A SYSTEMATIC APPROACH TO THE STUDY OF NONSTOP INTERPLANETARY ROUND TRIPS

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A general survey of nonstop round trips past Mars and Venus is presented. Analysis proceeds by stages of increasing complexity. At each step, promising areas of solutions are noted for further investigation, while large groups of less favorable possibilities are eliminated. Results are displayed on graphs from which all the following quantities of interest may be extracted: departure date, date of planetary passage, return date, departure speed, passage speed, return speed, and distance of closest approach to the pass planet.

An analysis is conducted of multilegged nonstop orbits passing both Mars and Venus during an uninterrupted trip. All likely mission areas in the 1970-1980 period are presented along with detailed studies of the more interesting trips.

A discussion of the advantages of applying vehicle propulsive thrust during planetary passage is presented, together with a procedure for locating the "best" point of impulse application.

Finally, requirements and performance data for a special class of nonstop round trips, pertinent to the scheduling of solar probe missions, are discussed. These trips include close-approach flights past Venus which enable nominally low-energy paths to be bent closer to the Sun.

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necessitate only modest additional fuel penalties beyond those usually quoted for "minimum-energy," one-way trips, while the compensating advantages of short communication distances and capsule recovery possibilities make consideration of such trips particularly worthwhile.

An orderly method of studying such missions (Ref. 1) is described, which not only affords the analyst a qualitative insight into the general nature of the solutions, but also provides an approach through which all possible classes of round-trip flights are taken into account.

The method proceeds by stages of increasing complexity. For preliminary purposes, the planets are first assumed to be traveling in coplanar circles about the Sun. The dynamical model is thereby rendered strictly periodic, and convenient generalizations may be drawn for all qualitative phenomena characterizing these trips. A modified form of Lambert's Theorem (Ref. 2) is applied in the present study by incorporating the relationship between transfer angle and trip time for each mission.

By the proof supplied in Ref. 2, this stipulation of both transfer angle and trip time [illegible] gorically produced and graphically displayed. The output data are studied at length;

promising areas of solutions are noted for further investigation, while less feasible groups of orbits are summarily rejected.

These surviving groups of solutions are to be regarded as nominal orbits, idealized in the sense that planetary perturbations during the encounter phases have thus far been neglected. By then extending the analysis to include gravitational effects resulting from close-approach maneuvers, many neighboring groups of solutions are introduced, all of which may be interpreted as having evolved from the nominal orbit families through a process of continuous perturbation, considering the periplanet distance as the generating parameter. In this intermediate phase of the study, the circular, coplanar model is still retained for the planetary motions so that close-approach effects may be isolated from additional complications due to planetary orbit inclinations and eccentricities. Although the absolute values of the numerical results obtained may therefore be subject to inaccuracies (especially for some trips past Mars, whose orbit eccentricity is comparatively large); nevertheless, the relative effects will remain valid. And, since the planetary geometry remains periodic, the results from this phase of the study retain their generality. The qualitative phenomena may be thoroughly explored, and further undesirable areas eliminated from future consideration.

Finally, after areas of interest have been located by the method described above, the cases still remaining are subjected to refinement by the adoption of a more realistic model for the planetary motions — one which accounts for their orbital eccentricities and inclinations. The data for this more sophisticated analysis lie reasonably near those predicted by the preliminary model. Accordingly, the three-dimensional, one-way trip tables computed in connection with the studies of Ref. 2 are searched near the dates indicated, and the proper pairs of one-way orbits thus selected are joined to fabricate the more accurate round-trip estimates. Where desired, these latter data may be used to form the basis for precise calculations in advanced vehicle navigation and guidance studies.

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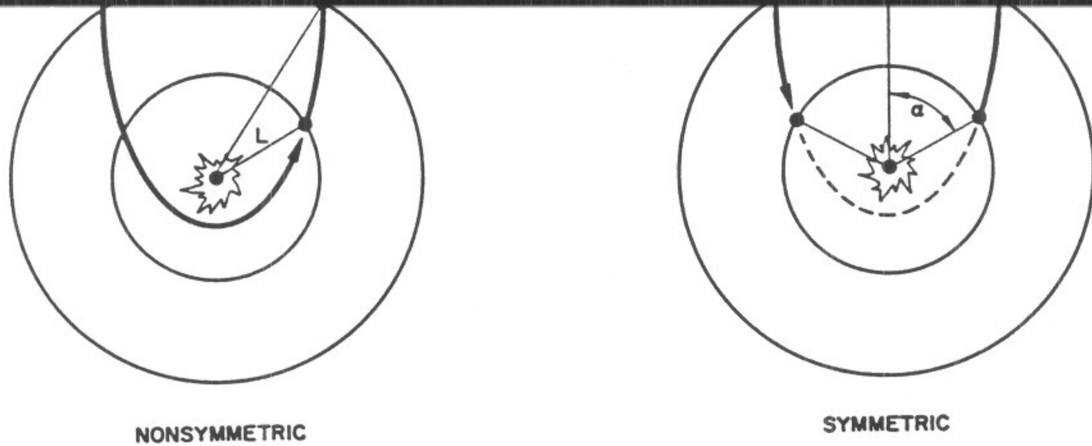

Fig. 1 Types of Nonstop Round-Trip Orbits

Nonsymmetric trips leave and encounter the Earth at times when it occupies one and the same position in space. Such missions are thereby limited to Sun-centered journeys whose flight durations span integral numbers of years. The próbe may circle the Sun several times before the planetary encounter; further, it may circle the Sun several more times before returning to Earth.

Vehicles on the Symmetric trips return to Earth in fractions of complete years. By geometrical considerations it can be seen that the transfer orbit's major axis must bisect the angle formed by the Earth's position vectors at the times of departure and arrival. Also, from considerations of symmetry it is evident that at the halfway point of each Symmetric trip, the Earth, Sun, and vehicle are always in strict alignment. Each Symmetric orbit will, in actual practice, require a midcourse plane change to achieve rendezvous at the points of departure, encounter, and arrival, since these points are normally noncoplanar with the Sun.

The Nonsymmetric round trips, however, require no such corrections, since the departure and arrival points coincide. In these cases, only three distinct points exist: Sun, Earth, and planet; and the orbit plane is well-defined.

For any orbit-plane changes which may be required in practical cases, either an impulsive velocity correction may be applied, or else a close approach to the planet may be utilized to modify the local asymptote direction. In fact, combinations of both types of maneuvers may be incorporated into trips which involve planetary reconnaissance operations as mission objectives.

Strictly speaking, any one-way flight may also be considered a round trip, since it must eventually encounter the Earth again at some future time. What the orbit analyst really seeks, however, are the most acceptable flights whose durations do not exceed some maximum length. This upper limit having once been specified, all flights whose durations do not exceed this value may be investigated fully. Included in the investigation would be not only all orbits whose periods are integral numbers of years, but also those having periods expressible as rational fractions whose numerators do not exceed the maximum trip time under consideration. Thus, on an orbit whose period is expressible as p/m years, a vehicle will negotiate m complete solar circuits in p years.

Similarly, for trips past Venus, the vehicle's aphelion distance must never fall short of Earth's orbital radius of 1.0 AU. For this case, $0.354 > P > p_{max}$ defines the limits for such orbits, and we may confine our search to this region only.

NONSYMMETRIC ROUND TRIPS

Nonsymmetric round trips, whose durations are expressible as integral numbers of years, are quite easily calculable, even by hand. Given an orbit period of P years, then the quantity $a = P^{2/3}$ specifies a semimajor axis length in AU. Corresponding to this value of a, a complete set of orbits may be generated by considering various values of eccentricity. Actually, any other quantity independent of a may be employed as the generator for each class of P-year orbits. In particular, let us choose L, the heliocentric angle swept out by the vehicle while travelling from Earth to the planet. (See Fig. 1). Then s, the semiperimeter of the triangle whose vertices are occupied by the Sun, the Earth at the departure date, and the planet at the pass date, is readily found. Using $E = -s/2a$, the form of Lambert's Theorem described in Ref. 2* is employed to find T, the modified first-leg trip time:

$$T = (-E)^{-3/2} \left[2m\pi + (f - \sin f) - (g - \sin g) \right] \quad (1)$$

where

$$\sin^2 \frac{f}{2} = -E$$

$$\sin^2 \frac{g}{2} = -E\left(1 - \frac{c}{s}\right)$$

m represents the number of complete orbital circuits traversed before the planetary encounter, and c is the chord connecting Earth at the departure date with the planet on the pass date. The actual first-leg trip time Δt is then obtained from the relation

$$\Delta t = \frac{T}{n}\left(\frac{s}{2}\right)^{3/2}$$

where n is the Earth's mean motion.*

The time Δt having been found, it merely remains to calculate the launch date, D_o, measured from the time of planetary alignment, using the expression

$$D_o = \frac{N\Delta t - L}{n - N} \tag{2}$$

where N is the mean motion of the pass planet. Complete information concerning the orbital parameters and the relative velocities involved at each terminal may now be derived using the equations cited in the Appendix of Ref. 2.†

The heliocentric departure speed V_1 for any mission is given by

$$V_1 = \sqrt{2 - \frac{1}{a}}$$

where a is the semimajor axis of the transfer orbit. For each family of Nonsymmetric (constant-period) trips, therefore, V_1 remains unchanged, since $a = P^{2/3}$ is also

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*See Ref. 2, Eq. (A-2).

†See Ref. 2, Fig. A-2.

symmetric Venus round-trip missions of less than two years' duration,* and all Nonsymmetric Mars trips lasting two years or less. These choices are dictated first by the desire for flights of reasonable length, especially in view of the ecology requirements for human occupants, and second because many of these journeys may easily be negotiated, even under the most pessimistic assumptions of propulsion availability during the next several years. Trips of longer span are therefore of questionable value, except for possible employment in special missions.

These constant-period curves are overlaid on contours of constant hyperbolic excess departure speed, similar to those described in Ref. 2, with the exception that Date of Planetary Passage has been substituted for First-Leg Trip Time as the ordinate.†

From one figure, then, the analyst may infer all of the following quantities of preliminary interest: departure date, first-leg trip time, total trip time, date of passage, departure speed, and return speed to Earth, the latter being equal to the departure speed, by symmetry.

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*Only first encounter passages of the 1/2-year Venus orbits are recorded in Fig. 2.

† This has been found to be a more convenient representation, and will be followed in all future work.

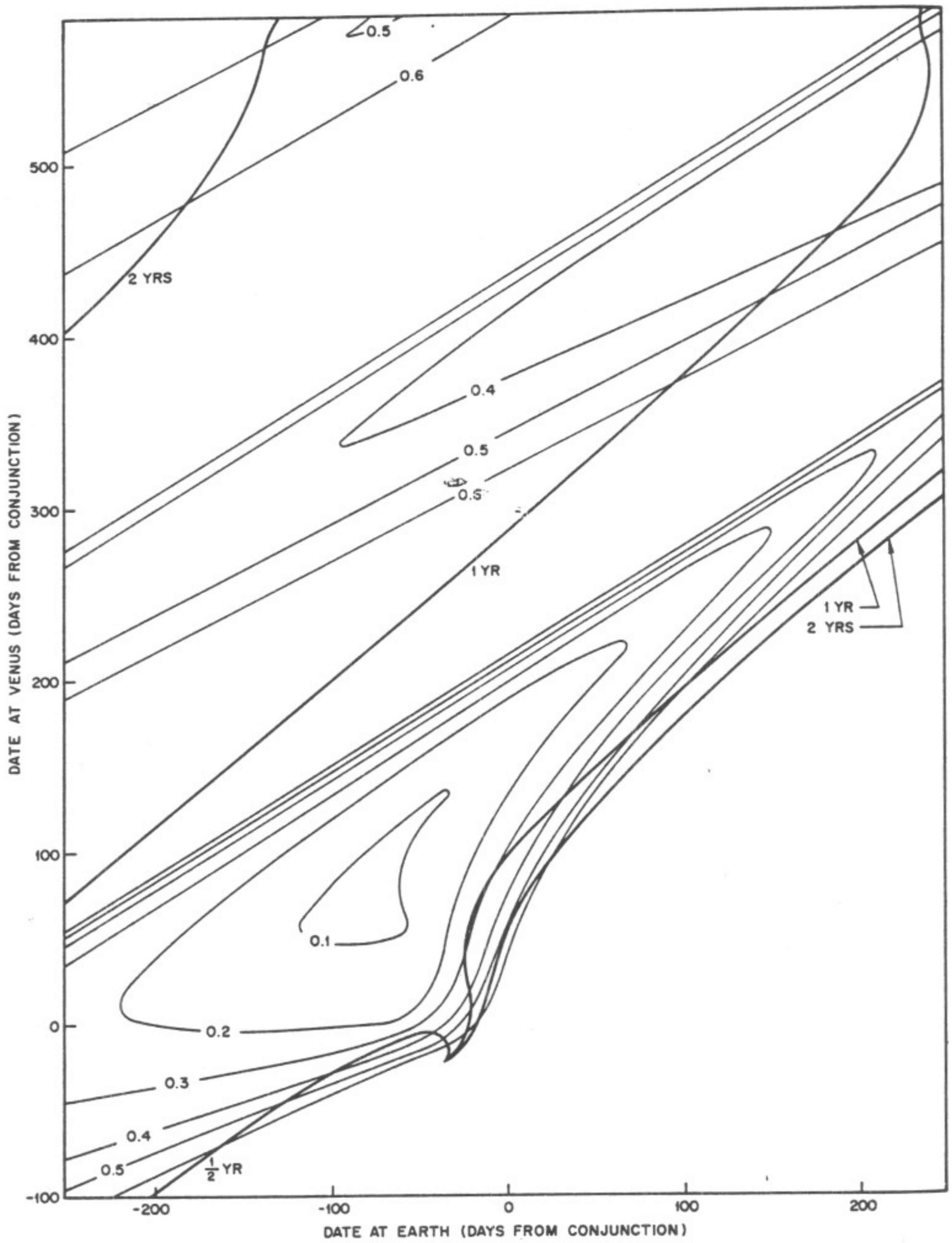
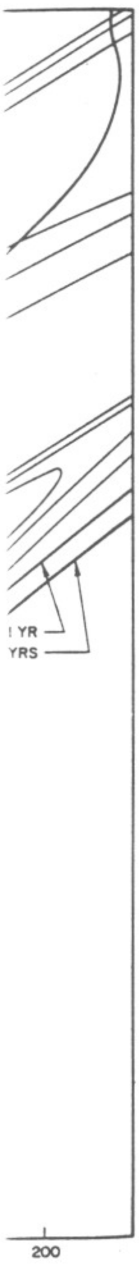


Fig. 2 Nonsymmetric Round Trips Past Venus, Overlaid on Constant Speed Curves



Speed Curves

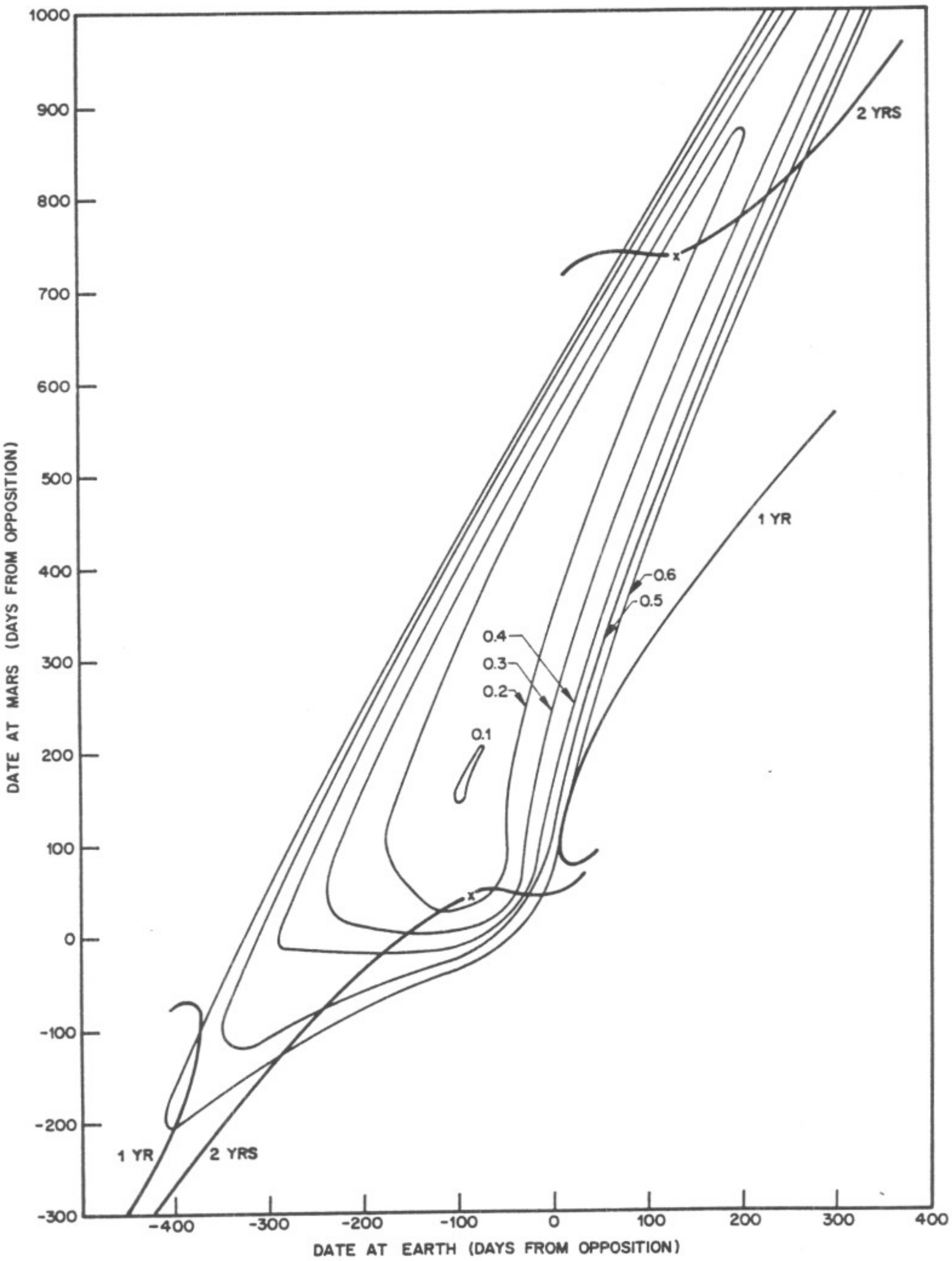


Fig. 3 Nonsymmetric Round Trips Past Mars, Overlaid on Constant Speed Curves

SYMMETRIC ROUND TRIPS

Symmetric orbits may also be obtained by employing Eq. (1). In this case, however, Lambert's equation must be inverted for the solution, and the use of a digital computer is dictated. To ensure contact with the Earth at the trip's conclusion, the total mission duration must be written as $\Delta t = 2(p\pi + \alpha)$, where 2α is the total transfer angle, mod (2π), from start to finish of the mission and p represents the number of complete years involved in the trip. Furthermore, the semiperimeter s, in the triangle formed by the Earth's positions at both the start and conclusion of the mission, and the Sun,* is equal to $1 + \sin\alpha$. Then Eq. (1) assumes the form:

$$\frac{2(p\pi + \alpha)}{[(1 + \sin\alpha)/2]^{3/2}} = (-E)^{-3/2}\,[m\pi + (f - \sin f) - (g - \sin g)] \qquad (3)$$

the nomenclature being that of Eq. (1). Here, we have the additional relationship $c = 2\sin\alpha$, from Fig. 1, for the chord length between the Earth's positions on the two dates considered.

Given values for p, the number of complete years elapsing, and m, the number of complete circuits negotiated on the orbit, then a complete family of Symmetric trips may be generated by allowing α to vary from 0 through π, and inverting Eq. (2) for E in each case. The remaining orbital parameters, as well as the relative velocities at each terminal, are again available from the formulas cited in Ref. 2.

Figures 4 and 5 summarize the results obtained by applying Lambert's Theorem to the study of Symmetric flights of less than 2 years passing Venus, and, of 2 years or less passing Mars. Round trips identified by the points marked "X" in these figures pass through heliocentric angles which are integral multiples of 2π. For such missions, the Symmetric round trips are identical with the Nonsymmetric, and these points, therefore, are common to both sets of curves associated with a given planet.

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*Note that the basic Lambert triangles for the Symmetric cases are not the same as those for the Nonsymmetric cases.

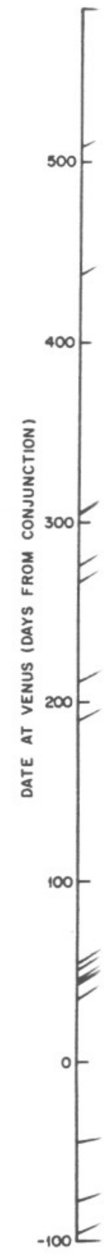

Fig. 4

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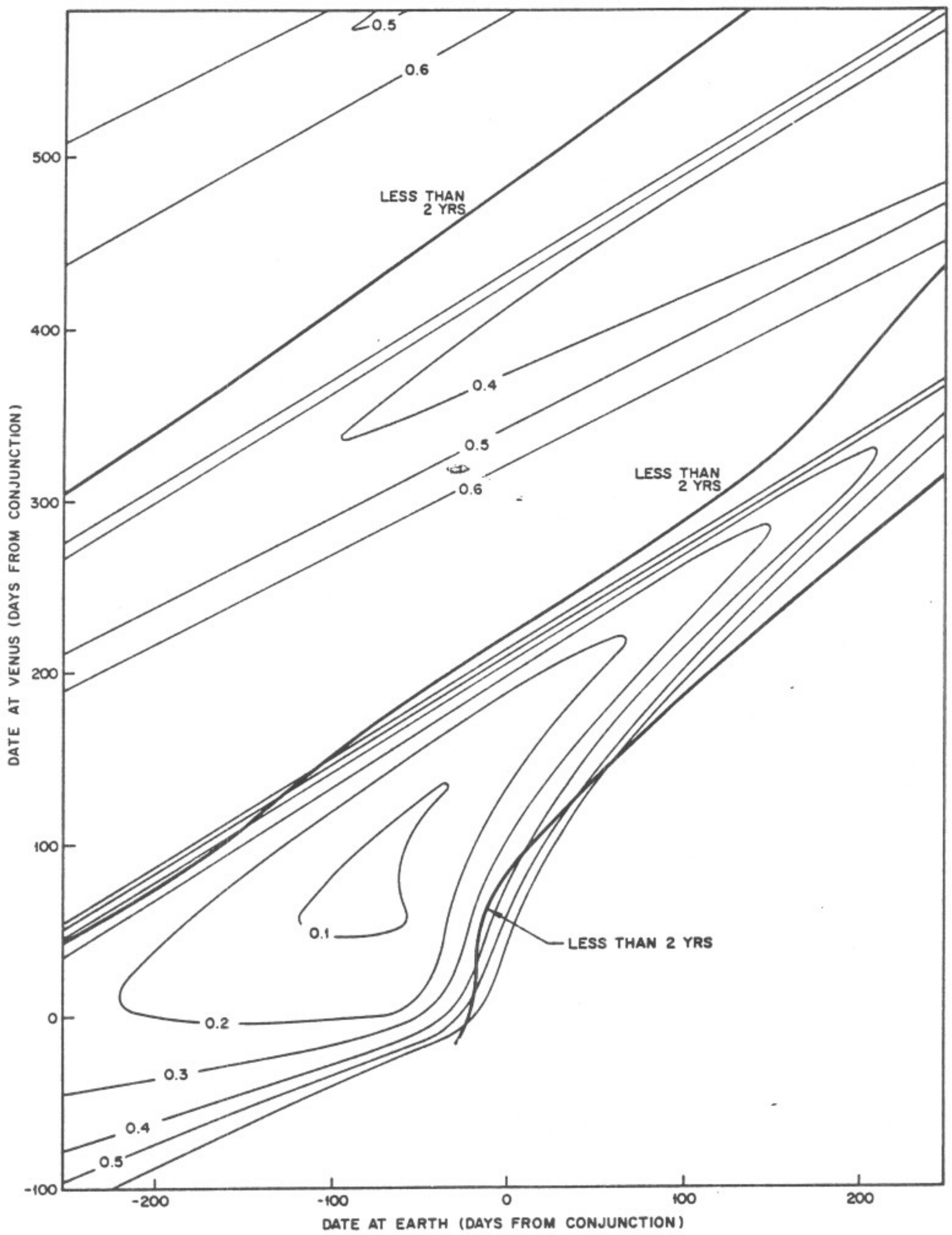

Fig. 4 Symmetric Round Trips Past Venus, Overlaid on Constant Speed Curves

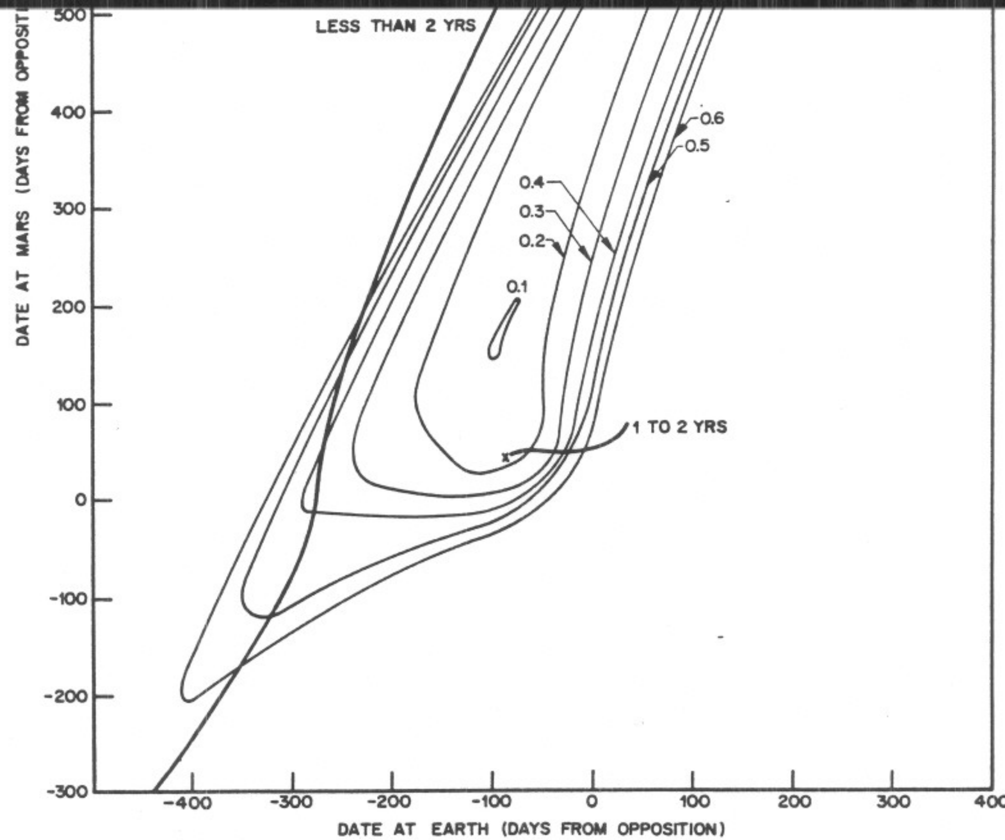

Fig. 5 Symmetric Round Trips Past Mars, Overlaid on Constant Speed Curves

RECIPROCAL PROPERTIES OF INTERPLANETARY ROUND TRIPS

We may immediately deduce the following useful theorem:

Theorem

If the planets are assumed to move in coplanar circles, then for each and every round-trip orbit there always exists a "reciprocal" orbit possessing the following properties:

- Departure and arrival speeds at Earth are interchanged between both orbits.
- Departure and arrival speeds at the planet are interchanged between both orbits
- Departure and arrival dates at Earth for the one are the negatives of the interchanged values for the other, all dates being measured from the time of planetary alignment.
- Departure and arrival dates at the planet for the one are the negatives of the interchanged values for the other, all dates being measured from the time of planetary alignment.

This theorem holds for both nonstop and stopover flights, and is valid also when the nonstop orbit involves a close planetary approach.

Not True

Proof

Numerous proofs may be advanced. Perhaps the most easily grasped is the following geometrical exposition. Considering any individual leg of the trip, let $-D_o$ be the launch date (i.e., D_o days before planetary alignment). On this date the planets are separated by an angle of, say, α_o. Let $-D_1$ be the arrival date, upon which day the planets are now separated by α_1.

Now, on date $+D_1$ the planets are again separated by α_1 (with the other planet leading), while on $+D_o$ the separation is α_o once more.

Thus, for both cases, the planetary configurations at each end are mirror images of each other and the transfer angles in both cases must be equal.

The trip times, likewise, are identical due to the choice of dates. We conclude therefore that the same elliptic segment connects the planets for both pairs of dates, namely ($-D_o$, $-D_1$) and ($+D_1$, $+D_o$). On $-D_o$ and $+D_o$ the segments meet the one planet, while on $-D_1$ and $+D_1$ they meet the other. However, the earlier date in each pair, namely $-D_o$ and $+D_1$, must of course be considered the departure date; it then follows that if the former segment is an outbound leg, the latter must be a return leg, and vice versa. And since any round trip may be decomposed into its component legs, the proof of the theorem is established.

Corollary

Any nonstop trip whose outbound and return legs are identical must pass the target planet on the date of planetary alignment. For, since the two legs are identical, the orbit is self-reciprocal, and therefore $+D_1 = -D_1$, which implies $D_1 = 0$, Q. E. D. Note that, by definition, all such orbits must belong to the class of Symmetric trips.

As an illustration, the above results may be applied to Fig. 2 through 5 in order to obtain return-leg information for these trips. That is, suppose a point ($-D_o$, $-D_1$) representing an outbound leg has been located on any of these figures. To find the return leg, note the point of intersection of that same orbit family curve with the passage date $+D_1$. Suppose that this new point has a corresponding launch date of $-D_2$. It represents another outbound leg, belonging to the same family of trips; however, by the Theorem, it can also represent a return leg, if the dates ($+D_1$, $-D_2$) are replaced by their negatives ($-D_1$, $+D_2$) and juxtaposed in meaning.*

Then we may consider the original outbound leg ($-D_o$, $-D_1$), and this return leg ($-D_1$, $+D_2$) as constituting the complete round trip. Departure speed on $-D_1$ is easily obtained from the constant speed contours, while the return speed on $+D_2$ may likewise be read from the constant speed contour passing through the reciprocal point.

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*The term $+D_1$ should be understood to signify $+D_1$ modulo S, the synodic period.

In the above manner, all of the following quantities may be easily read from a single graph: departure date, pass date, arrival date, departure speed, arrival speed. The power of this approach lies in its ability to be used rapidly in scanning, with a minimum of effort, the requirements for all possibilities within each family of missions. This approach will be employed again later in studying round trips which include close passages during planetary encounter.

CLASSIFICATION OF NOMINAL ORBIT FAMILIES

In Figs. 2 through 5, four principal families of orbits may be identified as bonafide mission possibilities: two refer to flights past Venus, two to flights past Mars. For flights past Venus, the following are of particular interest:

- Nominally high-energy trips, having durations of 1 to 2 years
- Short-period orbits, having periods equal to 1/2 year

In considering the nominally high-energy trips, both Nonsymmetric (Fig. 2) and Symmetric (Fig. 4) groups exist, the former involving trips of exactly 1 year, and the latter 1 to 2 years. The vehicle executes between one and two solar circuits during each Symmetric mission, and exactly one circuit during the Nonsymmetric flights. Generally speaking, the Nonsymmetric orbits of this class are superior to the Symmetric as regards speed and duration. These orbits, as will be shown later, constitue the most important family of trajectories for reconnaissance missions past Venus.

When the effects of close planetary approaches are evaluated, these curves will become quite complex; let us, therefore, undertake a thorough study of the idealized, unperturbed case under present consideration.

First we shall treat the Nonsymmetric high-energy orbits, labelled "1 yr" in Fig. 2. These have been noted by many previous authors, including Vertregt (Ref. 3). Two branches of the 1-yr curve are evident in Fig. 2. Each point on these branches corresponds to a trip of precisely 1 year on an orbit having a period of 1 year, and thus entails one complete circuit of the orbit.

Consider any particular ellipse from this class; it intersects Venus' orbit at two points and the Earth's orbit twice also. Therefore, there exist four outbound-leg possibilities for each ellipse in this group. These are:

(1, 2) The vehicle is launched inwards from the Earth and travels toward perihelion. It may meet Venus either before or after perihelion, and these two possibilities each contribute one point on each of the horns of the "1 yr" curve's lower branch.

(3, 4) The vehicle is launched outwards from the Earth and travels toward aphelion. After a comparatively long time, it meets Venus either before or after perihelion, creating one point on each of the horns of the upper "1 yr" branch.

As the eccentricity is decreased, the points in each pair move closer together and both pairs approach the center of Fig. 2. Finally, when some critical value of eccentricity is reached ($e = 1 - R_{♀} = 0.2767$) the transfer ellipse barely touches Venus' orbit, and the two points of each pair merge together at the nose of each branch. This nose is characterized by the property of minimal launch and arrival speeds. For smaller than critical values of eccentricity, no transfers are possible, since the ellipses do not reach to Venus' orbit.

Of parenthetical interest is the family of 4/5-year orbits (not depicted in Fig. 2). As eccentricity is decreased for this group, all four points approach the Hohmann point, and finally coalesce there. This is to be expected, since the Hohmann trip to Venus has a 4/5-year period.

Now, consider the Symmetric high-energy orbits, labelled "1 to 2 yrs" in Fig. 4. For this group, we set $m = p = 1$, in Eq. (3). Three branches are observed in the figure. This family of trips, as was explained earlier, is generated not by varying eccentricity, but rather by increasing the transfer angle 2α from 0 to 2π. Six representative phases in the generation of this family are depicted in Fig. 6.

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Fig. 6 Successive Phases in the Generation of High-Energy Symmetric Trips Past Venus

The vehicle is launched at point 1 and returns at 4 after completing one-and-a-fraction circuits around the Sun. As 2α increases from 0 we find one pair of solutions; namely, passage at 2, or at 3, Fig. 6a. This gives rise to the middle branch in Fig. 4. When the critical value of 2α is reached (Fig. 6b) the transfer trajectory barely touches Venus' orbit and points 2 and 3 coalesce. As 2α is increased still further, the vehicle no longer reaches Venus' orbit and solutions cease to exist (Fig. 6c).

In phases (a), (b), and (c) the transfer perihelion has been receding from the Sun, while the aphelion approached it. Finally, at phase (d), Fig. 6, these apsides interchange senses, and when the second critical value of 2α is attained (Fig. 6e), the vehicle's orbit once again touches that of Venus. Beyond this (Fig. 6f), the vehicle's path has two pairs of encounter points with Venus' orbit – points 2 and 3, or 5 and 6. The former occur during the first circuit, and the latter during the second. Corresponding to this phase, we find the remaining two branches of the "1 to 2 yrs" group in Fig. 4.

For the short-period orbits (about 1/2 year), both Nonsymmetric (Fig. 2) and Symmetric (Fig. 4) groups exist: the former involving trips of exactly 1 year, and the latter 1 to 2 years. The vehicle executes between two and three solar circuits during each Symmetric mission, and exactly two circuits during the Nonsymmetric flights.

Perihelia of all nominal orbits in this class fall rather close to the Sun, generally lying within 0.26 AU. Although obviating use of these orbits for many missions, this property nevertheless enhances their attractiveness for use as solar probes (Ref. 4). When it is recognized that the orbits of close solar probes, by their very nature, require great communication distances at the time of perihelion passage, and that solar background noise is apt to be intolerably large at such times, then the possibility of storing information for future readout during a close approach to Earth becomes well worth considering. It is primarily in this light that future views of this class of orbits will be considered.

The Nonsymmetric orbits negotiate precisely two solar circuits during each mission and, by reasoning analogous to that described above, provide eight different choices for outbound legs.

For flights past Mars, the following two regions are of particular interest:

- Nominally low-energy trips, having durations of 1 to 2 years
- Nominally high-energy trips, having durations of 1 to 2 years

In the nominally low-energy trips, both Nonsymmetric (Fig. 3) and Symmetric (Fig. 5) groups exist: the Nonsymmetric involve trips of exactly 2 years; the Symmetric represent trips of 1 to 2 years in length.

The Nonsymmetric trips are marked "2 yrs" in Fig. 3 and possess orbital periods of precisely this length. Four outbound-leg possibilities exist, and these are represented by the two branches of this family depicted in Fig. 3. As in the case of the Venus trips discussed earlier, each outward-launched pair of trips contributes two points to the left-hand branch, and vice versa for the other pair. Beyond the critical value of eccentricity, all transfer ellipses fail to reach the Earth's orbital radius, and no solutions exist. Particular orbits from this group have previously been treated by a number of authors, including Sternfeld (Ref. 5) and Vertregt (Ref. 3).

The Symmetric trips are labelled "less than 2 yrs" in Fig. 5. To generate this family, let $m = 0$, $p = 1$ in Eq. 3 and let 2α run from 0 to 2π; the orbits then produced perform up to one solar circuit in one-and-a-fraction years. All flights proceed outwards from the Earth towards Mars and return headed inwards from Mars. Figure 7 traces the generation of this family of orbits.

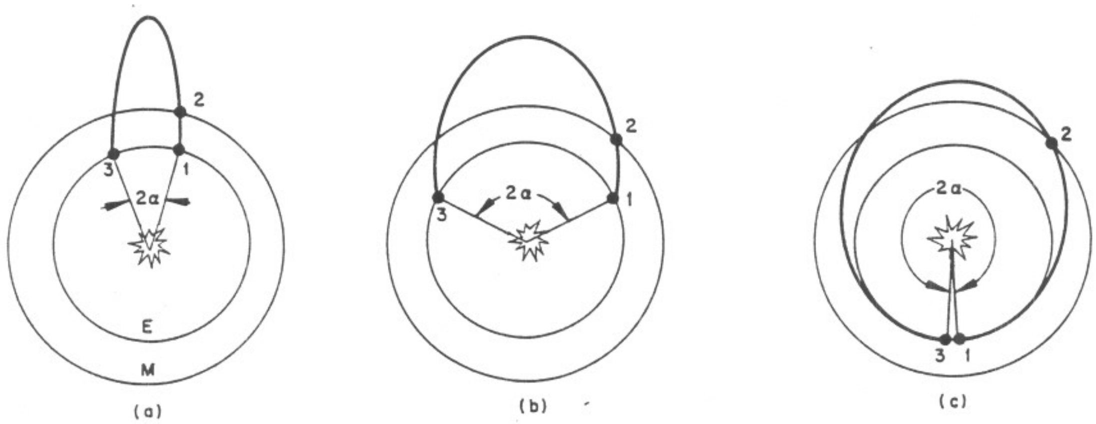

Fig. 7 Successive Phases in the Generation of Low-Energy Symmetric Trips Past Mars

When 2α is small (Fig. 7a), the trip time is approximately 1 year. Trip time increases with the travel angle until, at $2\alpha = 180$ deg, it reaches the 2-year limit (Fig. 7c) imposed earlier. This 2-year orbit passes through 360 deg and is, therefore, common to both the Symmetric and Nonsymmetric families; it is marked by an X at each of two reciprocal points, (-86, 40) and (-140, 464), in Figs. 3 and 5.

Note the abrupt termination of the Symmetric-trip curve at these two common points. For these orbits $2\alpha = 360$ deg. Were 2α to be increased still further, or, what amounts to the same, if m = 1, p = 2 in Eq. (3), these curves would have continued on smoothly beyond the common points. But this would have brought into consideration trips of greater than 2 years' duration, a violation of the assumed limitation on maximum trip time.

For the nominally high-energy trips (durations of 1 to 2 years), both Nonsymmetric (Fig. 3) and Symmetric (Fig. 5) groups exist: the Nonsymmetric group involves trips of exactly 1 year; the Symmetric represents trips of approximately 1-1/2 to 2 years.

The Nonsymmetric trips, marked "1 yr" in Fig. 3 are precisely the same set of orbits as the 1-year group investigated for flights past Venus in Fig. 2, however, they are recalibrated here for launch and arrival dates appropriate to Mars intercept, rather than for Venus. Beyond the critical value of eccentricity, $e = R_{♂} - 1$, the transfer ellipses fall short of Mars' orbit and, as before, no further solutions exist.

The Symmetric trips contribute only one branch to Fig. 5, this being labelled as "less than 2 yrs." To generate this family, let m = 1, p = 1, in Eq. (3), and let 2α run from 0 to 2π. The orbits thus produced perform one-and-a-fraction solar circuits in one-and-a-fraction years. (See Fig. 8.)

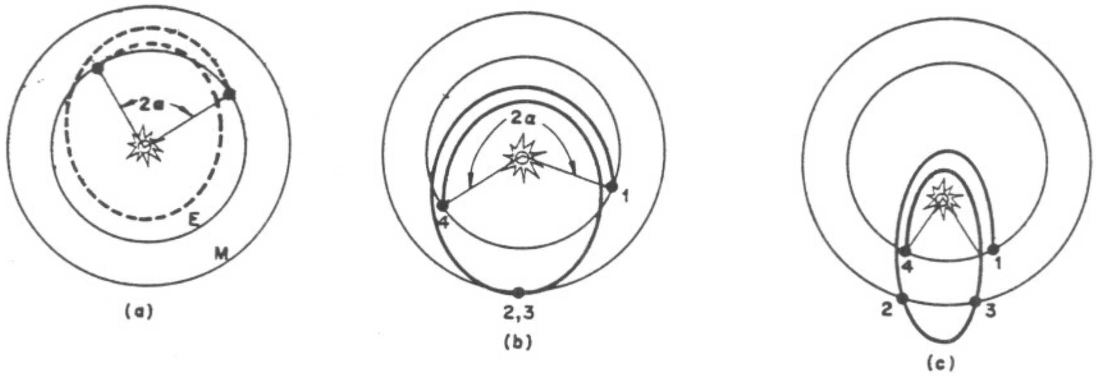

Fig. 8 Successive Phases in the Generation of High-Energy Symmetric Trips Past Mars

This set of flights is none other than the high-energy Nonsymmetric-trip group discussed above for flights past Venus. In the present case, however, all ellipses associated with small values of 2α fall short of Mars' orbit (Fig. 8a). Only after 2α reaches its critical value (about 200 deg) do legitimate solutions appear (Fig. 8b). These are produced in pairs, one for each point of encounter with Mars (Fig. 8c), and generate the single curve of this family in Fig. 5.

CLOSE APPROACH MANEUVERS

Having located, in a coarse sense, all areas of acceptable nominal flights, it now remains to attempt modifications in trip time, terminal speeds, and other factors by incorporating close approach maneuvers,* into the flight schedules. These maneuvers will naturally be desirable for the performance of reconnaissance operations during each encounter.

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*The guidance impulse requirements necessary to properly orient the pass hyperbola at the planet are assumed to be insignificant for present planning purposes.

In Figs. 9 through 12, each promising area is displayed on a magnified scale, with contours of various constant approach distance, measured in local planetary radii, and overlaid on hypobolic excess speed curves. Since the nominal flights discussed earlier were assumed to be completely unperturbed, the curves of Figs. 2 through 5 are to be interpreted here as contours of infinite passage distance.

Trajectories which are bent toward the Sun by the planetary encounter are viewed as having passed through local periplanet on the dark side of that planet. By the same token, an outward-bent trajectory will pass on the sunlit side of the planet. Thus, a curve marked, say, 1_D signifies passage at one planetary radius on the dark side, while 1_L stands for a lightside, 1-radius passage. It is further understood that by the term "lightside passage" we merely refer to the fact that periplanet occurs somewhere above the illuminated surface of the planet, whether it be in full sunlight or near the terminator; similar remarks apply to the "darkside" connotation. Any final decision on the degree of darkness tolerable during a given flight will depend upon the mission specifications as well as the time history of illuminated-surface visibility during that encounter. For this reason, we shall include both darkside and lightside passages in this phase of the study.

The close-approach analysis is performed by comparing separate one-way trips to and from each planet, evaluated on a grid of dates in the neighborhood of the nominal flights, still retaining the circular, coplanar orbit geometry. By noting the planetary right ascensions and declinations of the vehicle approaching and receding at the same speed on any given date, the asymptote deflection angle may be calculated using spherical trigonometry. If we call this deflection angle K , then (from Ref. 6) the periapsis distance of the pass hyperbola is readily shown to be

$$r_p = \frac{m}{m_\odot}\left[\frac{\csc\frac{K}{2} - 1}{\nu_\infty^2}\right]$$

where $m/m_\odot$ is the planetary mass in solar units, ν_∞ is the hyperbolic excess speed associated with the pass hyperbola, expressed in EMOS (Earth Mean Orbital Speed) units and r_p is delivered in AU .

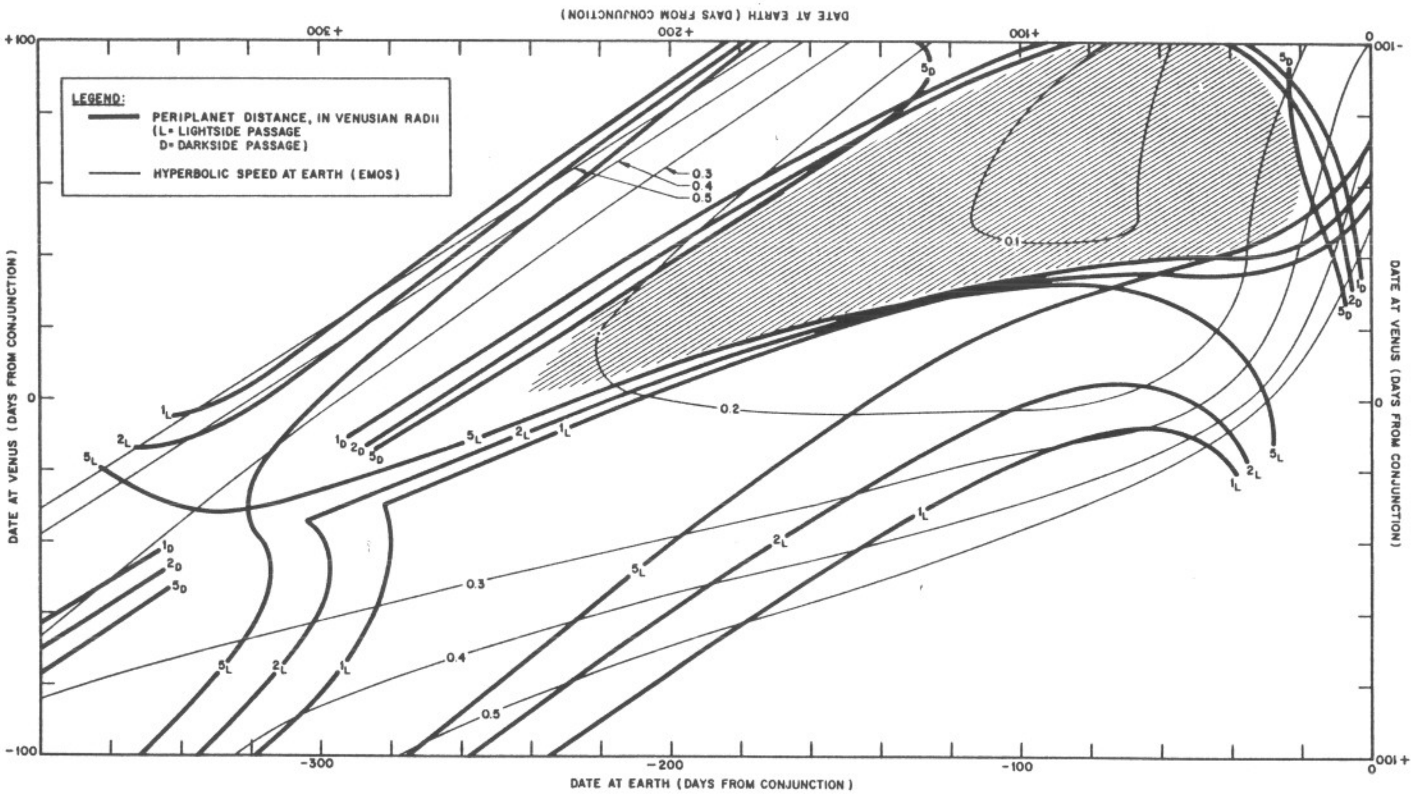

Fig. 9 High-Energy Trips Past Venus, Modified by Close Approaches at Various Distances

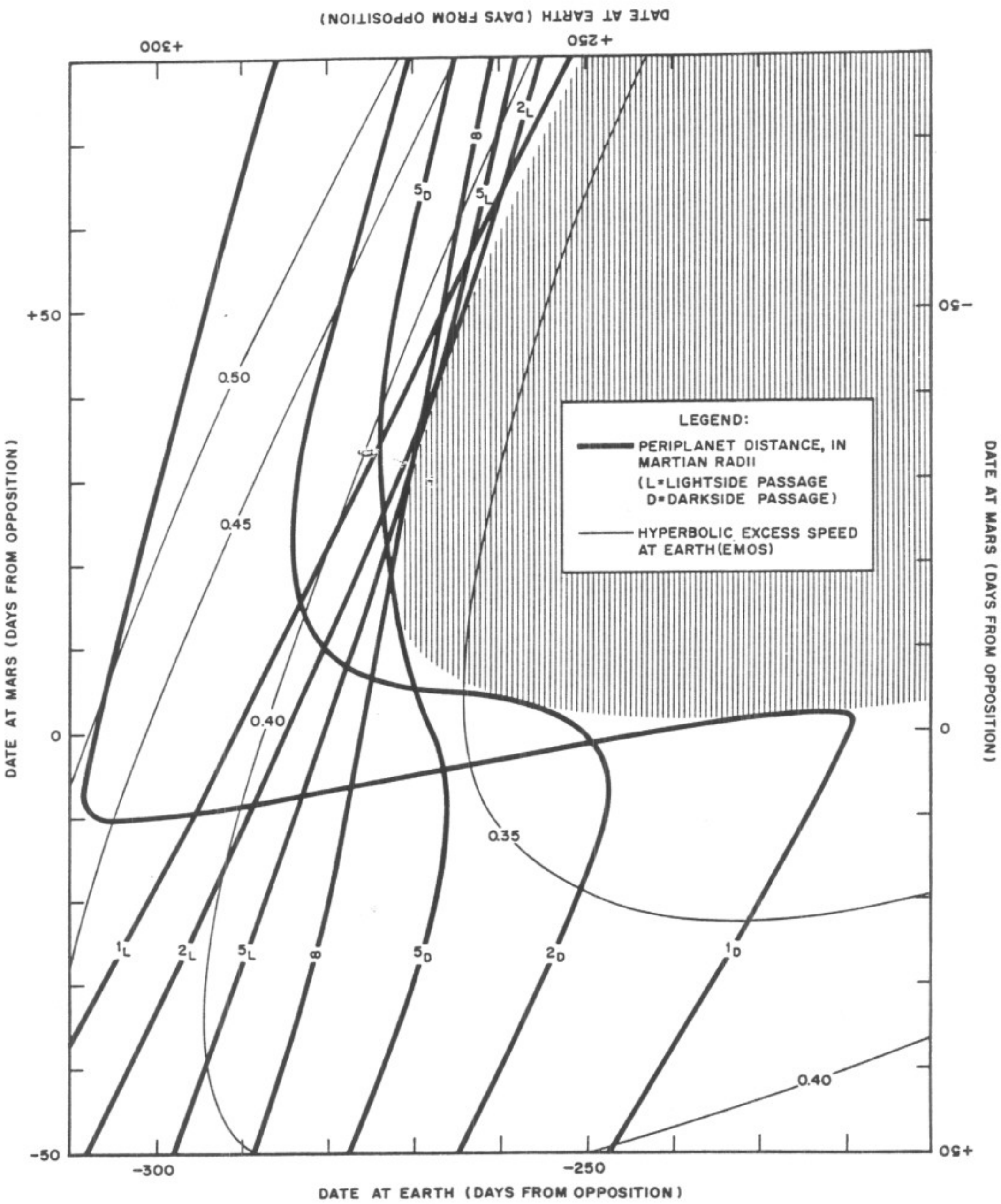

Fig. 10 Symmetric High-Energy Trips Past Mars, Modified by Close Approaches at Various Distances

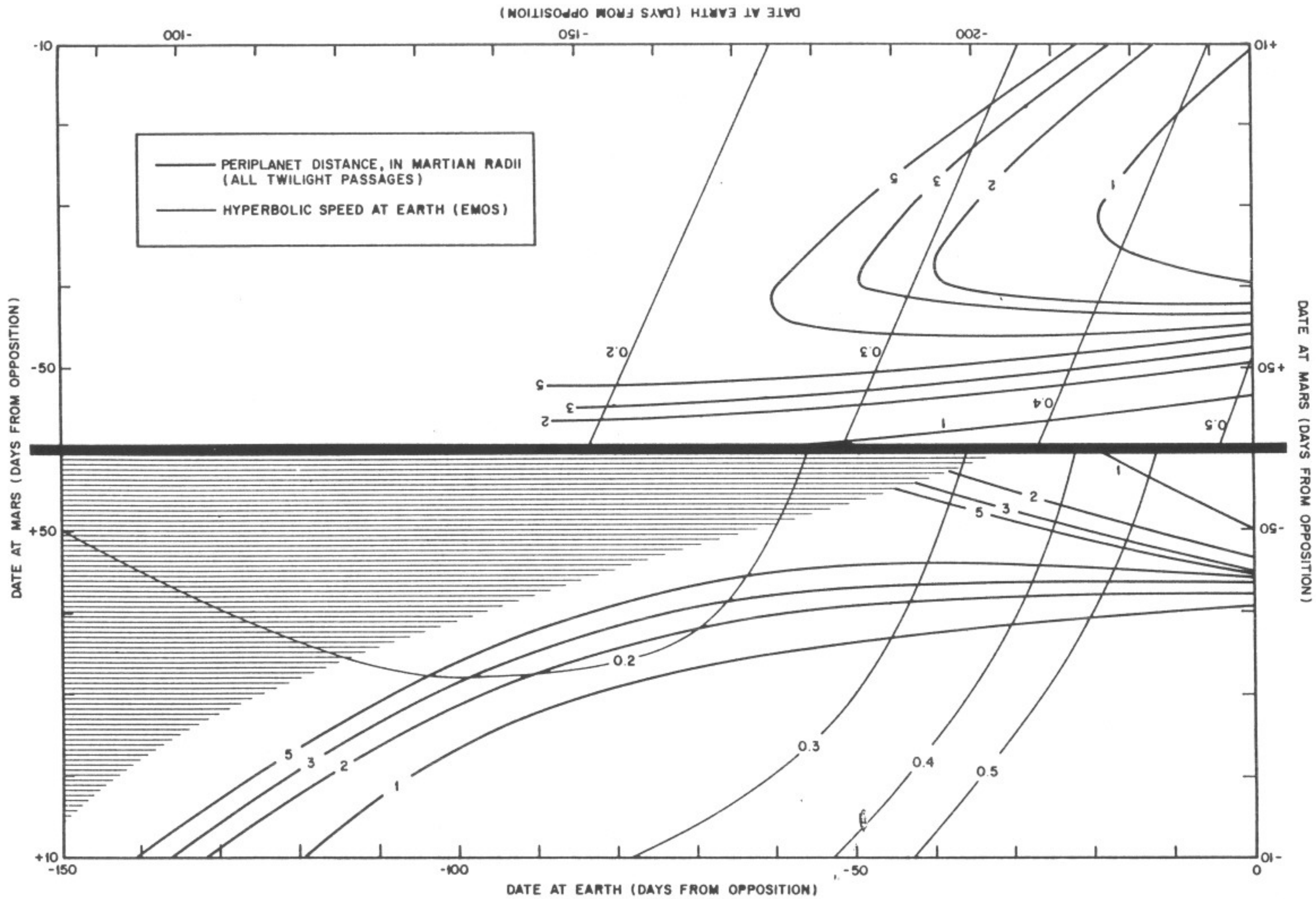

Fig. 11 Symmetric Low-Energy Trips Past Mars, Modified by Close Approaches at Various Distances

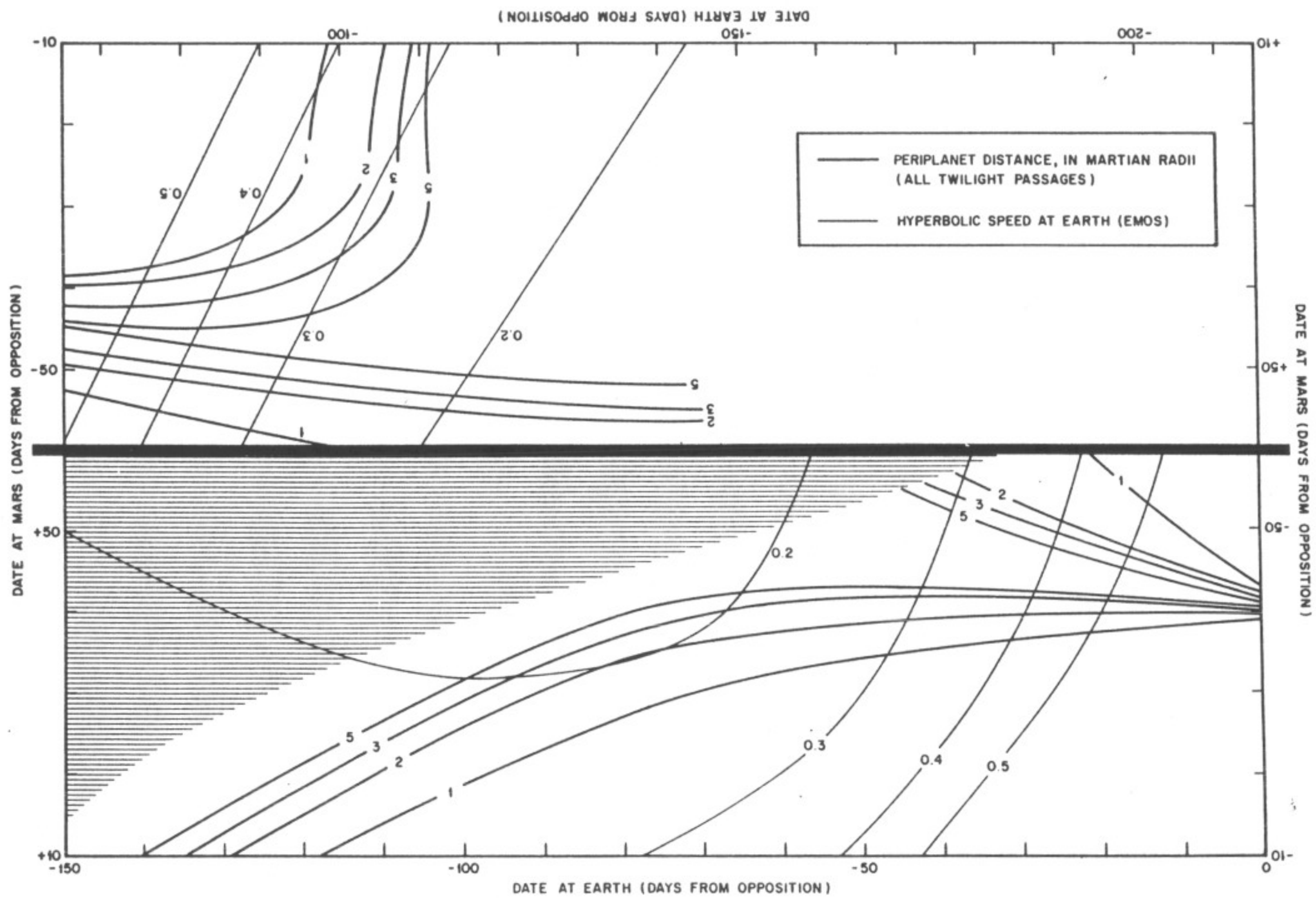

Fig. 12 Nonsymmetric Low-Energy Trips Past Mars, Modified by Close Approaches at Various Distances

Fig. 12 Nonsymmetric Low-Energy Trips Past Mars, Modified by Close Approaches at Various Distances

EXCLUDED REGIONS AND AN ENVELOPE THEOREM

For each family of flights there exists a region within which no nonstop round-trip trajectory may be found. These "excluded" regions are presented as shaded areas in Figs. 9 through 12. They occur whenever an outbound leg reaches the target planet with a speed so low that no return leg may be found leaving the planet on that day at the identical speed; i.e., all trips leaving the planet on that day fall short of the Earth's orbit. Analogously, portions of these regions are also generated when return-leg segments leave the planet at such a low speed that no outbound leg can reach it on that day with the same speed.

From inspection of Figs. 9 through 12, these regions appear to be bounded by the envelopes to the curves of constant close-approach distance. This is indeed the case, and may be demonstrated as follows:

Theorem

Each excluded region, within which no nonstop round-trip trajectory may lie, is bounded by the envelope to the curves of constant periplanet distance.

Proof

Consider the case for which the approach speeds at the target planet are so low that no matching return trips exist which would reach the Earth; the opposite case follows by similar reasoning.

Refer to Fig. 13, which represents passage speeds for all trips passing the target planet on date D_1. Departure dates are designated by D_o, and return dates at

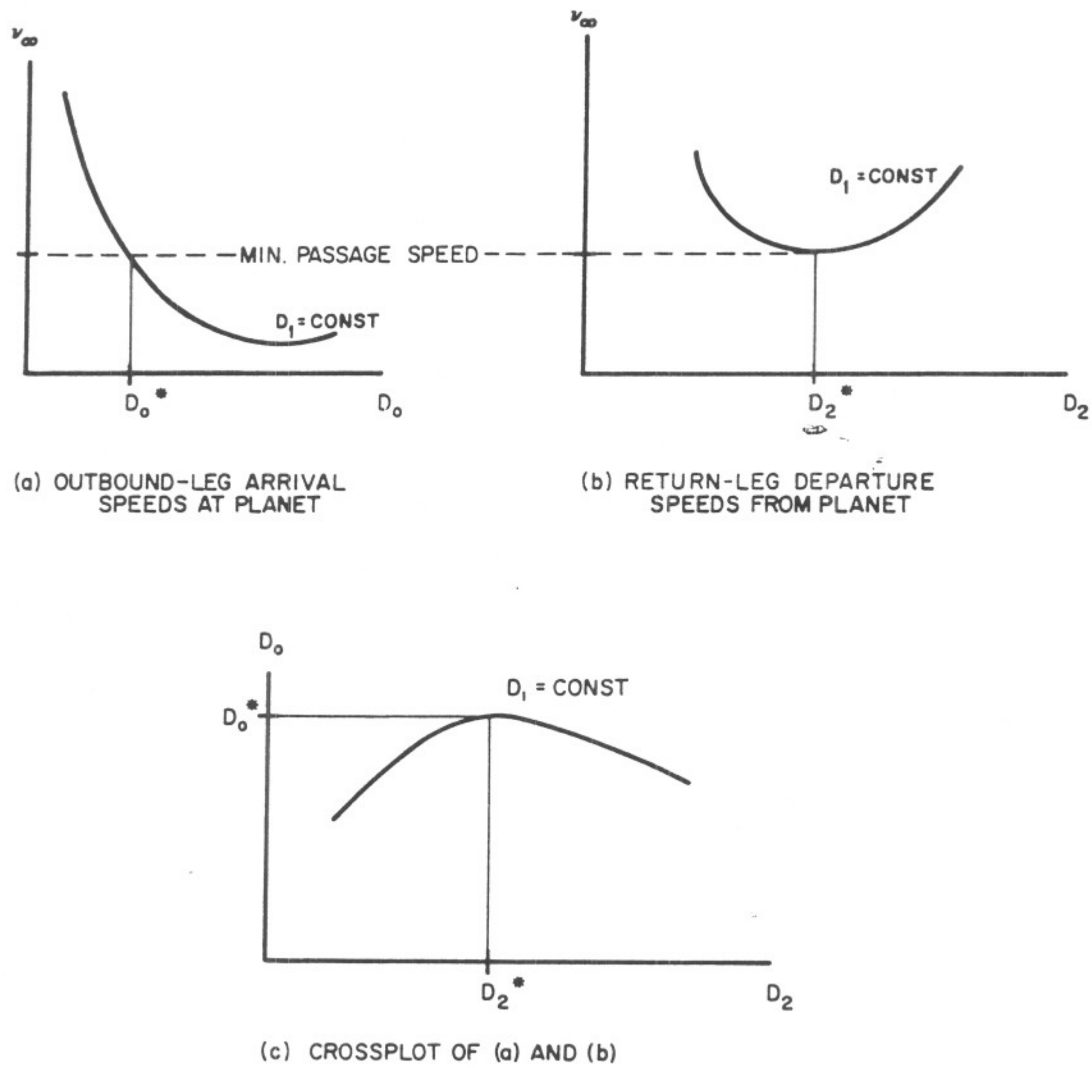

Fig. 13 Envelope Theorem Notation

Earth D_2 . Figures 13a and 13b portray the fact that, beyond departure date D_o^*, no return legs are available, since these require speeds of departure from the planet which are lower than the minimum in Fig. 13b. Now, since ν_∞ is double-valued in D_2 , while D_o is monotonic in ν_∞ , it therefore follows that D_o is double-valued in D_2 (Fig. 13c). Then the expression

$$\left(\frac{d\,D_o}{d\,D_2}\right)_{D_1} = 0$$

describes the boundary of the excluded region. Both D_o and D_2 may be viewed as continuous functions of r_p on any particular passage date, so that this latter condition may be written as

$$\left(\frac{d\,D_o}{d\,D_2}\right)_{D_1} = 0 = \frac{\left(\frac{d\,D_o}{d\,r_p}\right)_{D_1}}{\left(\frac{d\,D_2}{d\,r_p}\right)_{D_1}}$$

which in turn implies that at least the numerator vanishes.† But $\left(d\,D_o/d\,r_p\right)_{D_1} = 0$ is precisely the definition of the envelope to the curves of constant periplanet distance; this completes the proof.

HOW TO READ FIGS. 9 THROUGH 12

Before proceeding with the inspection of Figs. 9 through 12, it would be wise to review the procedure for extracting information from them. The method rests on the

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† The case where both numerator and denominator are infinite can be shown to be invalid.

Reciprocity principle outlined earlier, and presupposes that all planetary orbits are coplanar circles.

Turn to any one of these figures, and read only the numbers which appear right-side up. First select a distance, in planetary radii, and side (lightside or darkside) at which the point of closest approach is to occur. This locates a heavy curve, or curves, on the graph. Next choose a point on this curve which appears to be attractive in terms of speed, date or any other criterion. This point may then be characterized by (D_o, D_1) the dates of departure and passage, both measured from the day of planetary alignment. The speed of departure may be estimated by reading the thin (speed) curves between which this point falls.

Turn the figure upside-down, and again read only those numbers which appear right-side up. Following the same heavy curve selected above,* find the point corresponding to the passage date D_1 now. The abscissa of this new, "reciprocal" point represents the date of return to Earth,† and the return speed may, as before, be found by inspecting the nearby thin curves.

The following information has now been extracted: departure date, passage date, return date, departure speed, return speed, passage distance, and side of passage (viz. light or dark). Also, by implication, one may easily determine first- or second-leg trip time, and total trip time. Two or three practice exercises will suffice to acquaint one with this process, and the use of these curves will generally provide the analyst with a rapid, simple way to perform rough estimates of the properties of all trips in each family.

CHARACTERISTICS OF THE PERTURBED SOLUTIONS

Figure 9 presents the region of high-energy trips passing Venus, modified by planetary approaches at various distances. For these curves, the relatively large mass of

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*This may, in some cases, turn out to be the other branch of the curve chosen.
† Modulo S , the synodic period.

Venus has caused the Nonsymmetric family of orbits to merge with the shorter group of Symmetric trips (Fig. 6a). The most important result of this modification is that the resulting "stretched" contours now extend into the area near the Hohmann transfer point. Trajectories from this group have been studied previously by Battin (Ref. 6), Bollman (Ref. 7), and Gedeon (Ref. 8).

Viewed from a different aspect, we may consider that, by close approaches to Venus, many nominally unacceptable low-speed trajectories have been perturbed in such a way as to bring them back successfully to the Earth. Inspection of Fig. 9 reveals that the return speeds of trips in this neighborhood are not unduly high (about 0.25 to 0.30 EMOS) and also that the return speeds are almost insensitive to the choice of trajectories in that local area.

By reversing the choices of outbound and return-leg regions, a group of trips which leave at speeds of about 0.25 to 0.30 EMOS and return at almost-Hohmann values may also be obtained.

Total trip times for practical missions from this region range from about 300 days for the most ambitious to about 400 days for the simplest. These promising orbits are all characterized by lightside passages at Venus. Generally speaking, they represent perhaps the most attractive trips available for interplanetary exploration, combining low terminal speeds with good planetary visibility, ample launch intervals and very reasonable durations. And, since the orbits of Earth and Venus are nearly circular, we are certain to find some such trips during every synodic period.

Because these trips are so satisfactory, no further analysis will be made of the final group of high-energy missions, depicted in Fig. 6f, which involve considerably longer trip times and widely increased launch sensitivities.

Discussion of the short-period trips past Venus, which seem of interest primarily for the launching of solar probes, is discussed in the final section of this study.

Consider now trips passing Mars. Because of its low mass (about 0.13 that of Venus), the effect of close approach maneuvers will be much less pronounced here. Figure 10 presents the modified Symmetric high-energy trips. Previous inspection of the unperturbed results in Fig. 5 has shown that, because of the high terminal speeds associated with these orbits, only the area near the minimum values would be of interest. This area is shown in detail in Fig. 10. Both darkside and lightside passages are available, the former being more advantageous in terms of speeds and duration, the latter in terms of planetary visibility.

According to the discussion of the unperturbed case, we have here a group of orbits which pass Mars near their aphelia. When we include Mars' orbital eccentricity in later calculations, we can therefore expect the requirements for these missions to be highly sensitive to the year of launch. That is, since the transfer orbits barely reach Mars, the terminal speeds will depend strongly on whether this contact is made near Mars' own perihelion or aphelion. This variation is evident in the dated curves included in Ref. 10; such evidence is strong enough to remove this group of trips from practical consideration during all but two or three synodic periods during every 17-year cycle of oppositions.

By the corollary following the Reciprocity Theorem, all trips from this group which pass Mars at opposition ($D_1 = 0$) possess identical outbound and return legs; these trajectories preserve their symmetry even under the close-approach perturbation.

Figures 11 and 12 show requirements for modified low-energy missions past Mars; Fig. 11 displays Symmetric trips, Fig. 12 Nonsymmetric trips. Of the two groups, the former involves shorter trip times, as can be expected from the unperturbed results. Perihelia of these orbits all extend beyond 1.8 AU and periplanet always occurs near the terminator – i.e., "twilight" passage. Since planetary contact occurs well away from aphelion, a greater degree of yearly stability can be expected for these trips than for the high-energy group discussed previously. Terminal speeds for the least ambitious trips reach values reasonably near the minimum energy levels, and we may therefore expect this group of trips to play a prominent part in early round-trip flights to Mars.

Figure 14 shows various regions for modified Nonsymmetric high-energy trajectories passing Mars. All acceptable missions must lie outside both the excluded region and the region for $r_p < 1$. Inspection of the speed contours, and comparison with the results cited in Ref. 9 will verify that the mass requirements for these trips could be much more profitably applied to the performance of short-duration capture missions; this holds true even during the most favorable years, at which times the terminal speed requirements will not diminish by more than 0. 1 EMOS each. We shall therefore omit this group of trajectories from further consideration.

REDUCTION OF ACCEPTABLE AREAS

The arguments presented so far have served to focus our attention on three groups of trips: one past Venus and two past Mars.

For the Venus trips, two alternative regions are available: one involves very low departure speeds and moderate return speeds, while the reverse is true for the other, its reciprocal. Of these two, the former is preferable: first, because its lower departure requirements make it superior for missions which do not require payload recovery at Earth; secondly, because its shorter outbound legs assure a greater probability of partial mission success; and finally, because intensive study of recovery flights (Ref. 10) again confirms its superiority in terms of overall mass requirement. The moderate-departure, low-return speed group may therefore be eliminated.

Considering the low-energy trips past Mars, analysis of Figs. 11 and 12 reveals that the Symmetric trips affort shorter trip times than the Nonsymmetric, without suffering high increases in speed requirements. The Nonsymmetric group may thus be excluded from further study also.

Within the Symmetric group, there are again two reciprocal regions, namely a short outbound leg and long homebound leg, and vice versa. But by the same reasoning as was advanced in the discussion of the Venus trips, we may confine our attention solely to trips with short outbound legs.

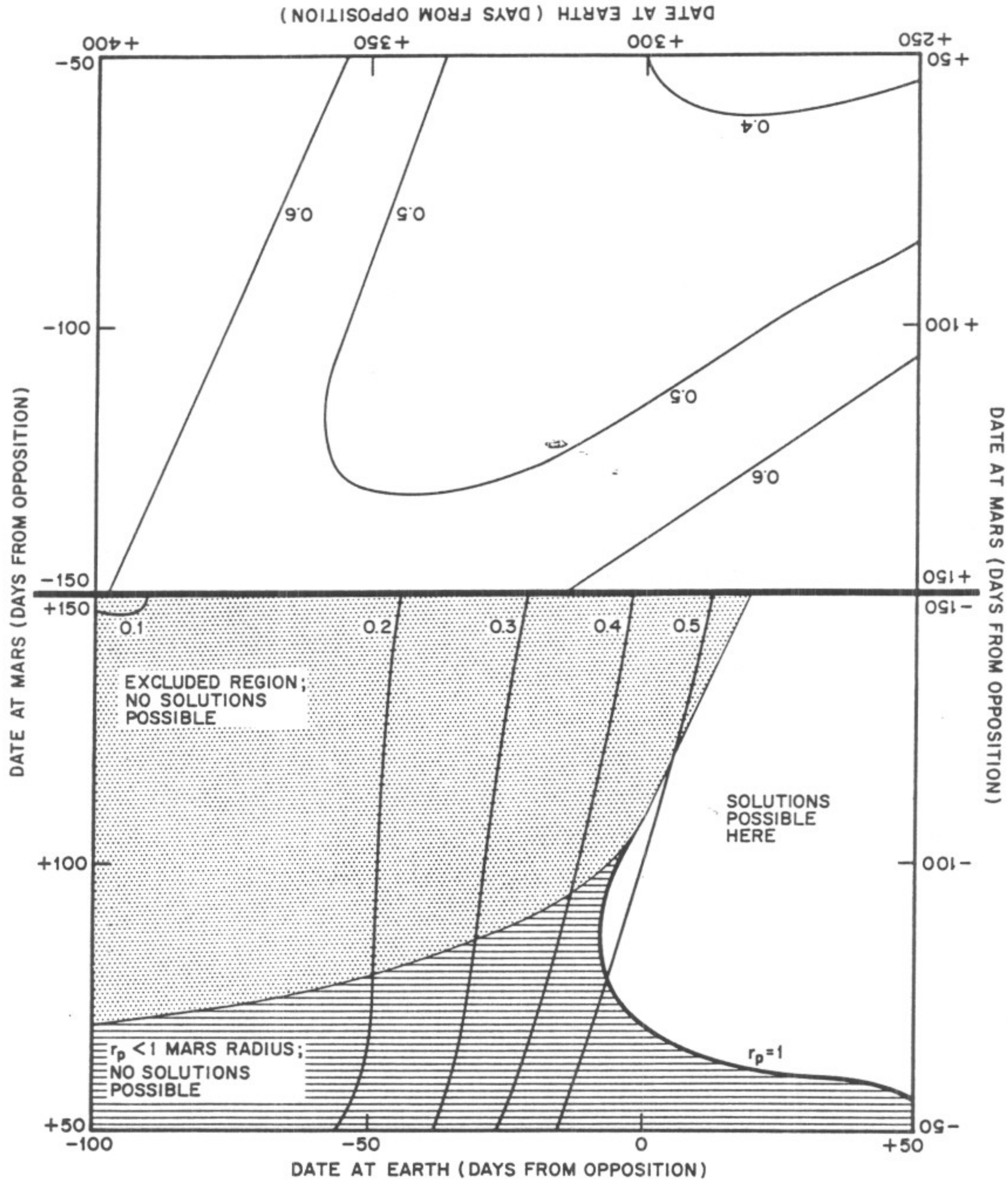

Fig. 14 Regions for Modified Nonsymmetric High-Energy Trips Past Mars

The high-energy Symmetric group shows promise for some missions at least, and will therefore be retained for future analysis.

The search for acceptable nonstop interplanetary round trips has now been narrowed to three small areas, one for Venus, and two for Mars. Curves presented in the Appendix display sample data for these areas during the years 1964 – 1965.

MISSION REQUIREMENTS FOR DATED PERIODS

The data presented in the Appendix takes into account the orbital eccentricities and mutual inclinations of the Earth, Venus and Mars. Because of this, the Reciprocity principle no longer holds, and outbound and return groups must now be displayed on adjacent, but separate, graphs. Use of these curves, however, is not difficult, and proceeds in a manner very similar to that employed previously. Turning to any one of these figures, it will be noted that a heavy diagonal line divides the page into two halves, the left pertaining to outbound legs and the right to return legs. As before, the process begins with the selection of a side and distance of passage. This choice locates one heavy curve on the left side of the page and another on the right. Choose a point on the left-hand curve; for the associated trip, the dates and speeds of departure and passage may be read as before. Now, instead of inverting the page to obtain return information, merely project the ordinate (date of passage) horizontally across the page until it meets the other heavy curve. This new point defines the return leg. All graphs in the Appendix are used similarly. To aid the mission analyst, speeds of passage have also been included on these charts. They are represented as light, dashed-line contours.

To reduce the amount of calculation required, natural symmetries and periodicities may be exploited. First we note that, at every sixth conjunction (about 8 years apart), the Earth and Venus occupy almost the same positions on their orbits. For present planning purposes, then, we need only calculate trajectories for an 8-year span, and be assured that the data will be valid also for the 8 years preceding and the 8 years following. The data for Venus presented in the Appendix is accordingly calibrated on three separate scales, referring to the periods 1962–1970, 1970–1978 and 1978–1986. Numerical checks have verified the accuracy of this substitution.

For the trips past Mars, a modified Reciprocity principle holds. Note that the opposition of 25.3 Feb. 1980 (Julian Date 244 4294.8) occurs when Mars is within 0.05 deg of its aphelion. Assume the Earth's orbit to be circular and Mars' to be a coplanar, confocal ellipse. Then, by reasoning analogous to the proof of the Reciprocity Theorem, it may easily be shown that any outbound leg with associated dates $(-D_0, -D_1)$ corresponds to a return leg $(+D_1, +D_0)$, where these dates are measured from the 1980 opposition; similar arguments hold for speed requirements, etc. This property also holds for trips with close approach maneuvers, and for stopover missions as well.

While this symmetry does not allow us to combine departure and return information on a single graph, it does permit us to present, with no extra labor, trip information for the years 1980–2000 by merely inverting the curves for 1960–1980. This recalibration is performed on the high-energy data in the Appendix; holding the page either right-side up or upside-down, the reader merely employs those numbers which appear erect.

We note in passing that although an approximate 15–17 year periodicity exists in the Mars opposition cycle, this repetition is not precise enough to permit its employment in the Appendix figures.

Some very brief generalities apply to the figures. First, note that the presence of the 180-deg "ridge" in the Venus graphs has caused nearby contours of constant approach distance to follow the ridge in one direction or another. This is due to the generation of new families of orbits in these localities. Such regions display high launch sensitivities, however, and are usually to be avoided.

The same phenomenon is evident in some localities within the low-energy Mars curves. Occasionally, this difficulty may be circumvented by employing a major midcourse maneuver to "break" the transfer plane near the line of nodes between the two planetary orbits (Ref. 11), although this technique may be of questionable value unless the marginal capability presently envisioned for these flights can be improved.

The severe yearly variation in requirements for the high-energy symmetric trips follows the earlier prediction. Dramatic changes in mass requirement for manned missions which employ these trajectories may be observed in the curves presented in Ref. 10.

MULTIPLE-PLANET FLYBYS

NONSTOP TRIPS PASSING BOTH MARS AND VENUS: THE INTERPLANETARY GRAND TOURS

Investigation of nonstop round trips to Mars or Venus quite naturally leads to the question of whether any orbits might be found which pass near both planets during an uninterrupted ballistic flight. Any round-trip orbit whose aphelion exceeds the maximum orbital distance of Mars, and whose perihelion falls within the minimum distance of Venus is a likely candidate for such a mission, providing that the analyst can afford to wait for the proper relative alignment of the three planets for that trip.

A more useful exercise, however, would be to search for all such trips which exist during any given calendar period, and to select those flights which show hope for execution, keeping in mind the realistic limitations of propulsion technology. To attain this end, it would be advisable to establish a preliminary model for the solar system in which all planets move in coplanar circles (as was done in the single-planet flyby). The relative geometry between any two planets would then be strictly repetitive and systematic generalizations could be made of the dynamical phenomena involved. The results from such a preliminary study will lie reasonably close to their true values, and may be used as first estimates for more realistic models for the planetary motions.

A number of symmetry features apply to these multiple-planet flyby flights. The synodic period between Earth and Venus is about 584 days, while for Earth and Mars it stands at 780 days. In 2336 days, then, four Venusian conjunction periods will occur, while three Martian cycles will span 2340 days. It may be concluded, therefore, that, to the degree of accuracy inherent in this preliminary model, the three planets will occupy the same relative positions about every 2338 days; i.e., about every 6.4 years. Having once located any group of trips for some particular year, one may then be assured of finding the same set of orbits at least during the previous cycle and during the next future cycle. In this way, at least twenty calendar years may be explored with little more effort than six. Of course, the new trips will agree with the old only in a phenomenological sense, since the eccentricities and inclinations of the true planetary orbits will cause the velocities and trip durations to vary somewhat from cycle to cycle. However, the method is accurate enough to serve as a coarse locator, and has been found quite useful in many practical instances.

Let us further suppose Earth and Venus to be moving in coplanar circles, but consider Mars' orbit to be an ellipse which is confocal and coplanar with the two circles. Then, if a date can be located upon which Mars occupies an apse of its orbit, while the other two planets lie on the major axis of Mars' orbit, it can easily be verified* that the Reciprocity principle holds for these multi-legged trips also.

Such a date having been found, only half of the 2338-day cycle need be explored, since the remainder of the trips may be generated by reciprocity. Once again it should be noted that the slight aberrations in the process due to the eccentricities of the two inner planets' orbits, as well as to their mutual orbital inclinations, will necessitate a final close study, but the area of search will have been greatly reduced by the exploitation of this symmetry.

On 17 August 1971 (JD 244 1180), the heliocentric longitudes of the three planets are found to be as follows:

Venus: 136°
Earth: 323°
Mars: 321°

This configuration is sufficiently close to the required alignment to adopt it as a practical reference date of symmetry. By then exploring the calendar interval from 1970 to 1974, the entire period 1962–1982 will have been covered. What remains now is to generate all multi-legged flights of interest. Again, a two-year maximum limit is imposed on trip duration. The method proceeds as follows: Suppose that a certain trajectory from any ellipse family passes Venus. Suppose also

*By reasoning completely analogous to that advanced for the single-planet flybys.

that another ellipse from the same family passes Mars. Suppose, finally, that the departure dates and the heliocentric departure angles associated with each of these two ellipses are identical. Then, since these two independent quantities as well as the parameter which defines the given ellipse family all agree, it follows that the two trips must be one and the same. That is, an unperturbed orbit passing both Mars and Venus during a single journey will have been determined.

By following the procedure adopted earlier for the ordinary (two-legged) round trips, we are assured that all nominal ellipse families lying within the required trip-time limit can be located. The following groups, in particular, will be investigated:

- High-energy symmetric trips.
- High-energy nonsymmetric trips. Contained in this group is the one-year round-trip trajectory studied by Crocco (Ref. 12).
- Low-energy nonsymmetric trips. The symmetric trips never dip inside the Earth's orbit and consequently cannot pass Venus.

The short-period group does not reach Mars' orbit and is, therefore, not considered here. The desired orbits may now be found graphically. For every trajectory family considered, all ellipses which pass from Earth to Earth in the specified time are generated using Lambert's Theorem. When the mission duration is an integral number of years, a complete family of ellipses (i.e., the Nonsymmetric trips) exists for each mission time specified, and these may be represented by the orbit period P . All members of each such family may be found, as before, by employing the Earth-Sun-Mars angle L as the generator. For the Symmetric trips, if p denotes the number of complete years in any trip, and m the number of complete orbital circuits, then the value of p/m again defines each set of flights.

For each family then, the heliocentric departure angle ψ may be plotted against departure date (measured from date of planetary alignment) in separate graphs for Mars and for Venus flights. By superimposing the corresponding Mars and

Venus plots for each family and aligning the conjunction and opposition marks to correspond in spacing to the calendar period of interest, points at which the two curves cross are acceptable orbits. That is, these intersection points are associated with identical values of departure date, departure direction, and ellipse factor for trips to Mars and to Venus; therefore, the orbits must be the same. Tables 1, 2, and 3, list the nominal trips in each of the three families.

It must be emphasized that the dates and speeds listed in the tables are only nominal. Modifications in both sets of values may be achieved by varying the synodic period and/or the close-approach distance at each planet.

From an examination of the tables, it appears that Trip No. 2 in Table 3 might be a promising choice. This trajectory's aphelion, it will be recalled, barely reaches Mars' orbit, near the date of opposition. Therefore, some reduction in speed requirement can be expected if the trip is chosen in the 1970–71 period, when Mars is near its perihelion at opposition. Further reduction may be possible by advantageous selection of the planetary approach maneuver.

For illustrative purposes, a brief search was carried out in the region indicated; the following sample trajectory was determined:

Depart Earth:	3 Dec 1970	(JD 244-0945);	speed 0.278
Pass Mars:	10 Nov 1971	(JD 244-1265);	speed 0.215
Pass Venus:	7 Jun 1972	(JD 244-1475);	speed 0.354
Return Earth:	21 Aug 1972	(JD 244-1550);	speed 0.167

Mars passage occurred at 1.6 planetary radii, on the sunlit side, while Venusian passage was at 3.0 radii, also on the sunlit side.

Table 1

NOMINAL 1-YEAR MULTILEGGED FLYBYS

Trip No.	Depart Earth	Pass Venus	Pass Mars	Pass Venus	Arrive Earth	Departure Speed
	(All dates are JD minus 244-0000)					(EMOS)
1*	1161	–	1291	1498	1526	0.540
2	1323	1376	1431	–	1688	1.333
3	1371	–	1616	1674	1736	1.101
4	1464	1481	1705	–	1829	0.881
5	1596	1688	1861	–	1961	0.565
6	1606	1696	1881	–	1971	0.591
7	2001	–	2051	2298	2366	0.966
8	2083	–	2277	2368	2448	0.782

*Crocco's trajectory (Ref. 12) is contained in this group.

Table 2

NOMINAL 2-YEAR MULTILEGGED FLYBYS

Trip No.	Depart Earth	Pass Venus	Pass Mars	Pass Venus	Arrive Earth	Departure Speed
	(All dates are JD minus 244-0000)					(EMOS)
1	0896	0930	1052	–	1620	0.582
2	0904	0943	1063	–	1634	0.530
3	1000	–	1627	1674	1730	1.249
4	1170	–	1212	1827	1900	0.893
5	1206	1265	1901	–	1936	1.177
6	1318	1367	1412	–	2048	1.402
7	1467	1482	1583	–	2197	1.068
8	1578	1655	1723	–	2308	0.733
9	1710	–	2320	2373	2440	1.025
10	1916	–	1981	2596	2646	0.508
11	1941	–	1977	2655	2671	1.003
12	2290	2357	2916	–	3020	1.028

Table 3

NOMINAL HIGH-ENERGY SYMMETRIC FLYBYS

Trip No.	Depart Earth	Pass Venus	Pass Mars	Pass Venus	Arrive Earth	Departure Speed
	(All dates are JD minus 244-0000)					(EMOS)
1†	0906	0959	1259	1421	1474	0.437
2†	0922	0960	1309	1405	1500	0.529
3	1058	–	1619	1668	1729	1.179
4	1318	1369	1404	–	2028	1.419
5	1435	–	1556	2032	2092	1.113
6	1466	1534	1592	–	2105	0.978
7	1588	1618	1756	–	2181	0.646
8	1771	–	2276	2392	2410	0.978
9	1790	1852	2336	–	2451	1.154
10	1905	1956	2520	–	2615	1.419
11	2145	–	2242	2793	2843	1.362

† Trips 1 and 2 each pass near Venus on both first and second solar circuits. Passage of Venus either during one or both circuits is possible.

By the Reciprocity principle, we can expect to find another trajectory involving the dates (244-0810, 244-0885, 244-1095, 244-1415); a further search located the following actual trip:

Depart Earth:	9 Sep 1970 (JD 244-0838);	speed 0.127 ≈ 3.57 km/sec
Pass Venus:	5 Dec 1970 (JD 244-0925);	speed 0.230
Pass Mars:	13 Jul 1971 (JD 244-1145);	speed 0.191
Return Earth:	3 May 1972 (JD 244-1440);	speed 0.288

The foregoing trajectories were chosen on the basis of low terminal speed requirements, although many other nearby trajectories exist which may be more preferable under different constraining conditions. For instance, if short trip time is of primary importance, the following choice may be more suitable:

Depart Earth:	11 Aug 1972 (JD 244-1540);	speed 0.238
Pass Venus:	24 Dec 1972 (JD 244-1675);	speed 0.413
Pass Mars:	9 Sep 1973 (JD 244-1934);	speed 0.352
Return Earth:	18 Nov 1973 (JD 244-2004);	speed 0.336

A reciprocal trajectory is predicted at (244-0376, 244-0446, 244-0705, 244-0840); one has been found at:

Depart Earth:	9 May 1969 (JD 244-0350);	speed 0.377
Pass Mars:	18 Jul 1969 (JD 244-0420);	speed 0.381
Pass Venus:	13 Apr 1970 (JD 244-0689)	speed 0.426
Return Earth:	15 Sep 1970 (JD 244-0844);	speed 0.197

Trip No. 2 in Table 3 passes near Venus on both first and second solar circuits. This creates the supposition that a single trajectory might be found which makes close contact with Venus on each circuit, as well as Mars. Such a trip has been located:

Depart Earth:	6 Sep 1970 (JD 244-0835); speed 0.158
Pass Venus:	20 Nov 1970 (JD 244-0910); speed 0.347
Pass Mars:	9 May 1971 (JD 244-1080); speed 0.330
Pass Venus:	4 Mar 1972 (JD 244-1380); speed 0.235
Return Earth:	22 Jul 1972 (JD 244-1520); speed 0.155

In the above case, the three planetary passes (all within 2 planetary radii) have been parlayed into terminal speed reductions which bring the departure and arrival speeds down almost to parabolic values. This reduction is not gained without some penalty, however, as the increase in trip time indicates. A reciprocal trajectory exists:

Depart Earth:	21 Sep 1970 (JD 244-0850); speed 0.221
Pass Venus:	19 Jan 1971 (JD 244-0970); speed 0.251
Pass Mars:	15 Dec 1971 (JD 244-1300); speed 0.330
Pass Venus:	7 Jun 1972 (JD 244-1475); speed 0.387
Return Earth:	6 Aug 1972 (JD 244-1535); speed 0.216

To conclude, we should make note of the apparent possibility of "combining" Trips 1 and 2, plus their reciprocals, into a single mission which would pass Venus four times and Mars once, all within a 1-1/2 to 2-year span. This possibility is currently under investigation.

OPTIMAL IMPULSE APPLICATION DURING PLANETARY PASSAGE

PHILOSOPHY OF APPROACH

In many instances, the application of moderate propulsive thrust as the vehicle rounds the pass planet may reflect worthwhile savings at the points of arrival and departure. The thrust may often be combined with a navigational correction during the planetary passage, and it is of interest, therefore, to choose the "best" place in the vicinity of the planet for executing this maneuver.

In analyzing the problem, it should be noted that the employment of nonstop trajectories constitutes only a temporary phase in any complete plan for interplanetary travel. Viewed in perspective, the nonstop mission appears useful only while the means are lacking for performing capture and landing operations at the planets. Nonstop trips, then, assume significance only when limited systems potential exists for interplanetary voyages. It follows that these missions will be performed with marginal capability, and that the total impulse available for a planetary maneuver will probably be quite limited. It is assumed, therefore, that the acceleration applied during the planetary passage will be small enough to cause only differential changes in the orbit.*

Because of this assumption, the modified trajectory will not differ sharply from the original; this, in turn, suggests that the search for trajectory improvement should begin with a nominal orbit representing the best mission choice which does not involve an impulse application. Having located the points corresponding to this nominal orbit on the contour charts, the analyst may then attempt to improve the choice by gradually moving these points in the directions indicated by the improvement criterion adopted for the mission.

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*A recent treatment of the large-impulse analysis may be found in Ref. 13.

Each new pair of points so selected determines new values for the speeds and directions of the hyperbolic asymptotes at the planet. For these new choices, the incoming and departing hyperbolic speeds will no longer coincide, nor will the angle between the asymptotes remain unchanged; the propulsive maneuver is performed for the very purpose of matching the proper incoming hyperbolic segment with the required departing hyperbolic segment.

The encounter trajectory will, in general, lie in a plane formed by the two asymptote vectors and the planet's center. In some cases it may be desirable to shift the perifocus point to a new position which requires "breaking" this plane, but that case will not be treated in this study. The present problem therefore remains two-dimensional in nature.

To ensure that proper conditions exist for planetary reconnaissance, the point of closest approach will be set at some predetermined value. This point represents one of the most critical places for gathering data during the entire mission. Since an impulse may upset experiments in progress, we shall omit from consideration all trajectories for which the thrust is applied at this close-approach point. This immediately rules out the case where neither hyperbolic segment includes a perifocus.

The only remaining possibilities, then, are where either or both hyperbolic segments contains a perifocus which occurs at the stipulated close-approach distance.*

In summary, then, the main assumptions governing the optimal impulse analysis are:

- Planar motion
- Single impulse application
- Differential changes of the encounter orbit
- Either or both hyperbolic segment contains a perifocus of specified distance from the planet

---

*When both segments contain perifoci, the nearer one must satisfy the close-approach condition.

MATHEMATICAL ANALYSIS

The approach and departure speeds, ν_{∞_-} and ν_{∞_+}, determine a differential speed requirement, $d\nu_\infty = \nu_{\infty_+} - \nu_{\infty_-}$. The total asymptote deflection angle, together with the perifocus distance, r_p, determines a differential asymptote angle requirement, $d\Omega$ (see Fig. 15). That is, $d\Omega$ represents the increment in asymptote deflection required over and above that which is caused by planetary bending when the vehicle passes at a distance of r_p.

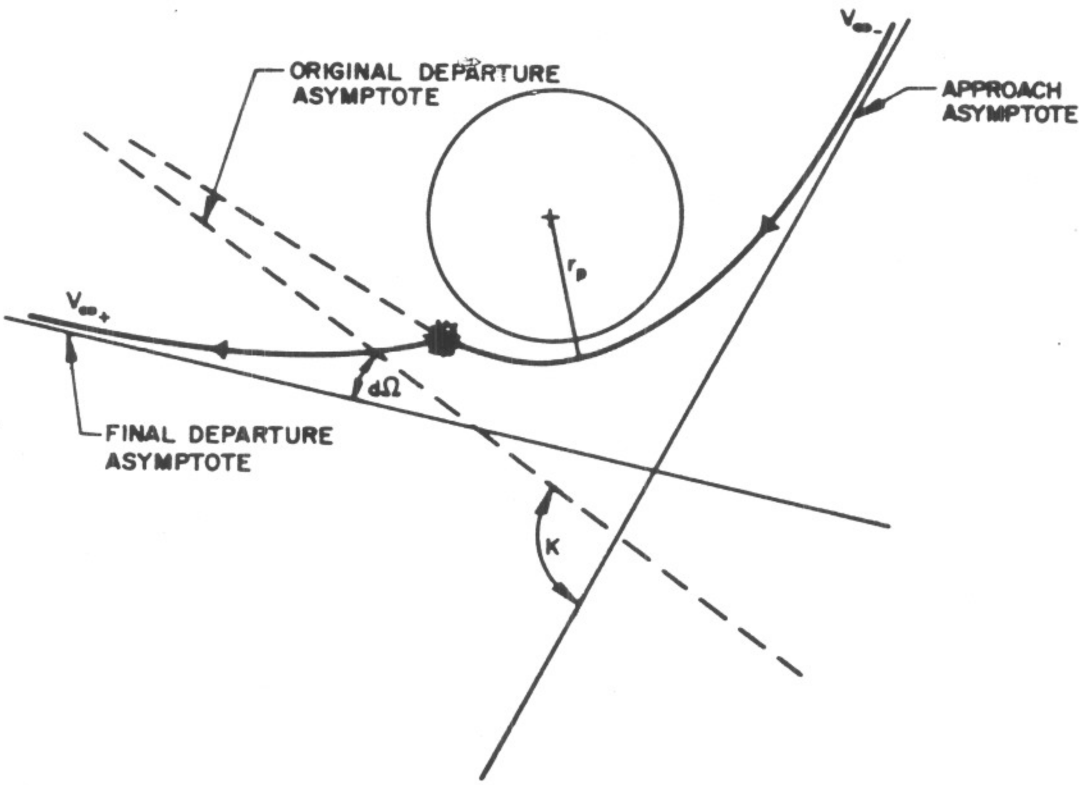

Fig. 15 Nomenclature for Corrective Impulse

Both $d\nu_\infty$ and $d\Omega$ may be related to changes of a, e, ω during passage,* as viewed in the local planetary coordinate system. By Conservation of Energy, $\nu_\infty = \sqrt{-\frac{\mu}{a}}$ and therefore, $d\nu_\infty = \frac{n}{2} da$.

Two effects contribute to $d\Omega$. The first is dK, which alters the angle between the asymptotes; the second is $d\omega$, a change in perifocus position, which serves to rotate both asymptotes as a rigid frame. For the total effect, then, $d\Omega = -\frac{dK}{2} - d\omega$. But, as is well-known, $\sin\frac{K}{2} = \frac{1}{e}$, so that

$$\frac{dK}{2} = -\frac{de}{e\sqrt{e^2 - 1}}$$

Now, if we assume that the thrust is applied impulsively during planetary passage, then the "perturbing accelerations" in the Variation of Parameters formulas may be treated as impulse functions whose time integrals are the appropriate velocity components. Since the thrust acts instantaneously the coefficients of these "perturbing accelerations" retain their instantaneous values at the time of thrust application, and the formulas for $\dot{a}$, $\dot{e}$, $\dot{\omega}$ may be integrated directly to yield values for da, de, $d\omega$, and hence, for $d\nu_\infty$ and $d\Omega$.

By performing the algebraic manipulations, we obtain

$$d\nu_\infty = \frac{e \sinh E}{e \cosh E - 1} d\nu_r + \frac{e}{\sqrt{e^2 - 1}}\left[\frac{e - \cosh E}{e \cosh E - 1} + \frac{1}{e}\right] d\nu_s$$

$$\nu_\infty d\Omega = \frac{\sqrt{e^2 - 1}}{e}\left[\frac{(e - \cosh E) + \sinh E}{e \cosh E - 1}\right] d\nu_r + \frac{1}{e}\left[\frac{(e - \cosh E) - (e^2 - 1)\sinh E}{e \cosh E - 1} + (\cosh E - \sinh E)\right] d\nu_s$$

---

*a is the semi-major axis, e, the eccentricity, ω the perifocus position.

These above equations may be inverted for $d\nu_r$ and $d\nu_s$, the radial and circumferential velocity increments. The total velocity increment may then be expressed as $d\nu = \sqrt{d\nu_r^2 + d\nu_s^2}$. Again performing the manipulations indicated, we find

$$\left(\frac{d\nu}{d\nu_\infty}\right)^2 = \frac{1}{\Delta(e, E)}\left[F(e, E)\tan\psi_\infty - \frac{G(e, E)}{F(e, E)}\right]^2 + \left[\frac{1}{F(e, E)}\right]^2 \qquad (4)$$

where

$$\Delta(e, E) = 1 + \frac{\exp(-E)}{e}$$

$$F(e, E) = \frac{e\cosh E + 1}{e\cosh E - 1} = \frac{\nu}{\nu_\infty}$$

$$G(e, E) = \frac{2\sqrt{e^2 - 1}}{e}\;\frac{\exp(-E)}{e\cosh E - 1}$$

and

$$e = 1 + \frac{r_p \nu_\infty^2}{\mu}\;;\quad \tan\psi_\infty = \frac{\nu_\infty\, d\Omega}{d\nu_\infty}$$

All variables in Eq. 4 are nondimensional, and may be applied to any planet. By the assumptions stated above, both e and ψ_∞ are stipulated, and it only remains to choose E, the eccentric anomaly, in the most advantageous manner. By this we usually mean that $d\nu$, the velocity increment, should be minimized, although constraints may sometimes be imposed which prevent the impulse from being applied too close to the perifocus.

If the perifocus of the second hyperbolic segment is also traversed during the passage, it should not come closer than the stipulated value of r_p; this possibility must be checked for each case under consideration.

Figure 16 displays speed requirements for probes launched from Earth onto Hohmann trajectories towards the Sun – the so-called "direct descent" method.

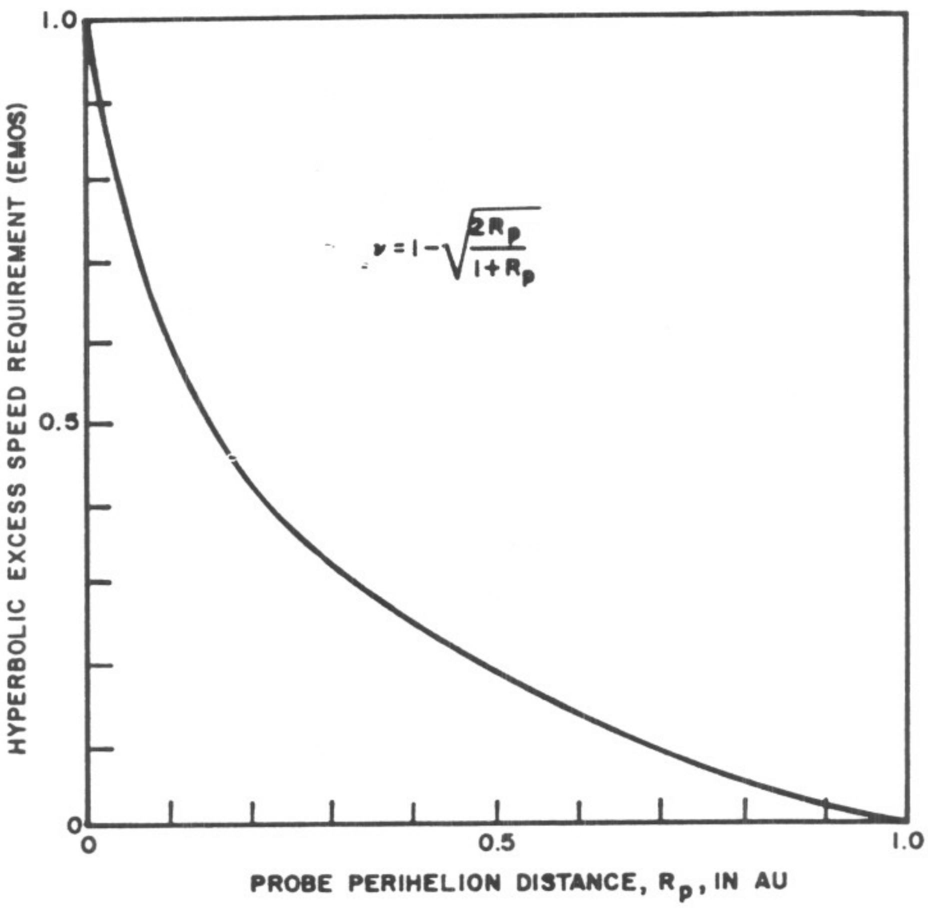

Fig. 16 Speed Requirements for Solar Probes on Hohmann Orbits

When perihelion distances within about 0.4 AU are sought, however, this technique becomes impractical because of the high launch speeds which are required.

One alternative method for establishing perihelia in the 0.1- to 0.4-AU range is to employ close passages at Venus so that nominal probe orbits will be deflected nearer to the Sun. This method combines the desirable properties of moderate speed and short mission durations. The close-approach technique would allow reconnaissance of Venus as well as the Sun during a single mission.

Figure 17 displays the characteristics of these trajectories during 1965–66, the period when the earliest solar probe flights might be expected. Only that portion of the launch cycle when this technique displays a worthwhile advantage over direct descent has been considered. The shaded areas, overlaid on contours of constant launch speed, denote perihelion distances for perturbed orbits which employ peri-planet passages of between one and two Venusian radii. The upper boundary of each shaded region specifies mission requirements for the one-radius passages; the lower limit for two-radius passages. Tick marks extending from the right-hand ends of these regions show the characteristics associated with passages at intermediate distances between the two limits.

Close solar probes involve large communication distances as well as high background noise levels at the times of perihelion passage. It would be of interest, therefore, to schedule a return to the vicinity of the Earth during such flights and to attempt a second readout of data under more favorable communication conditions.* The 1/2-year curve depicted in Fig. 2 passes within the region of trip dates under consideration. This 1/2-year curve is depicted in greater detail in Fig. 18. For solar round-trip missions, actual physical recovery of the probe may not be required if the vehicle can be brought into the general vicinity of the Earth where it could be easily interrogated. With the removal of the Earth impact requirement, close-approach contours

*Assuming that the payload can be protected from heat damage during the solar passages.

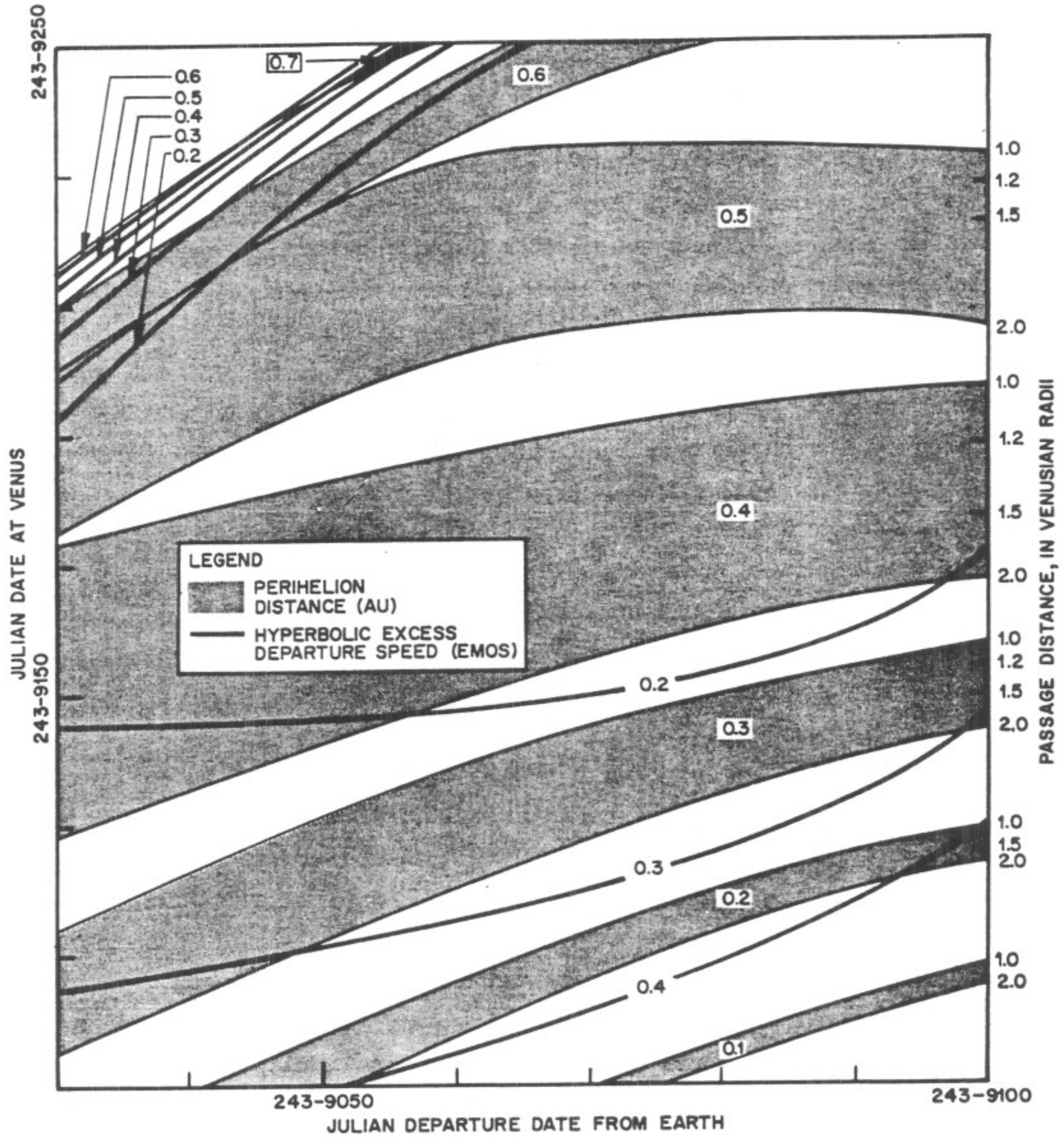

Fig. 17 Solar Probe Perihelion Distance After Venusian Passage at Various Planetary Radii

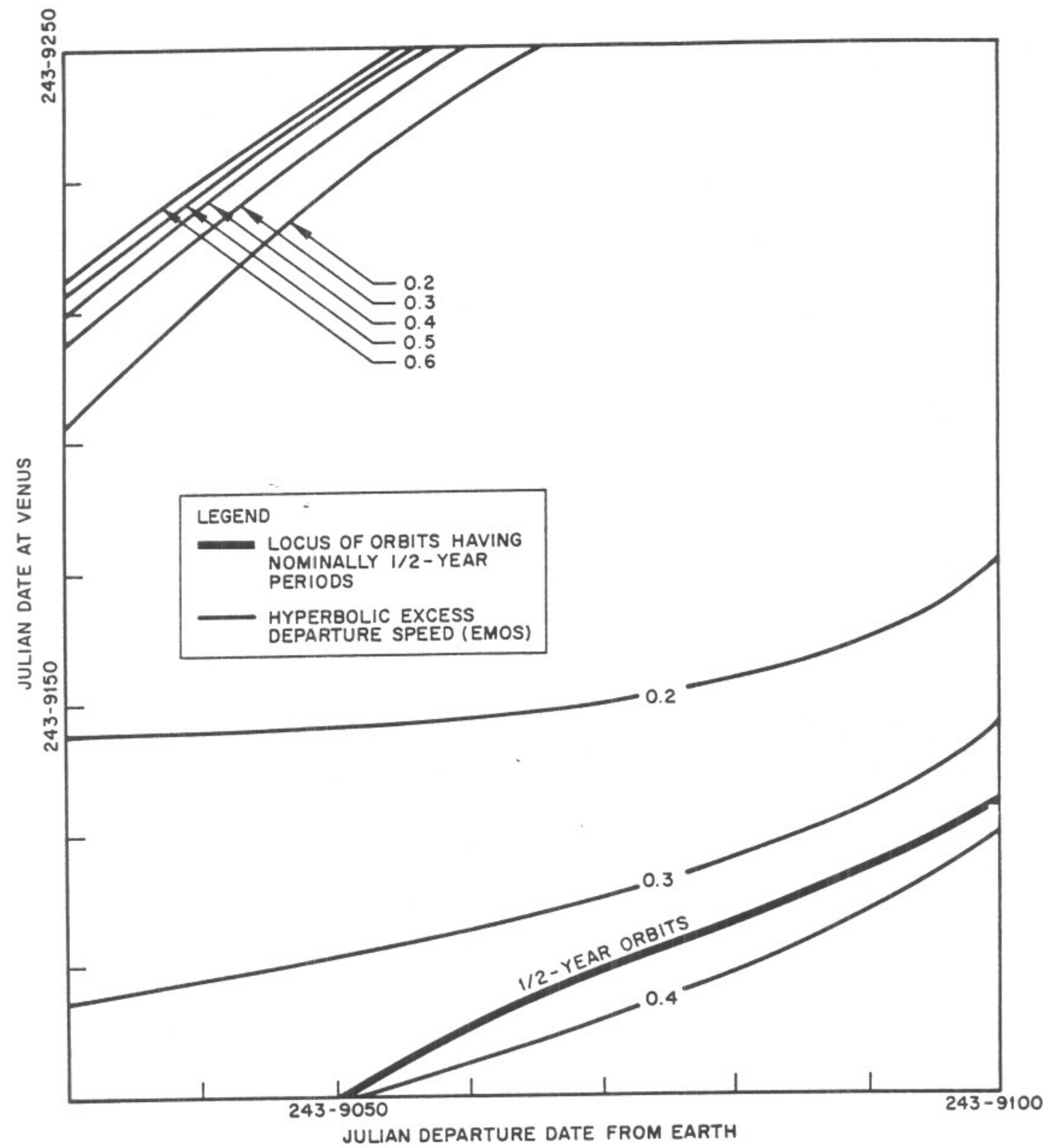

Fig. 18 Half-Year Solar Orbits

lose their significance and a wide region of acceptable perturbed trips becomes available. The extent of this region depends primarily upon the communication distance tolerable at the mission's termination.

It should be noted that all trajectories which require close planetary approaches must assume the weight burden of additional guidance instrumentation for the passage maneuver. Just how much of the nominal performance gain will be nullified by this weight penalty is a matter which must be determined for each mission under consideration.

The possibility of using Mercury, as well as Venus, for performing solar probe missions does not appear feasible. Mercury's extremely small mass together with its high orbital speed would present excessively stringent navigational requirements for the missions contemplated.

ACKNOWLEDGMENT

This work was performed under NASA Contract NAS8-2469 (Interplanetary Trajectory Handbook Study), for the George C. Marshall Space Flight Center, Huntsville, Alabama. To Richard Brower, John Cottrell, Robert Farquhar, Craig Olsen, David McKellar and Jacqueline Schroedter of Lockheed Missiles & Space Company, go my sincere thanks for their unselfish devotion and good-natured patience in helping with data reduction.

A number of contour maps of the type presented in the Appendix are available in the Final Report to Contract NAS8-2469, dated 2 March 1963. These maps cover the periods 1962-1984 for trips past Venus, 1965-1980 for low-energy trips past Mars, and 1960-2000 for high-energy trips past Mars.

Appendix

CLOSE-APPROACH CONTOURS FOR 1964-1965

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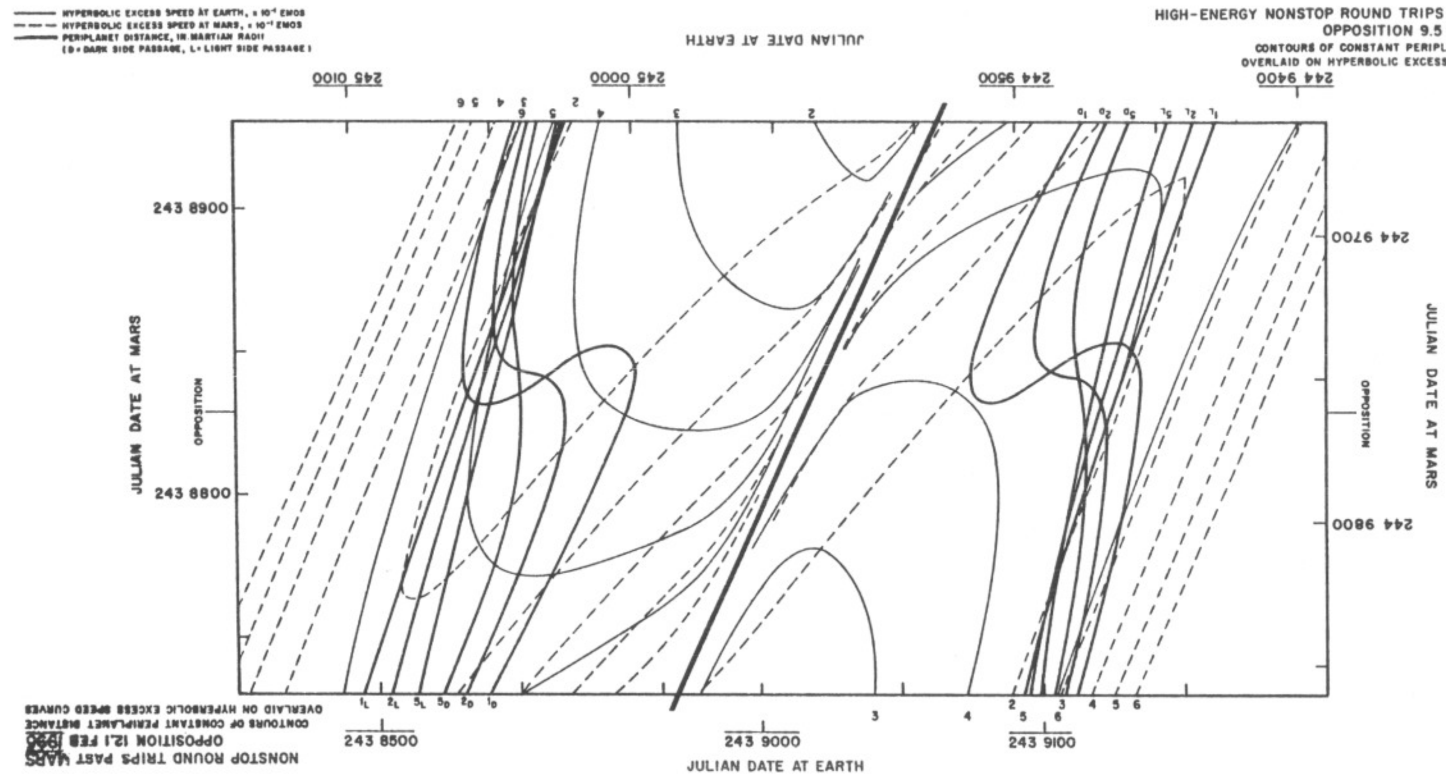
HIGH-ENERGY NONSTOP ROUND TRIPS PAST MARS
OPPOSITION 9.5 MARCH 1965
CONTOURS OF CONSTANT PERIPLANET DISTANCE
OVERLAID ON HYPERBOLIC EXCESS SPEED CURVES



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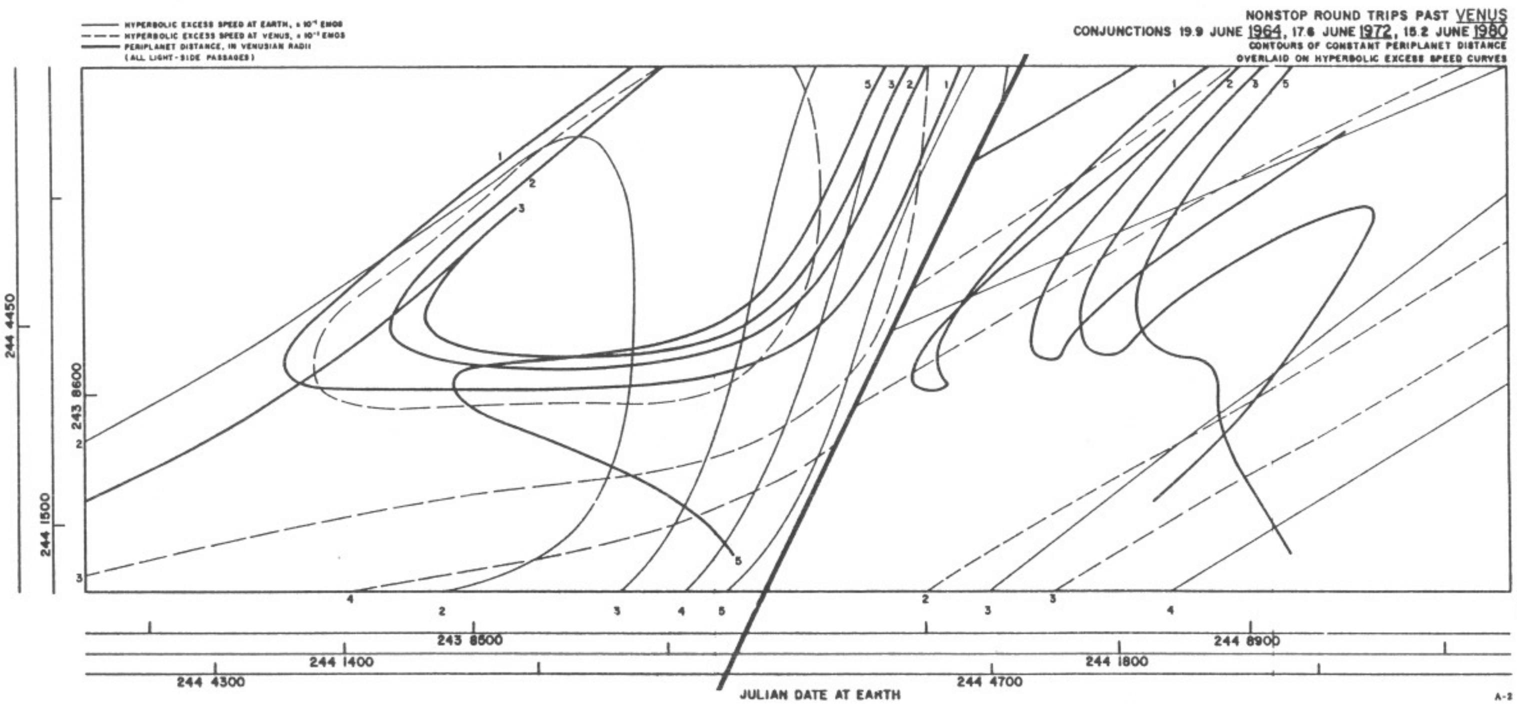
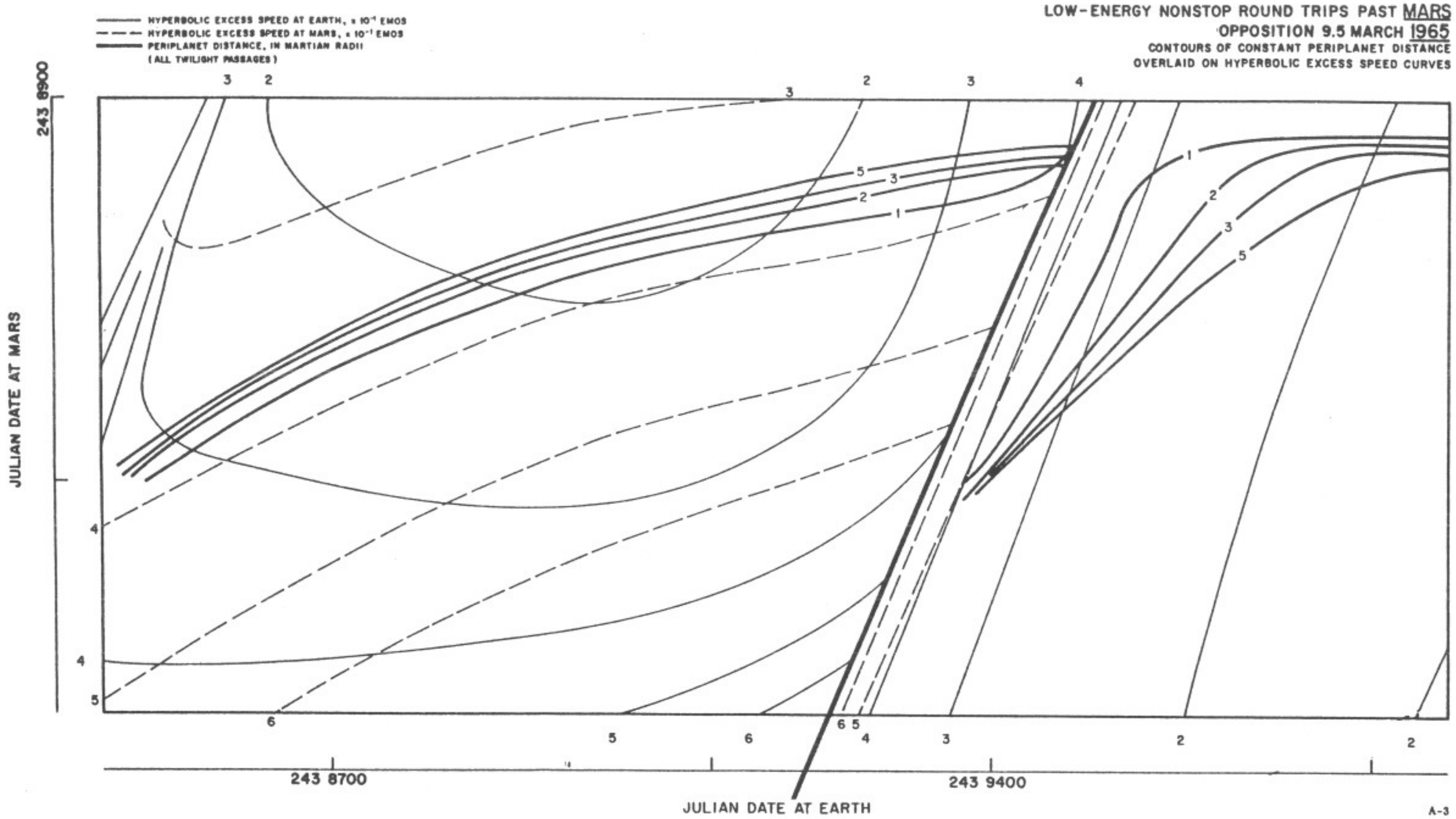

NONSTOP ROUND TRIPS PAST VENUS
CONJUNCTIONS 19.9 JUNE 1964, 17.6 JUNE 1972, 15.2 JUNE 1980
CONTOURS OF CONSTANT PERIPLANET DISTANCE
OVERLAID ON HYPERBOLIC EXCESS SPEED CURVES
HYPERBOLIC EXCESS SPEED AT EARTH, × 10⁻¹ EMOS
HYPERBOLIC EXCESS SPEED AT VENUS, × 10⁻¹ EMOS
PERIPLANET DISTANCE, IN VENUSIAN RADII
(ALL LIGHT-SIDE PASSAGES)
JULIAN DATE AT VENUS
244 4450
243 8600
244 1500
JULIAN DATE AT EARTH
243 8500
244 8900
244 1400
244 1800
244 4300
244 4700
A-2

LOW-ENERGY NONSTOP ROUND TRIPS PAST MARS
OPPOSITION 9.5 MARCH 1965
CONTOURS OF CONSTANT PERIPLANET DISTANCE OVERLAID ON HYPERBOLIC EXCESS SPEED CURVES



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