t_2)V_{2_z}
\end{aligned}
\qquad (4.7)
$$

The use of the superposition concept is more valuable as a visualization aid than it is to deduce Equation (4.7). The results of the first velocity increment can be considered as the motion of a fictitious vehicle which only made one velocity change. The results of the second velocity increment can be interpreted as the motion of the true vehicle relative to the fictitious vehicle. The sum of the two results gives the motion of the true vehicle relative to the Earth. Each individual result can be visualized by reference to Figure 5 as before. This concept will be useful in the discussion of bi-elliptical transfer in Chapter 7.

4.7 Other Uses of the Model

Besides its application to mission planning the model can be useful wherever visualization of orbital characteristics is desirable. As mentioned earlier, it was originally adapted to the problem of orbital rendezvous[13]. The model is also useful in the study of navigational corrections. Stern[19] has developed an analytical expression for any mid-course velocity correction. The model described here allows a visualization of the effect of those corrections. It also offers a quick approximate check of any results derived from more accurate models.

CHAPTER 5

THE ROUND TRIP

5.1 Conditions Necessary to Go

In order that man can go to Mars he must have a vehicle capable of taking the necessary payload to the destination and back. If the vehicle is an impulse-type rocket then the requirement can be expressed by two curves. One is the velocity increment required* for the mission as a function of the total mission time and the other is the weight of the required payload as a function of the total mission time. For some total mission time the vehicle must be capable of imparting the velocity increment to the corresponding payload weight.

The required velocity increment curve is obtained in this manner. First, represent all the possible orbits to Mars and back in some convenient manner (such as that outlined in Chapter 3, for example). Second, select a total time for the mission and choose from all those orbit pairs that give the selected total time the one pair that gives the minimum required velocity increment. With this required velocity increment and total mission time plot

*For this analysis the velocity increment required is the sum of the velocity increments at all four terminals of the interplanetary transfers.

one point of the curve. This procedure has been carried out by Dugan[12] and Johnson and Smith[16] for orbits which start and terminate in close circular orbits about Earth and Mars. Both works assumed circular, coplanar, orbits about the Sun for Earth and Mars. Dugan's results are shown in Figure 6. The characteristic shape can be seen to have relative minima near 450 days and 1,000 days. Decreasing the total trip time below 450 days requires an ever increasing velocity increment.

The payload weight curve is obtained by considering the weight of the crew and their equipment including all the items necessary for life support outside the protecting cover of the Earth's atmoshere. The major contribution of the weight of the payload appears to be the radiation shielding for the crew. The weight requirements for passive shielding have been studied by Wallner and Kaufman[17] and the best figures available are only estimates because of the basic lack of knowledge about both the existing solar radiation and man's tolerable dosage. Kash and Tooper[18] have indicated the possibility of major weight reductions through the use of active shielding but the techniques are far from developed. Because of the unknown information about the major contribution to the weight of the payload the curve is estimated by bounding lines shown in Figure 6. The maximum slope is near one month. The increase is due to the increased weight of shielding necessary to provide protection from a major solar flare. The probability of encountering such a flare at some time during the trip increases rapidly if the trip time is greater than one month. The question of how to shield against the possibility of a giant solar flare is largely a conjecture at this time. Also, little is known about the shielding requirements for equipment such as the electronic components. The weight of food, oxygen and water is estimated at about 15 pounds per man per day. In general the slope will be greater than this value because the weight of the required shielding also increases with time. It is possible to have a negative slope as the time of flight gets extremely short if the power plant is of the nuclear type. The increased thrust requirement for very short missions increases the radiation from the engine. If the engine does not offer a radiation hazard then extremely small payloads are required for very short duration missions.

Because the interplanetary vehicle discussed above will have to first be boosted into an Earth orbit, its initial weight is a controlling factor. The easiest and consequently the first capability for man to go to Mars will be with the lightest vehicle that meets the requirements of the two curves.

For an impulse type rocket assuming infinite staging:

$$\Delta v = c \ln \frac{W_i}{W_f} \qquad (5.1)$$

where:

Δv = velocity increment given the payload

c = exhaust velocity of the propellant

W_i = initial weight of the rocket

W_f = final weight of the rocket.

Rewriting Equation (5.1)

$$c \ln W_i = \Delta v + c \ln W_f \qquad (5.2)$$

Assume that W_i and W_f are continuous functions of the mission time, t_m. For a relative minimum:

$$\frac{\partial \Delta v}{\partial t_m} + \frac{c}{W_f} \frac{\partial W_f}{\partial t_m} = 0 \qquad (5.3)$$

For the comparison of any two relative minima (or any two missions) designated 1 and 2, the first will be the true minimum if

$$\Delta v_1 - \Delta v_2 < c \ln \frac{W_{f_2}}{W_{f_1}} \qquad (5.4)$$

Set c at 30,000 ft./sec. which is a nominal value for a nuclear rocket. The first rocket capable of taking man to Mars will probably be a nuclear rocket. The conditions of Equation (5.3) and (5.4) can now be used with the curves of Figures 6 and 7 to select the mission that gives the minimum vehicle weight. This occurs with a mission time of around 400 days. Therefore the easiest and hence the first capability to go at all will involve a mission time of around 400 days. Faster trips are more desirable from almost all viewpoints except the velocity required, so it is concluded that the actual mission will be 400 days or less. The figures to reach this conclusion are obviously approximate but in any case it appears that they justify the use of the branch of the required velocity curve in Figure 6 to the left of the 450 day minimum.

For all missions taking less than 450 days an increase in the stay time at Mars causes an increase in the required total velocity increment. For the 400-day mission Ref. (12) shows this is about 300 feet per second per day at Mars. For the minimum velocity trip there is relatively little choice. Stay time on Mars will have to be near 450 days or there is no advantage in resorting to this type of trip for the manned mission. In terms of a "scram" capability, the short missions can depart Mars prior to the planned time with less fuel than anticipated for the mission. The longer missions must remain at Mars 450 days or else expend more fuel than was planned for the mission.

Several types of continuous-thrust engines are now appearing as possible competitors to the impulsive-type rocket[20]. A curve analogous to the velocity-increment-required curve could be developed for a continuous-thrust engine. However, since the continuous-thrust engines are in an early stage of development the impulsive-type rocket will almost certainly be the first to offer a manned Mars capability. If a continuous-thrust engine is to compete it will have to offer the capability of a 400-day mission or less.

The conclusion drawn here is that man cannot go to Mars until the capability exists. Once the capability exists man will probably go to Mars with as short a mission time as possible. That mission time will be about 400 days' duration or less.

5.2 Discussion of the Conclusion

The analysis of Section 5.1 is crude. The fact that the payload weight curve is not well bounded is not too serious because the criteria for a minimum depends more on the slope, and the slope can be estimated within a tighter bound than the curve itself. The assumption that all velocity changes will be accomplished with thrust is a poor one because the use of atmospheric braking at either terminal can afford weight savings. Atmospheric braking will be discussed further in Chapter 9. The assumption of a nuclear rocket is critical. The optimum mission time depends directly on the exhaust velocity of the rocket propellant. If the nuclear rocket is not available and chemical rockets ($c \cong 10,000$ ft./sec.) must be used, then the optimum mission time by the analysis of Section 5.1 will be longer unless there is some way to reduce the required velocity.

There are other reasons why shorter missions are desirable besides the decreased payload weight. One major factor is the effect of the long mission duration on the crew. A year's flight in a space capsule could be compared to a year in prison. The history of aviation has shown that man's inherent impatience has continually caused him to develop faster and faster aircraft. The traffic on airlines indicates that man is willing to pay a higher fare in order to reach his destination in less time. A faster trip to Mars would mean lower durability and reliability requirements on the equipment. From practically all viewpoints, except the velocity increment required, shorter trips are to be desired. Since favorable launch opportunities come about every 780 days the results of a longer mission would not be available for application to a new trip for over four years. The results of a shorter mission would be available for use during the next "Mars season".

The discussion of this Chapter should introduce the nature of the many trade-offs involved in this study. While an analytical method is presented for choosing an optimum mission time there are subjective arguments for deviating from the optimum. In summary, it is concluded that if the nuclear rocket is available, the easiest capability to go to Mars will be with a mission time of 400 days or less.

The analyses in this Chapter and Chapter 6 are based on single-elliptical transfers between Earth and Mars. The conclusions which are drawn are needed to introduce the newer concepts of bi-elliptical transfer and transfer via Venus. Consequently the statements made in these two chapters do not necessarily apply to missions of the latter type.

CHAPTER 6

PATHS IN SPACE

6.1 An Earth-centered Rotating Frame

The orbital characteristics of round-trip missions to Mars are visualized most easily in an Earth-centered or a Mars-centered coordinate frame. Figure 8 shows the motion of Mars relative to the Earth-centered frame described in Chapter 3. Mars appears to rotate clockwise with a period of about 780 days. The position of Mars is shown at monthly intervals before and after opposition. Each pair of trajectories that represents a point on the curve of Figure 6 can be plotted in this frame. The opportunity for executing the mission represented by any one point occurs only once every 780 day period. These missions fall into two basic types. The minimum velocity mission is shown by a dotted line in Figure 8 and represents one type. It is seen that the major portion of the mission is spent on Mars while Mars is distant from the Earth. The other type of mission arrives at Mars close to the opposition position. This type includes all the missions of less than 400 days' duration. All these missions arrive at Mars between the two quadrature positions. A typical mission of this type is indicated by a solid line in Figure 8. For a circular Mars' orbit and a given total mission time the total velocity increment is a minimum when the arrival at Mars is displaced either side of the opposition position. The total velocity increment does not change much when the arrival position is varied either side of the minimum. Because of this insensitivity to time of arrival the arrival date can vary either side of the opposition date by as much as one quarter of the round-trip travel time with less than 15% velocity penalty. This allows the arrival point at Mars to be shifted away from the opposition position towards perihelion and thus take advantage of the decreased radial distance from the Sun. This result can also be used to pair orbits so that the out-of-plane component does not constitute a penalty. This means that a typical mission will probably arrive at Mars on the perihelion side of the current opposition but in any case not further from opposition than the quadrature position.

6.2 An Earth-centered Inertial Frame

Figure 9 shows the position of Mars relative to an Earth-centered frame that is non-rotating with respect to inertial space. Position is indicated at monthly intervals before or after opposition. The minimum velocity mission is indicated in the Figure by a dotted line. Again, this mission requires the crew to remain at Mars for 450 days. The conclusion concerning the manned mission to Mars appears obvious when the motion is observed in this frame. For fast missions to Mars it is easiest to visit near the opposition position. In this frame all the missions of less than 400 days arrive at Mars inside a flat 15° cone about the position of Mars at opposition.

6.3 A Mars-centered Rotating Frame

Figure 10 shows the position of Earth relative to a Mars-centered frame which rotates so that the Sun has constant direction. The position of Earth is indicated at monthly intervals before or after opposition and the minimum velocity mission is shown by a dotted line. The launch date from Earth and the return date to Earth are indicated by the position of Earth at either time relative to the opposition. While Figure 8 shows the major effect of changing the arrival date at Mars, Figure 10 shows the major effect of changing the launch or return date at Earth. For a given total mission time a change in the date of Earth launch must be accomplished by a change in the date of Mars arrival or there will be an increase in the required velocity increment. The many trade-offs are best evaluated using the charts suggested in Chapter 3 where all the parameters of all the possible orbits to Mars are displayed together.

Figures 8-10 are drawn assuming circular orbits for Earth and Mars. The true orbits will bring the planets closer or further apart than shown depending upon the opposition date.

6.4 The 400-Day Trips

If the mission time is constrained to be about 400 days, the mission profile that uses the minimum total velocity increment looks very similar to the one shown by the solid line in Figure 8. The going trajectory launches with a velocity relative to Earth that is pointed almost directly toward the Sun. As shown in Figure 5, this causes the vehicle to go towards the Sun and ahead of Earth before going out to intercept the Mars' orbit. The time of flight to Mars is about 240 days. In a Sun-centered, inertial frame the heliocentric trip angle would be about 270 degrees. As shown in Chapter 4, this is an uneconomical way to get to Mars, but it is chosen because one of the two halves of the round trip must get ahead of Earth if the other half gets behind. This aspect of round-trip mission planning is what makes it so complex in comparison with interplanetary trips which only go one way.

Other observations from Figure 8 are that the spaceship never gets further than one astronomical unit from the Earth and during favorable oppositions could be kept within a half astronomical unit. The point of closest approach to the Sun occurs at a distance which is approximately equal to Venus' orbital radius.

Similar observations can be made for the reciprocal mission which uses the shorter transfer for the outbound leg and the longer transfer for the inbound leg.

CHAPTER 7

BI-ELLIPTICAL TRANSFER

7.1 Possible Savings through Bi-elliptical Transfer

The following observations are made based on the simple models already presented. Low energy transfers to the outer planets are characterized by the fact that the spaceship falls behind Earth while low energy transfers to the inner planets cause the spaceship to run ahead of Earth. The manned mission to Mars will probably be accomplished with a mission duration of about 400 days or less. Missions of this duration which use two single-elliptical transfers require the use of one uneconomical transfer in order to get ahead of Earth. By starting a trip to Mars with a low energy transfer toward Venus, the spaceship will run ahead of Earth and be in a position for two low energy transfers, first to Mars and then back to Earth. The reciprocal mission is also possible by making a low energy transfer to Mars with a bi-elliptical transfer on return.

The above argument in favor of a bi-elliptical transfer may be justified on more rigorous grounds by considering

Figure 11. Suppose that E'πM is the optimum single-ellipse transfer to Mars for some round-trip mission. Barrar[21] has given analytical proof that transfer by two impulses, one at E and one at π, is always more economical than a single impulse transfer at E' to achieve the orbit πM. In addition the EπM transfer will place the spaceship farther ahead of Earth at arrival than the E'πM transfer. It is true that the EπM transfer will have a longer flight time than the E'πM transfer but this can be partially made up on the return trip by virtue of the more favorable distance ahead of Earth on arriving at Mars.

7.2 Analysis

The simple model is already equipped to handle bi-elliptical transfer by virtue of the superposition principle explained in Section 4.6. The approximate coordinates of the spaceship are given by Equations (4.7). The effect of each individual velocity increment can be visualized using Figure 5. It is possible to minimize the total velocity increment under any set of constraint conditions by using the model together with the Calculus of Variations as in Appendix A. The method is straightforward but it requires the solution of a simultaneous set of equations of high order for most realistic constraint conditions.

In the analysis of a single-elliptical transfer between Earth and Mars it was pointed out that the specification of a launch date and an arrival date was sufficient to uniquely determine a trajectory between the two planets. In the case of a bi-elliptical transfer the number of possible pairs of ellipses for a given launch and arrival date is infinite. Furthermore, the choice of the ellipses involves several trade-offs. To make the required launch velocity increment a minimum the angle $ES\pi$ (see Figure 11) should be near 180 degrees. To make the time of flight a minimum the angle $ES\pi$ should be as small as possible. Similarly, the angle πSM should be near 180 degrees for minimum velocity relative to Mars at arrival but as small as possible for a short time of flight. In order to make comparisons between the two types of transfer it appears desirable to fix the configuration of the bi-elliptical transfer so that the specification of a launch and an arrival date will uniquely determine a trajectory. To this end, assume throughout the analysis that the two ellipses $E\pi$ and πM have a common periapse at π. Now if the angle πSM is specified, the choice of a launch date and an arrival date is sufficient to determine a unique bi-elliptical transfer. The velocity relative to Mars at arrival is directly related to the size of the angle πSM. If this velocity is to be dissipated by means of atmospheric braking then the choice of the angle πSM will depend directly on the limitations imposed by drag braking in the Martian atmosphere. In addition to offering fuel savings at launch bi-elliptical transfer gives the mission planner control over the arrival velocity at Mars making it possible to fully exploit the use of atmospheric braking at Mars. Atmospheric braking will be discussed in greater detail in Chapter 9. Once the desired arrival velocity at Mars is selected the angle πSM is chosen to give approximately that arrival velocity. Specification of the launch and arrival date will now give a unique bi-elliptical transfer which can be compared with the conventional, single-ellipse transfer corresponding to the two dates.

7.3 Advantages of Bi-elliptical Transfer

The fuel saving afforded through bi-elliptical transfer is very great if drag brakes can dissipate high approach velocity at Mars without heavy weight penalty. It makes possible the full utilization of the soft Martian atmosphere to terminate a fast economical transfer to Mars. Even with thrust-braking at both Mars and Earth the technique still offers savings over the best pair of single-elliptical transfers of equivalent mission duration. The saving is the largest for mission durations of 400 to 500 days. Bi-elliptical transfer will always provide a trajectory at least as good as single-elliptical transfer because the latter is the limiting case of the former when the mid-course velocity increment approaches zero. The fact that the bi-elliptical transfer allows independent adjustment of the terminal velocities in both magnitude and direction is important. Many of the best single-elliptical missions in 1971, for instance, launch almost directly into the Sun. This factor could be a distinct disadvantage during early tracking from Earth-fixed observatories. An additional fuel saving is possible in that the first navigational correction and the second orbital transfer can both be made in one thrust maneuver.

7.4 Planetary Environment

One factor that limits the gains anticipated with bi-elliptical transfer is that the second orbital transfer is accomplished while beyond the influence of any planetary gravity. As shown in Chapter 8, the planetary environment allows a velocity increment measured relative to a planet to produce a greater than one to one ratio velocity change when evaluated relative to the Sun after planetary escape. In crude terms, it "costs" more fuel to generate a velocity change when distant from the planetary environment. This means that the bi-elliptical transfer would be even more effective if the second orbital transfer could be accomplished during an encounter with Venus while en route to Mars. By making a small velocity change close to Venus at high velocity and low potential energy, a large orbital energy change will be generated to place the spaceship on a trajectory to Mars. The disadvantage of this type of mission is that it requires a specific orientation of Earth, Venus and Mars, the synodic period of their relative positions being about six years. Because of the timing requirement the bi-elliptical transfer to Mars via Venus encounter will be treated in Chapter 11 as a three-legged mission of single-elliptical transfers. The Venus encounter mission is mentioned here primarily because it was conceived as an extension of the bi-elliptical transfer.

CHAPTER 8

PLANETARY ENCOUNTER

8.1 Analytic Model

The analytic model for the planetary encounter assumes a planet-centered, inertial, reference frame. The only force assumed to be acting is due to the planet's gravitational field. All orbital transfers are accomplished by impulsive velocity changes. This is consistent with conventional patched-conic trajectory analysis.

There are three basic maneuvers that will be of interest. The first maneuver is the direct descent from a solar orbit at effectively an infinite distance from the planet down to the top of the planet's atmosphere via hyperbolic transfer. The direct ascent from the top of the planet's atmosphere to escape from the planet's gravitational field via hyperbolic transfer is considered as the same basic maneuver. The second maneuver is the transfer into or out of a circular parking orbit from an infinite distance via hyperbolic transfer. The third maneuver is a flyby of the planet with a velocity impulse applied at the point of closest approach to the planet. The inbound trajectory and the outbound trajectory are both hyperbolas with a common periapse.

All three maneuvers involve hyperbolic trajectories and in each case the velocity increment will be accomplished by a thrust impulse applied at the periapse of the hyperbolic path. In practice the thrust increment cannot be applied in an exact impulse at the exact periapse position. Zee[22] has reported an introductory investigation into the errors associated with the assumption of impulsive thrust application. He indicates that the assumption of impulsive thrust will give results sufficiently accurate for mission planning.

The unit for distance in this analysis will be the planet radius. Velocity will continue to be expressed in EMOS units. The following quantities are defined for the analysis of the three basic planetary-encounter maneuvers.

r_π = the radial distance of the pericenter of the hyperbola from the center of the planet.

V_π = the velocity of the vehicle relative to the planet at the pericenter of the hyperbola.

V_H = the hyperbolic approach or departure velocity of the vehicle relative to the planet after escape from the planet's gravitational field.

V_C = circular satellite velocity at the planet's surface.

V_S = circular satellite velocity at the radial distance, r_π.

V_E = escape velocity at the radial distance, r_π.

δ = the angle between the vectors $\overline{V}_\pi$ and $\overline{V}_H$.

ΔV = the velocity increment applied at the periapse.

ϵ = the eccentricity of the hyperbola.

a = the semi-major axis of the hyperbola.

p = the semi-latus rectum of the hyperbola.

In the case of the flyby maneuver the subscript I or O will be added to refer the parameter to the inbound or outbound hyperbola, respectively.

8.2 General Hyperbolic Motion

The main parameters of the hyperbolic motion which are needed in each of the basic planetary-encounter maneuvers are V_H, V_π, r_π and δ. The parameter, r_π, can be conveniently expressed in terms of the circular satellite velocity, V_S, at that radial distance by

$$r_\pi = \left(\frac{V_C}{V_S} \right)^2 \qquad (8.1)$$

V_C, the circular satellite velocity at the planet's surface, is a constant for each planet and is given below for Earth, Venus and Mars.

$$V_{C\oplus} = .266 \text{ EMOS}$$