JET PROPULSION LABORATORY

INTEROFFICE MEMORANDUM
MVM'73-VCC-74-27

RECEIVED

DEC 17 1974

OFFICE OF PATENT COUNSEL

TO: L. P. Speck

16 December 1974

FROM: V. C. Clarke, Jr.

SUBJECT: SUBMITTAL OF AWARD NOMINATION FOR "GRAVITY-ASSIST TRAJECTORIES"

REFERENCE: Standard Practice Instruction 1-22-1, NASA Monetary Awards, dated 17 September 1974.

In accordance with the reference SPI, I am submitting an award nomination for M. Minovitch and myself for the development and application of the gravity-assist technique for interplanetary trajectories. The award nomination is not for the invention of this technique but for extensively exposing its great utility in permitting missions which would otherwise be impossible or cost prohibitive with present-day launch vehicles. For example, a 50 percent launch energy reduction was gained on Mariner Venus/Mercury 1973 by using gravity-assist at Venus. This mission would have been impossible in 1973 without this energy reduction and if flown later would have required the much more expensive Titan/Centaur launch vehicle. Also, it would have been a "Mercury only" mission.

M. Minovitch performed the bulk of the development work, which evolved from work I had been doing on round trip gravity-deflected trajectories in early 1961, as reported in Research Summaries 36-9 and 36-10. In June 1961 I assigned Dr. Minovitch the task of extending analysis and computations of round trip trajectories. He went a step further and generalized the problem to include gravity-assisted transfer between n planets. The results of his extensive calculations are reported in Technical Report 32-464, "The Determination and Characteristics of Ballistic Interplanetary Trajectories Under the Influence of Multiple Planetary Attractions," 31 October 1963.

As a result of this basic development, numerous other applications are in use, such as, the Grand Tour, extension of Pioneer 11 flight to Saturn, and the Mariner Jupiter/Saturn flight. The use of the technique, thought of as a "wild idea" in 1961, is now a standard and important part of interplanetary space flight.

VCC:jm

V. C. Clarke, Jr.

cc: T. H. Coyne
R. J. Parks

1. Title of Contribution: Gravity-Assist Trajectories

2. Contributors: Name(s)	Employee? (✓)	Specify Contributions of Co-Contributors
M. Minovitch	Ex	Extensive analysis and development of gravity-assist trajectories
V. C. Clarke, Jr.	✓	Initiated and supervised development of gravity-assist trajectories

3. Source of Support For Contribution:
Were NASA funds expended in the work resulting in this contribution? [X] Yes [ ] No
a. If yes, identify the program, task, project, grant, or consulting agreement under which the work was done:
Work done under JPL supporting research and development funds, 1961-64

b. If no, state source of funds ____________ Approx. Amount $ ____________

and explain the relation of this contribution to aeronautical and space activities:

4. Past Honors, Awards, Nominations For This Contribution:
a. Awards and Honors: Kind Exceptional Service Medal for M. Minovitch

Source NASA Date 11-9-72

b. Last Nomination: Title ____________

Contributor(s) ____________ Date ____________

5. JPL Sponsorship: Why does this contribution deserve JPL sponsorship for a NASA monetary award?
The use of gravity-assist trajectories has greatly increased mission return on MVM'73 and promises to on Pioneer II and MJS 77. Additionally, many millions of dollars were saved on MVM'73 by permitting use of the Atlas/Centaur rather than the Titan/Centaur to reach Mercury. Truly, the use of the gravity-assist trajectory opened a new era in planetary space exploration.

6. Description of Contribution: Attach description and sketches. Existing articles, manuscripts, drawings, diagrams, photos, may be used. TR 32-464, RS 36-9 and 10

Prepared By	Date	Approved by (Division/Office Manager)	Date
V. C. Clarke, Jr. (signed) V. C. Clarke, Jr.	12-5-74	(signature)	12/6/74

Research Summary No. 36-9, Volume I

for the period April 1, 1961 to June 1, 1961

JET PROPULSION LABORATORY
CALIFORNIA INSTITUTE OF TECHNOLOGY
PASADENA, CALIFORNIA

July 1, 1961

SYSTEMS DIVISION

II. Systems Analysis

A. Trajectory Analysis

V. C. Clarke

1. *Interplanetary Trajectories*

As reported in *RS 36-8*, a computer study is in progress to determine the characteristics of ballistic interplanetary trajectories from Earth to Mars, Venus, Mercury, and Jupiter. The computations for Venus from 1962 to 1970, for Mars from 1962 to 1977, for Mercury from October 1967 to January 1969, and for Jupiter from December 1969 to February 1970 are now complete. A table of minimum energy transfers was published in *RS 36-8*. Final additions to that table are given in Table 1. These trajectories have been computed using actual planet positions obtained from an ephemeris tape. Thus, inaccuracies arising from assuming coplanar, circular motions of the planets are nonexistent.

Selected parameters of the trajectories have been saved on magnetic tapes. These tapes are being used to generate graphs of the parameters on an automatic plotting machine. The results of this study, with about 200 graphs, will be reported in Reference 7. Publication target date is September 1961.

A second phase of the study has begun with the computation of return ballistic trajectories from Mars and Venus to Earth. The results of these computations will also be saved on magnetic tape, and graphs will be prepared. In addition, the return trajectories will be combined with the Earth-to-target planet trajectories in a merging program to obtain round trip transfers. These transfers will include both ballistic flyby and stopovers of various duration at the planet. Completion of this phase of the study is projected to November 1961.

2. *Out-of-Ecliptic Trajectories*

A study is nearing completion of a special class of trajectories in which a space probe is launched from Earth in a direction perpendicular to the ecliptic plane, flies above (or below) the plane for a period up to 6 months, and finally returns to Earth. A cursory look at such trajectories from two-body (Sun-probe) mechanics indicates that the distance above the ecliptic that can be

Table 1. Characteristics of minimum-energy transfer trajectories

Planet	Trajectory type	Launch date	Flight time, days	Geocentric injection energy,* $m^2/s^2 \times 10^6$	Heliocentric central angle, deg	Earth–planet distance at arrival, 10^6 km	Celestial latitude of planet at arrival, de[g]
Mars	I	5/24/71	210	0.079	156.0	164	−0.352
	I	7/30/73	192	0.146	141.4	179	1.16
	I	9/15/75	206	0.187	144.7	221	1.85
	I	10/19/77	224	0.170	152.9	244	1.44
Jupiter	I	1/3/70	985	0.753	177.9	744	0.004
	II	12/30/69	995	0.754	182.5	757	−0.007

*Twice the total energy per unit mass, or the vis viva integral.

attained by a probe is directly proportional to the injection energy or hyperbolic-excess speed according to the relations

$$Z = R_E \tan \beta$$

and

$$\sin \frac{\beta}{2} = \frac{V_h}{2V_E}$$

where Z is the distance above the ecliptic, R_E is the Sun–Earth distance, β is the celestial latitude of the probe when it reaches its maximum distance above (or below) the ecliptic, V_h is the hyperbolic-excess velocity (directed normal to the ecliptic), and V_E is the heliocentric speed of the Earth.

In addition, for return to Earth, two-body mechanics dictates a 6-month flight time. A severe launch restriction is associated with this class of trajectories. For launchings from Cape Canaveral, it is found that launch azimuth is restricted to extreme northerly (<27 deg) or extreme southerly (>153 deg) azimuths. The reason for this is that the declination of the hyperbolic-excess velocity vector must be approximately 66.5 degrees or 90 degrees minus the obliquity of the ecliptic. The relation governing the limiting launch azimuths is

$$\sin \Sigma_L = \frac{\cos 66.5^\circ}{\cos \phi_L}$$

where Σ_L is the launch azimuth measured positive east of true north, and ϕ_L is the launcher latitude. Because of the azimuth restriction, it is very likely that range safety considerations would prohibit launches from Cape Canaveral for this type of trajectory. However, launchings from the launch facilities of the Pacific Missile Range are feasible.

Departure from two-body mechanics to exact simulation of these trajectories reveals several interesting characteristics. These are illustrated in Figures 17 and [...] Figure 17 is a plot of flight time vs injection energy. I[...] it is seen that, as injection energy is decreased to z[...] the flight time falls to just over 4 months, instea[...] 6 months. Any further decrease of energy would re[...] in even lower flight times. This result cannot be [...] dicted by two-body mechanics. In Figure 18, the [...] of range from Earth center vs time shows that f[...] hyperbolic-excess speed of 3 km/sec (Venus-type sp[...] a height of almost 15 million kilometers above the ecl[...] can be attained, in good agreement with two-body res[...]

Another property of this class of trajectories is tha[...] approximate analytical solution to the four-body (S[...] Earth–Moon–probe) problem can be obtained. This s[...] tion is currently being evaluated and compared to [...] exact solution from the JPL six-body integrating tr[...] tory program on the IBM 7090. Results of this compar[...] will be reported in the next *Research Summary*.

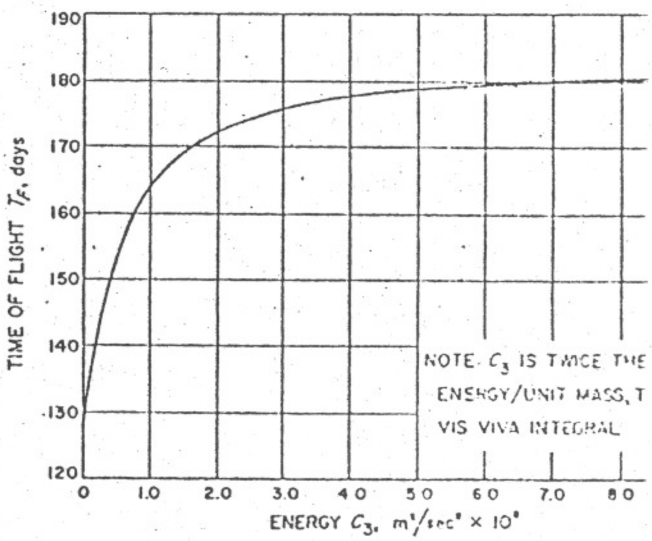

Figure 17. Time of flight vs injection energy

SYSTEMS DIVISION

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Interplanetary Trajectories

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ıs part of the continuing effort in ballistic interplane-ɿ trajectory analysis, a library of round-trip trajec-es, both flyby and stopover, will be computed. A ıputer program is now being written to accomplish phase of the analysis. A formal publication on the ılts of the round-trip trajectories should be ready ut the end of 1961.

2. Out-of-the-Ecliptic Trajectories

In *RS 36-9*, preliminary results of a study of a special class of trajectories, called "Out-of-Ecliptic Trajectories," were given. For this class of trajectories, the primary motion is normal to the ecliptic plane, or along an Earth-centered radial direction which has a declination of about 66.5 degrees and a right ascension of about 270 degrees. An Earth-based observer would, at first, observe a space probe launched on this type of trajectory traveling outward along the radial direction with decreasing speed until it came to rest. The maximum distance of outward travel (or altitude above the ecliptic) would be proportional to the hyperbolic-excess speed (i.e., injection energy). After reaching maximum distance at zero-speed, the probe would turn around and return to Earth along the same radial, but with increasing speed. An illustration of the speed profiles for various hyperbolic-excess speeds is shown in Figure 1. The independent variable in this figure is time of flight from injection. From this figure it can be seen that the total flight time is also dependent on the hyperbolic excess speed, varying from 128 days for $V_h = 0.1$ km/sec to 182 days for $V_h = 3$ km/sec.

A Sun-based observer, who sees the Earth revolving around him once per year, would notice the probe rise from the Earth and fly directly above it with increasing

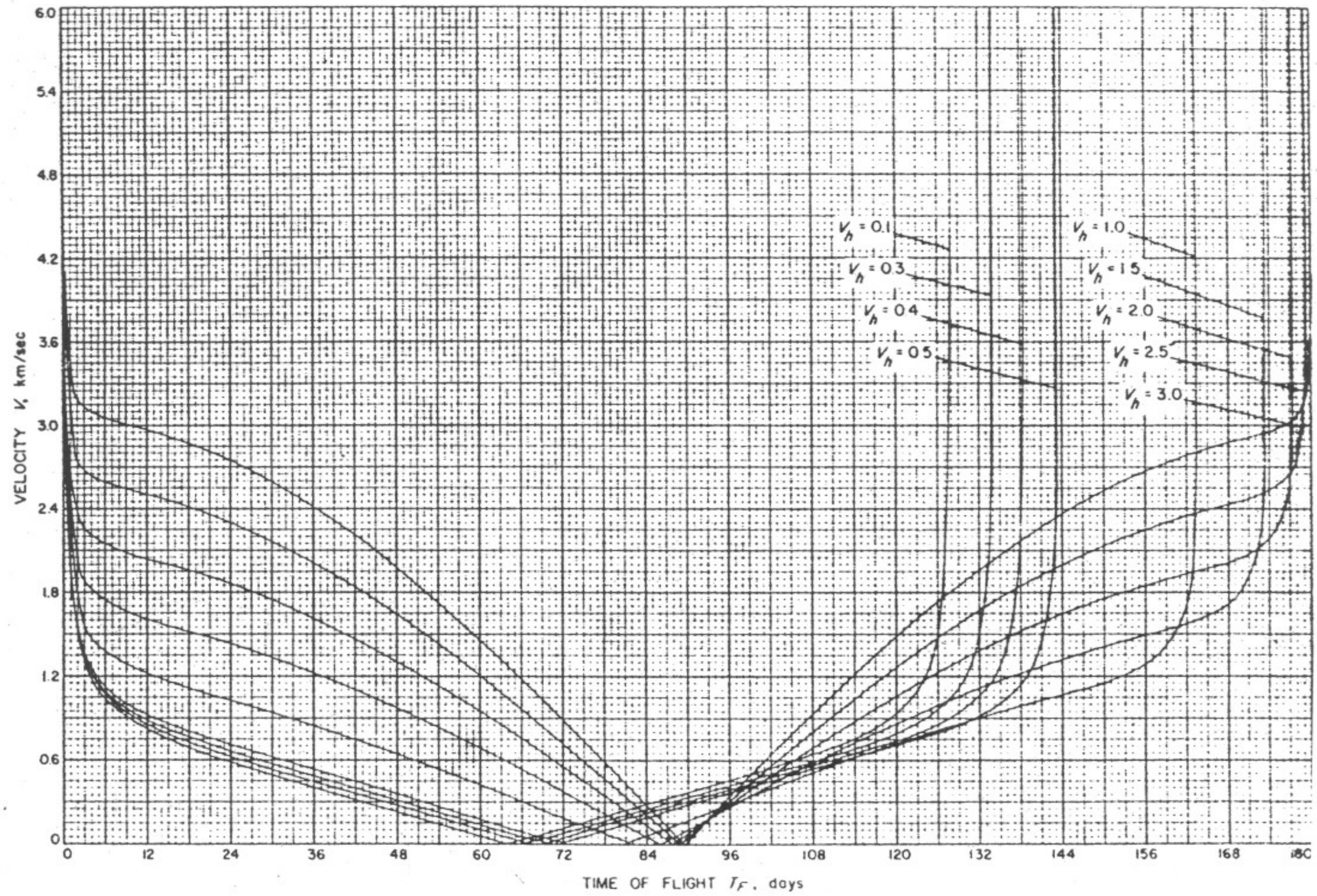

Figure 1. Velocity vs time of flight for various hyperbolic-excess speeds V_h

height until it reached a maximum point. It would then begin to descend back to Earth, and return to Earth in the same time as it took to reach maximum height from launch. The Sun-based observer would find the speed of the Earth and probe to be equal.

Either method of visualization is helpful to understand the motion of this class of trajectories. However, the former provides a simplified basis for an approximate solution to the restricted four-body problem. The Earth-based observer, using a cartesian coordinate system with the X and Y axes lying in the ecliptic plane (X being in the direction of the vernal equinox, and Z along the north ecliptic pole), sees all the motion along the Z direction and almost none along X and Y. Thus, the only equation of motion for the probe is

$$\ddot{Z} = -\frac{GM_E}{Z^2} - GM_s\left[\frac{Z}{(Z^2 + R_s^2)^{3/2}}\right] - GM_m\left[\frac{Z}{(Z^2 + R_m^2)^{3/2}}\right] \qquad (1)$$

where G is the universal gravitational constant; M_E, M_s, M_m, are the masses of the Earth, Sun, and Moon, respectively; R_s is the mean Earth–Sun distance (149,598,50 km); and R_m is the mean Earth–Moon distance (384,40 km). In Equation (1) it is assumed that the orbit of th Earth is circular about the Sun with mean distance R_s and the Moon's orbit is circular about the Earth wit mean distance R_m. In addition, the Moon's orbit lies i the ecliptic plane. This is the restricted four-body prob lem. Equation (1) can be integrated to give

$$\dot{Z}^2 = \frac{2GM_E}{Z} - \frac{GM_sZ^2}{R_s^3} + \frac{2GM_m}{(Z^2 + R_m^2)^{1/2}} + C \qquad (2$$

In this equation, it is assumed that the distance Z i always much less than the Earth–Sun distance R_s, so tha

$$\frac{1}{(Z^2 + R_s^2)^{1/2}} \simeq \frac{1}{R_s}\left[1 - \frac{1}{2}\left(\frac{Z}{R_s}\right)^2\right] \qquad (3$$

. is interesting to note that the form of Equation (2) . similar to (and can be reduced to, by eliminating the un and Moon) the vis viva integral of two-body theory. ın important difference, however, is the presence of the ıegative solar term, $-2GM_s Z^2/R_s^3$. It is essentially the .ction of this term which causes the probe to slow to ero speed at maximum distance and return to Earth, ven though its two-body energy far exceeds the two-ody escape condition. Equation (2) is useful (if the unar term is eliminated) in that if C is known, the maxi-num distance can be found by equating $\dot{Z}$ to zero and olving a resulting cubic in Z. Solution of the cubic pro-ides *three* roots or values of Z where $\dot{Z} = 0$. This result s not predicted from physical reasoning and may well e a spurious one because of the approximate analysis. Further numerical evaluation is necessary to determine he significance of the three roots. Some numerical eval-ıation of Equation (2) has been accomplished. Values of Z and $\dot{Z}$ obtained from an exact integrating trajectory program were substituted into Equation (2) to test the "constancy" of C. Figure 2 shows a plot of C as a function of flight time. The constant was evaluated both with and without the lunar term. It is evident that the results improve if the lunar term is included. C has an average value of about 0.08 km²/sec² and a maximum variation of 15%. The cause of this variation is presumed to be due to the approximate analysis used, since circularity of Sun and Moon orbits was assumed and Earth's oblateness was neglected. In addition, it was noted that the variation reduces to as low as 4% for higher energy orbits such as $C = 2.25$ km²/sec².

Further effort is suggested on this problem because of the existence of the cubic in Z. It has been determined numerically that one of the roots coincides with the condition, $\dot{Z} = 0$. What the other two roots mean is yet to be determined.

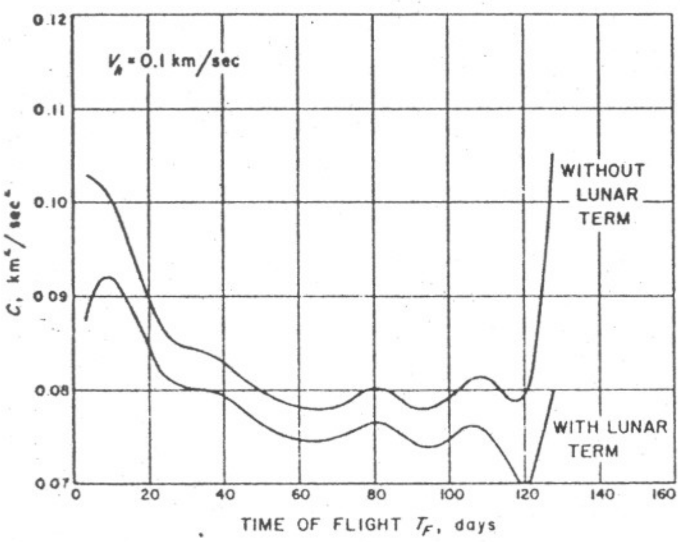

Figure 2. Test of the constancy of C

B. Addition of Range in Orbit Determination of Venus Probe

J. D. Anderson

A preliminary investigation of the use of an additional block of range data in the orbit determination of a Venus probe has been completed. This block of data could be defined by a time tag, the time of the first range point for example, and by an interval of time over which data is obtained at some prespecified rate. Then the improvement in certain orbital parameters could be specified for various blocks of data as well as for various combinations of blocks. Clearly the orbital parameters would ultimately be defined in terms of mission objectives and might take the form of miss components at Venus. Then, if an error description of these mission objective parameters had been obtained from some assumed data distribution, consisting of angular and range rate measurements, the degree of improvement in the orbit determination because of the addition of range measurements would follow. In particular, if only one block of range data defined as above is available, it would be possible to determine the optimum time point with respect to mission objectives at which the range block should be taken.

This particular problem is the one that is investigated here. The time interval of the block is assumed to be sufficiently small so that it is legitimate to treat the range data as a single measurement at a time t of observation. Further, the analysis is designed only to yield a qualitative result, the point in the trajectory where a range block is most useful in reducing the terminal miss. Also, a rigorous solution of the problem is avoided to eliminate lengthy numerical computations; for example, a numerical integration of the equations of motion for the probe. Therefore, a considerably simplified model of the orbit is employed based on the Hohmann ellipse. This orbit is a minimum-energy two-body trajectory where the probe is launched at aphelion, the probe's greatest distance from the Sun, and intercepts the orbit of Venus at perihelion, the distance of the probe's closest approach to the Sun. In effect, the motions and influence of the

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