PRENTICE-HALL INTERNATIONAL SERIES IN SPACE TECHNOLOGY

C. W. Besserer and Floyd E. Nixon, Editors

PRENTICE-HALL, INC.
PRENTICE-HALL INTERNATIONAL, UNITED KINGDOM AND EIRE
PRENTICE-HALL OF CANADA LTD., CANADA

ALEXANDER AND BAILEY *Systems Engineering Mathematics*
BAZOVSKY *Reliability Theory and Practice*
ELLIS AND LUDWIG *Systems Philosophy*
FOGEL *Biotechnology: Concepts and Applications*
KOLK *Modern Flight Dynamics*
LINDEN *Printed Circuits In Space Technology*
LLOYD AND LIPOW *Reliability: Management, Methods and Mathematics*
LOH *Dynamics and Thermodynamics of Planetary Entry*
MACOMBER AND FERNANDEZ *Inertial Guidance Engineering*
NELSON AND LOFT *Space Mechanics*
ORDWAY, GARDNER, AND SHARPE *Basic Astronautics*
ORDWAY, GARDNER, SHARPE AND WAKEFORD *Applied Astronautics*
SAMARAS *Applications of Ion Flow Dynamics*
SAMARAS *Theory of Ion Flow Dynamics*
SCHLESINGER, *et al.* *Principles of Electronic Warfare*
SLAGER *Space Medicine*
STEARNS *Navigation and Guidance In Space*
STILTZ (ED.) *Aerospace Telemetry*

PRENTICE-HALL INTERNATIONAL, INC., *London*
PRENTICE-HALL OF AUSTRALIA, PTY., LTD., *Sydney*
PRENTICE-HALL OF CANADA, LTD., *Toronto*
PRENTICE-HALL FRANCE, S.A.R.L., *Paris*
PRENTICE-HALL OF JAPAN, INC., *Tokyo*
PRENTICE-HALL DE MEXICO, S.A., *Mexico City*

NAVIGATION AND GUIDANCE IN SPACE

EDWARD V. B. STEARNS

Manager, Advanced Program Development
Space Systems Division
Lockheed Missiles and Space Co.

PRENTICE-HALL, INC.
ENGLEWOOD CLIFFS, N. J., 1963

6.1 INTERPLANETARY ORBITS

The selection of an interplanetary orbit involves more than the simple orbit to orbit transfer. It is necessary to select an orbit which will carry the spacecraft from a specific point or body moving in one orbit to another specific body which is moving in its own orbit. The relative positions of the two planets (origin and destination) will be changing with time before initiation of the transfer as well as after. The selection of an orbit for the transfer will, or course, be related to this initial configuration of the planets. The classical Hohmann orbit, for example, depends upon waiting until a certain planetary configuration is obtained so that at the end of one half period for the transfer orbit, the destination planet will be at a position with longitude $\pi/2$ different from the longitude of the original planet at the time of initiation of the transfer. Transfer between inclined orbits is even more sensitive to the starting point. If the transfer is to be accomplished from a point on the line of nodes to another point on the same line, there will be no requirement for maneuvers which change the orbital plane.

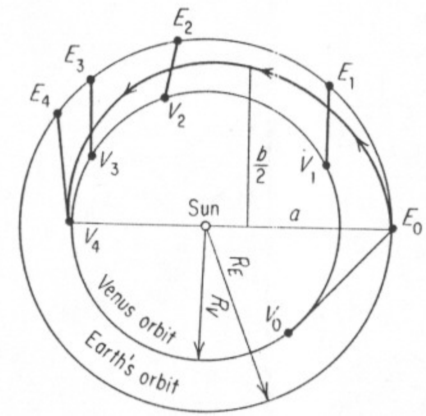
Situations may arise where the interplanetary mission at hand can not tolerate the delays which would be required in waiting for planetary configurations of the kind involved if transfers like those described above are to be used. Similarly the mission may require transfer times which are less than those which are indicated by the minimum energy examples. The characteristics of reduced flight time trajectories will be discussed in the following section.

Before proceeding to discuss the factors which influence the selection of a transfer orbit, it seems to be appropriate to discuss the meaning of the term *minimum energy transfer orbit.* Such an orbit is one which demands the expenditure of least energy from the propulsion system in order to transfer from the orbit in which a body is traveling, to place it in a transfer orbit, and then to match its orbit with that of a destination body. Depending on the nature of the specific transfer, the orbit which requires the least characteristic velocity may, or may not, be the transfer orbit with the least energy in the sense that energy is defined by Equation (2-13) and implicitly by Equation (2-22). A minimum energy transfer orbit is generally considered to be the one that involves the least characteristic velocity for the over-all maneuver.

Transfers are established by the definition of a series of velocity maneuvers; it is also necessary to define the direction in which these velocity increments should be added to the initial velocity of the craft in order to place it in the desired orbit. This requires the solution of an orbit which passes through two definite points in space and which has a transfer time which is appropriate to the specific situation at hand. While this problem can not be solved in general terms we shall discuss some practical applications of its resolution which will be useful in the practice of space navigation.

6.1.1 Selecting an Orbit

A major consideration in planning for interplanetary flights is the tremendous energy which is required from the propulsion system to effect the transfer of a spacecraft from the vicinity of one planet to that of another. If possible, then, it would be desirable to make such a transfer using the minimum energy in the maneuvers. For the special case of transfer between coplanar circular orbits, the minimum energy transfer defines a class of paths known as *Hohmann orbits* (Reference 6.2). These orbits are illustrated in Figure 6.3. A summary of the properties of these orbits is shown in Table 6.2. When the initial and final orbits are not coplanar and eccentricities are introduced, the minimum energy



Properties of transfer orbit
Major axis, $a = R_E - R_V$
Minor axis, $b = \sqrt{R_E R_V}$
Transfer time $= \pi\sqrt{\frac{K}{a^3}}$

Fig. 6.3. The Hohmann orbit.

maneuver will depart from that which is described by the Hohmann orbit. The theory of minimum energy orbits is beyond the scope of this text and represents an area of specialized study in itself. For the details of this special problem, the reader is referred to works of analysts of celestial mechanics, for example Reference 6.5. It will provide some useful insight into the navigation problems of interplanetary space to consider the following generalization of the problem.

The studies reported by Hohmann demonstrated that the application of tangential impulses at the apsides of a cotangential transfer orbit will require minimum expenditure of rocket fuel in order to transfer between coplanar circular orbits. Since that time, it has been recognized that the same maneuvers

Table 6.2. HOHMANN ORBIT PROPERTIES FOR INTERPLANETARY FLIGHT

Planet	*Hohmann launch vel.*	*Hyperbolic excess vel.*	*Transfer time*
Mercury	44,000 ft/sec	26,600 ft/sec	100 days
Venus	38,000	10,900	150
Mars	38,000	10,900	260
Jupiter	46,000	29,800	2·7 years
Saturn	49,000	34,300	6·0
Uranus	51,000	37,100	16·0
Neptune	52,000	38,000	31·0
Pluto	53,000	39,800	46·0

will minimize fuel consumption for transfer between certain other coplanar orbits where the departure and arrival both occur at the apsides of the two orbits. The considerations leading to the Hohmann orbit customarily consider only one or two impulse maneuvers. When three or more impulses are considered, some special examples may be found in which transfer may be accomplished for less total transfer energy than with two impulse maneuvers. For example, in transfers where the final orbit is many times larger, or smaller, than the initial orbit, fuel can be saved by transferring beyond the final orbit, applying retro-thrust at the apogee of this ellipse and entering the final orbit at the perigee of a second transfer ellipse. See Figure 6.4. The absolute minimum fuel consumption in this case would necessitate escape from the original orbit along a parabola followed by an infinitesimal impulse which is applied after escape and which causes the vehicle to return along a new parabola. The maximum possible savings in this maneuver is about 8 per cent of the total characteristic velocity. (Reference 6.6.) This saving would probably not justify the added time and complexity involved in the use of such a maneuver.

Although the study of Hohmann orbits and coplanar transfers has proved useful in order to estimate the magnitude of the propulsion required for space travel, a realistic evaluation of the maneuvers required makes it necessary that the effects of orbit inclination and the eccentricity be taken into account. The flight mechanics of such a transfer have been discussed in Reference 6.7 where the maneuver has been evaluated in terms of the *hyperbolic excess velocity* required to obtain interplanetary transfer. The hyperbolic excess velocity, v_∞, can be obtained from the energy equation as expressed in Equation (2-14); it is

$$v_\infty = \sqrt{v^2 - \frac{2k}{r}} = \sqrt{v^2 - v^2_{\text{esc}}} \qquad (6\text{-}2)$$

The contour chart of Figure 6.5 shows a contour map that indicates the hyperbolic excess velocity required for trips to Mars (♂) from the Earth (⊕). The contours have been drawn in a plane where the abscissa represents the

dates of departure from and arrival at the Earth (shown as the Julian calendar date). The ordinates show the dates of arrival and departure when reckoned from Mars. The departure speeds (as indicated by hyperbolic excess velocity) from the Earth are shown by the solid contour lines at the left of the chart and the speed of arrival at Mars is shown by the corresponding dotted contour. To the right of the chart, Mars departures are indicated by the solid contours and the arrival speeds at the Earth are shown by the dotted lines. The velocity values shown have been normalized using one tenth of the Earth's mean orbital speed as a unit. (1 EMOS = 29.77 km/sec = 18.5 mi/sec, 0.1 EMOS = 1.85 mi/sec.)

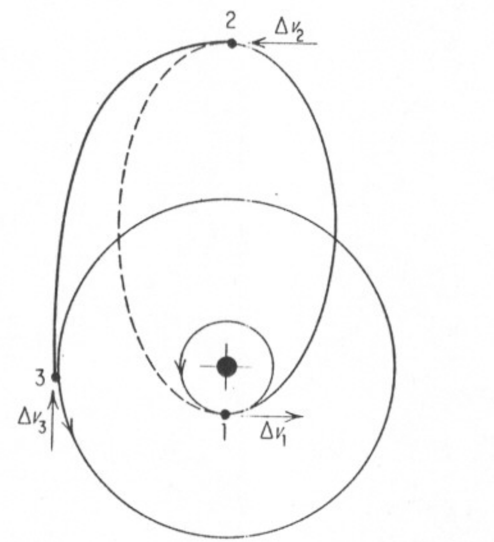


Fig. 6.4. Transfer by three impulses: (1) escape from inner orbit to first transfer orbit, (2) transfer at apoapsis of first transfer orbit to second transfer orbit, (3) transfer to final orbit.

Flight durations are shown in days and are indicated by the difference between the Julian dates that are shown on the ordinate and the abscissa. A one-way transfer is represented by a point that has convenient values for the departure and arrival speeds. The dates for nodal transfer are indicated on the reference axes. The calculations leading to these charts has been described in the literature, Reference 6.7; the calculations include the effects of planetary orbit inclinations as well as the eccentricity of the orbits. Charts such as those shown in Figure 6.5 may be used to plan a complete journey. An example has been indicated on the chart.

Departure from a node, for example Julian date 244 0903, *and* arrival at a node, for example J–244 1083, are both required in order for the flight to be in the ecliptic plane. No ecliptic plane flights are possible when the initial or terminal points are not on the nodes. Hohmann transfer requires ecliptic plane flight, *and* the appropriate initial configuration for 180 deg transfer between the two planets; no such flight opportunity occurs in the time period that is shown in the chart of Figure 6.5. An example of the use of the chart is shown on the face of the Figure. The key points of the example are as follows:

	Jul. Date	*Cal. Date*	*Duration*	*Speed*
Leave ⊕	244 0880	10/21/70	—	0.3470 EMO's
Arrive ♂	1100	5/29/71	220 da	0.2828
Leave ♂	2130	6/28/71	30	0.1965
Arrive ⊕	1260	11/5/71	130	0.1361
		Duration of journey: 380 days		

6.1.2 Characteristic Velocity and the Energy of Transfer

The principles of rocket propulsion were discussed briefly in Section 1.7 where the relations between fuel consumption and velocity increments were developed. Since the fuel for propulsion of a rocket frequently represents a major fraction of its total mass, it is customary to discuss the maneuvers of a rocket system in terms of its *characteristic velocity*, which may be related to fuel requirements. This term refers to the sum total of all the velocity increments that a rocket must develop, or dissipate, by fuel consumption in the course of a given maneuver or journey. The characteristic velocity is useful in evaluation of interplanetary flight plans and in estimation of the capability of a given rocket system to meet the indicated flight plan. Let us assume that a spacecraft is to be launched from the Earth to a landing on a distant planet. The complete sequence of maneuvers will be defined by a series of steps as follows:

1. Escape from the Earth: v_{e0}
2. Orbit plane change to the plane of the selected transfer orbit: v_{t0}
3. Change of hyperbolic velocity relative to the Earth, placing the craft in transfer orbit: v_{t0}
4. Deceleration in the vicinity of the destination reducing the hyperbolic excess velocity relative to the destination planet: v_{tf}
5. Change in plane to that of the orbit of the destination from that of the transfer orbit: v_{tf}
6. Reduction of velocity gained in falling through the gravity field of the destination planet: v_{ef}.

The characteristic velocity for the journey may be calculated on a one-way or a round trip basis. For a one-way journey the characteristic velocity will be given by the sum of each of the individual terms listed above.

$$v_c = v_{e0} + v_{t0} + v_{t0} + v_{tf} + v_{tf} + v_{ef} \tag{6-3}$$

Depending upon the manner in which these individual increments of velocity are introduced into the system the total energy which is required of the propulsion system may be reduced. In any case of practical interest, the transfer may be resolved into two periods of maneuvers. The first maneuvers will be at the point of origin and the second group of maneuvers will occur in the vicinity of the destination. The first three velocity increments may then be combined in some convenient fashion and the final three maneuvers may be similarly grouped. If this is done, a reduced *characteristic velocity* will be obtained which reflects the vector summation of the groups of velocity increments. The fuel consumed and the equivalent kinetic energy of the transfer must be calculated on the basis of this reduced characteristic velocity which will depend upon the specific program of maneuvers which is elected for the mission at hand.

As the spacecraft approaches the atmosphere and the gravitational field of a planet, accelerations may be applied to the craft which perturb and distort its trajectory. These accelerations may be used to introduce, in part, the maneuvers indicated in the second group of velocity increments. Similarly, there are effects of gravity loss and drag loss which act to increase the total velocity required in the vicinity of the originating planet.

Inter-orbital maneuvers will involve one, or more of the following orthogonal components:

- Maneuvers involving a change in the tangential velocity; $\Delta v = v_{\text{tangential}}$
- Maneuvers in the orbit plane, but which involve changes in the radial component of the velocity. These are related to the orbital flight path angle $\Delta\varphi = \Delta v_{\text{radial}}/v_c$
- Maneuvers which involve changes in the orbit plane. An orbit plane change of angle $\Delta\psi$ will result from a velocity increment which is applied normal to the orbit plane; $\Delta\psi = \Delta v_{\text{normal}}/v_c$

The point at which a plane change maneuver is made will always be the node of the former orbit and the new orbit. This point and the point at the center of the force field for the orbit will define the line of nodes and hence the relative orientation of the new orbit. Plane change maneuvers which are commanded at a point other than one on the desired line of nodes will require two successive plane change maneuvers, each of which will be larger than the single plane change which would be required if the maneuver had been executed at the node.

The characteristic velocity for the maneuver may be calculated from the expression

$$v_c = \sqrt{\Delta v^2{}_{\text{tang.}} + \Delta v^2{}_{\text{rad.}} + \Delta v^2{}_{\text{norm.}}} \tag{6-4}$$

This may also be calculated from the values of the velocity vector of the craft before and after the maneuver, that is

$$\mathbf{v}_c = \mathbf{v}_f - \mathbf{v}_0 \tag{6-5}$$

where

$\mathbf{v}_f$ = the final velocity vector
$\mathbf{v}_0$ = the velocity vector before the maneuver.

From the geometry shown in Figure 1.6, it will be seen that the characteristic velocity is given by

$$v_c^2 = v_0^2 + v_f^2 - 2v_0v_f \cos \Delta\varphi \tag{6-6}$$

The angle between the initial velocity vector and the direction of the thrust axis during the maneuver will be called the *thrust angle*, α, it is given by

$$\alpha = \sin^{-1}\left(\frac{v_f}{v_c}\right) \sin \Delta\varphi \tag{6-7}$$

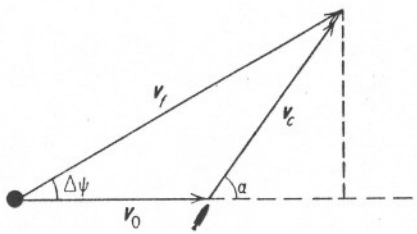


Fig. 6.6. Departure maneuver.

As a general rule, great flexibility will be available to navigators of spacecraft in the selection of their maneuvers. If no limitations were placed on the transfer, it would be possible to wait until the spacecraft occupies a point on the line of nodes of the two orbits; he would then make the transfer using an optimum impulse. In this case, minimum characteristic velocity, and hence minimum energy, is used to intersect, or osculate with the destination orbit. In the event that the destination planet is not at the point of intersection of the two orbits upon the arrival of the craft, the spacecraft might choose to continue to follow the transfer orbit as long as necessary for interception to occur. Alternatively, the navigator might elect to transfer the craft to an orbit in the plane of the destination and with a period only slightly perturbed from that of the destination body. The spacecraft would then await the time to be overtaken by its destination, or vice versa. In reality, it will usually be required that more dispatch be exercised in the transfer. Indeed, for missions involving living payloads, there will be a premium associated with the prompt completion of the mission. Consequently the work that follows will assume that interplanetary transfers will be made from a configuration of planets such that the first common point of the transfer and destination orbits will be occupied by the destination planet upon the arrival of the spacecraft.

6.1.3 Setting the Course for an Interplanetary Transfer

In this section we shall discuss a procedure which will permit the space navigator to determine the maneuver which he will require in order to effect an interplanetary transfer. As a first step, the navigator must select a heliocentric orbit which will carry him from a selected point on his present orbit to a closure point, or rendezvous, with his destination which, itself, is in an orbit about the Sun. It will be shown that the choosing of a transfer orbit can not be done without setting up a criterion for the choice of the orbit. Typical examples of suitable criteria are minimum characteristic velocity, cotangential departure (or arrival), fixed transfer time, or minimum transfer time. The choice of these criteria will be discussed at the conclusion of the section.

The initial configuration of the spacecraft and the planets will be set by the time of the departure of the spacecraft from its orbit of origin, t_d. The terminal configuration will be set by the time of arrival, $t_d + \tau$. This situation is complicated since the interplanetary transfer orbit must ordinarily be preceded by an escape ascent trajectory and hyperbola leaving the originating planet. Similarly, the transfer must be concluded with a capture and descent hyperbola as it approaches the destination planet. While these extensions of the transfer orbit affect the complete time for the transfer and present a problem of transition from planetocentric hyperbolae to heliocentric ellipse, it will be convenient to consider only the heliocentric elliptical sector of the transfer orbits.

As a first step in navigation, see Figure 6.7, it will be necessary to determine the time relations of the flight. The *departure time* is usually a matter of operational importance and not a decision left to the navigator. The *transfer time* is the time which elapses while the spacecraft is in transfer flight from departure to destination. *Arrival time* is, of course, determined by the time of departure and the transfer time. The transfer time may be computed by application of the time Equation (2-38)

$$(t - t_p) = \frac{1}{n}(A_e - e \sin A_e) \tag{2-38}$$

This equation specifies the time which has elapsed since the spacecraft was at periapsis, the transfer time will be given as the time difference between the times for the initial and final positions on the transfer orbit. The point of departure may not be the periapsis (in the case of interplanetary flight, the perihelion)