NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
CONTRACT NO. NAS 7-100

Technical Report No. 32-273

Constants and Related Data Used in Trajectory Calculations at the Jet Propulsion Laboratory

Victor C. Clarke, Jr.

C. R. Gates
TWH
C. Gates, Chief
Systems Analysis Section

JET PROPULSION LABORATORY
CALIFORNIA INSTITUTE OF TECHNOLOGY
PASADENA, CALIFORNIA

May 1, 1962

CONTENTS

I. INTRODUCTION

The purpose of this report is to set forth the list of constants and related data currently in use for space trajectory computation for the *Ranger, Mariner,* and *Surveyor* programs. In the past, efforts have been made by trajectory design personnel within NASA to standardize a set of constants. Such an effort resulted in a meeting held at Marshall Space Flight Center on May 4, 1961, and attended by representatives of Goddard and Marshall Space Flight Centers, Jet Propulsion Laboratory, Langley and Lewis Research Centers, and what was then Space Task Group. The minutes of this meeting were published by R.K. Squires of GSFC and contain the values and sources of the data that were agreed on as the tentative NASA Standard Set of Constants. To the author's knowledge, this set has never been officially sanctioned or published. However, as trajectory work must proceed, this set (in slightly modified form) has been adopted and is used by JPL and by those contractors associated with the *Ranger, Mariner,* and *Surveyor* programs, which JPL administers. Again, to the best of the author's knowledge, this set of constants has been adopted and is being used by GSFC and STG. Thus, in spite of the lack of official sanction, these constants are in widespread use.

For those interested in adopting this set of constants, it should be noted that their chief qualification is *standardization*. However, a real attempt was made to select a set which might be termed the "best available at the time." The plain truth, especially with regard to lunar and planetary constants, is that there is a fair degree of uncertainty as to their values, and different experimenters have determined values of the same constant which differ widely and whose probable errors do not even overlap. Fortunately, a fair activity is at present being stimulated within NASA to make new and better determinations of these constants. One such activity was the recent precise determination (Ref. 1) of the astronomial unit by JPL using the Goldstone Tracking Station. The results of that determination are reflected below in that the value given is based on the data reduction as of May 31, 1961. The reported (and more final) value as of March 8, 1962, is 1 au = 149,598,845 ± 250 km. This is only slightly different from the "standardized" value given below.

In the area of lunar and planetary ephemerides, an intense effort is being conducted at JPL by P. Peabody, D. Holdridge, and N. Block, in cooperation with the Nautical Almanac Office, to improve the quality of the ephemerides. Already significant improvements in both position and velocity ephemerides have been made by these workers.

II. EARTH CONSTANTS

A. Potential Function*

The potential function of the Earth is expressed by

$$\Phi(R, \phi) = \frac{GM_E}{R}\left[1 + \frac{JR_E^2}{3R^2}(1 - 3\sin^2\phi) + \frac{H}{5}\frac{R_E^3}{R^3}(3 - 5\sin^2\phi)\sin\phi\right.$$

$$\left.+ \frac{D}{35}\frac{R_E^4}{R^4}(3 - 30\sin^2\phi + 35\sin^4\phi)\right]$$

where

$$GM_E = 3.986032\ (\pm 0.000030) \times 10^5\ \text{km}^3/\text{sec}^2$$

$$R_E = \text{equatorial Earth radius}$$

$$= 6378.165\ (\pm 0.025)\ \text{km}$$

$$R = \text{geocentric radius}$$

$$\phi = \text{geocentric latitude}$$

$$J = 1.62345\ (\pm 0.00030) \times 10^{-5}$$

$$H = -0.575\ (\pm 0.025) \times 10^{-5}$$

$$D = 0.7875\ (\pm 0.0875) \times 10^{-5}$$

*Recommended, in slightly different form, by W. Kaula of GSFC at the MSFC meeting, May 4, 1961.

The values of GM_E, J, and R_E are consistent with the values of the geodetic parameters of flattening f and gravity g_e,

$$f = 1/298.30$$

$$g_e = 0.00978030 \text{ km/sec}^2$$

The values of R_E, f, and g_e are consistent with those specified in the Department of Defense (DOD) World Geodetic System 1960.

B. Rotation Rate

The Earth's rotation rate ω_E is given by

$$\omega_E = \frac{360}{86164.09892 + 0.00164T} \text{ deg/sec}$$

where T is the number of Julian Centuries of 36525 days from 1900 Jan 0.5 UT (Julian Date = 2415020.0). For the year 1962

$$\omega_E = 4.1780742 \times 10^{-3} \text{ deg/sec}$$

C. Atmospheric Model

The reference atmosphere for boost vehicle trajectory calculations for launches from Cape Canaveral, Florida, is given in Ref. 2, 3, and 4. Intensive studies of this subject have been conducted by O.E. Smith of MSFC.

D. Miscellaneous Constants

1. Conversion Factors. The National Bureau of Standards chart reproduced in Fig. 1 gives the necessary conversion factors between the English and metric systems.

2. Velocity of Light. The velocity of light c (Ref. 5) in a vacuum is based on the determination of Froome

$$c = 299792.5 \text{ km/sec}$$

3. Ephemeris – Universal Time Reduction. The relation between Ephemeris Time (ET) and Universal Time (UT) is given in Ref. 6. For 1962, the difference is

$$\Delta T = \text{ET} - \text{UT} = 34 \text{ sec}$$

4. Radar Station Coordinates. An extensive tabulation of the geographic coordinates and other pertinent data of 102 tracking and telemetry stations scattered throughout the world is given in the "Tracking Station Directory," maintained by P.A. Tardani of JPL.

III. LUNAR CONSTANTS

1. Earth-Moon Mass Ratio

$$M_E/M_m = 81.335$$

2. $GM_m = 4.900759 \times 10^3 \text{ km}^3/\text{sec}^2$

3. Mean Lunar Radius

$$R_m = 1738.09 \text{ km}$$

4. Moments of Inertia About Principal Axis

$$A = 0.88746 \times 10^{35} \text{ kg meters}^2$$

$$B = 0.88764 \times 10^{35} \text{ kg meters}^2$$

$$C = 0.88801 \times 10^{35} \text{ kg meters}^2$$

These values were calculated from the values $a = 1738.57$ km, $b = 1738.21$ km, $c = 1737.49$ km taken from Ref. 7 using

$$A = \frac{M_m}{5}(b^2 + c^2)$$

$$B = \frac{M_m}{5}(c^2 + a^2)$$

$$C = \frac{M_m}{5}(a^2 + b^2)$$

where

$$M_m = \frac{M_E}{81.335}$$

and

$$M_E = 5.975 \times 10^{27} \text{ gm}$$

5. Selenographic Coordinates. The formulas and expressions for computing the transformations to selenographic coordinates are given in Ref. 8. It should be noted, however, that the formulas for the nutation in longitude and obliquity given in this reference do not apply. Better expressions for these quantities are given by Woolard (Ref. 9).

IV. PLANETARY CONSTANTS

1. Astronomical Unit

$$1 \text{ au} = 149.599000 \times 10^6 \text{ km}$$

2. Mass Ratios and Gravitational Constants. The mass ratios and gravitational constants are given in Table 1. The gravitational constants were computed using the Gaussian constant and the Sun-to-planet mass ratios from

$$GM_{planet} = \frac{GM_{sun}}{M_s/M_{planet}}$$

V. LUNAR AND PLANETARY EPHEMERIDES*

A. Current STL – JPL Ephemeris Tapes

The heliocentric and geocentric tapes were derived from the same sources, the geocentric values being obtained from the heliocentric data by vector addition. Only positional data are available directly on the magnetic tape, the velocity being obtained by numerical differentiation. The coordinates of the various planets have been obtained from a variety of source works with different degrees of accuracy and slight inconsistencies in the mathematical models of the solar system.

The coordinates of Mercury were taken from the British Planetary Coordinates, which give the rectangular coordinates to 3 decimals (in astronomical units). Since the mean distance of Mercury is 0.387099, the accuracy is not quite 2½ significant digits.

The coordinates of Venus, the Earth – Moon barycenter, and the Earth were obtained from Herget's evaluation of the positions as determined from Newcomb's tables (not the theory), after applying small corrections to the mean anomaly of each body (Ref. 12, 13). Comparison of these values with those obtained by direct evaluation of the Newcomb theory yields residuals in longitude with a mean error of $0\overset{''}{.}3$, with occasional deviations as large as $0\overset{''}{.}9$.

The coordinates of Mars were obtained from a numerical integration by Herget in which the orbit was fitted by least-squares to agree with a second-order theory by Clemence.

The coordinates of Jupiter, Saturn, Uranus, Neptune, and Pluto were taken from Ref. 14, which contains values obtained by simultaneous numerical integration of the five outer planets. In this computation the masses of the inner bodies were lumped with that of the Sun. Hence, the positions obtained are *not* heliocentric, but rather represent the coordinates of five bodies of a mathematical model which does not depart far from the true gravitational model. To obtain truly heliocentric values it is necessary to apply corrections given by Clemence in Ref. 15.

The positions of the Moon were computed from data given in the *Improved Lunar Ephemeris* (Ref. 16) according to Brown's theory (not the tables) and suitably corrected to remove the effects of aberration and certain emperical terms.

---

*This section is by N. Block of JPL.

B. Ephemeris for the JPL Space Trajectories Program

The coordinates of all bodies except the Earth – Moon barycenter and Venus are the same as described in Section V-A. For Venus and the Earth – Moon, a Cowell integration scheme was used to provide a double-precession calculation over a 10-year arc from 1960 to 1970, using a least-squares procedure to iterate upon the injection values by computing residuals from the ephemeris values described in Section V-A. The iterative procedure was continued until no further reduction in the standard deviation over the 10-year arc could be realized. The coordinates of the Earth are obtained from those of the Earth – Moon and the Moon by a simple calculation involving the Mars ratio. The Cowell scheme provided "smooth" values of both position and velocity for the period covered. The residuals between this "tracked" ephemeris and the ephemeris described in Section V-A show clearly the presence of nongravitational effects of both a periodic and secular nature. The periodic effect is sinusoidal with a period approximately equal to the sidereal period of the body and is known to be mostly due to the omission of perturbation terms in the latitude.

The secular effect appears to be parabolic or cubic and is not clearly understood, but it is probably due to the inadequacies of Newcomb's theory – which is only of first order – to represent a gravitational model which is known to require at least a third-order theory to represent it to the accuracies desired. Nonetheless, the "tracked" ephemeris provides perhaps the "best" ephemeris values currently available, allowing for possible variations of injection conditions. The provisional ephemeris described herein was adopted on March 20, 1962, for engineering calculations using the Space Trajectories Program.

C. Future Plans for Improved Ephemerides

The coordinates of Mercury can be greatly improved by the direct evaluation of Newcomb's theory, which would provide values good to the seventh decimal of an au. It should be noted that the theory is only of first order.

The coordinates of Mars can now be obtained from Clemence's third-order theory of Mars by evaluation of the Fourier terms according to the Hansen method.

The coordinates of the five outer planets can be corrected as described in Section V-A to provide truly heliocentric values.

The coordinates of the Moon can now be computed directly from Brown's improved lunar theory, using a computer program written by N. Block of JPL.

It is proposed that the coordinates of the bodies obtained as just described next be "tracked" over sufficiently long arcs, and that a procedure be evolved to join such segments into a consistent ephemeris over the entire interval to be covered. Such a procedure will also provide velocities.

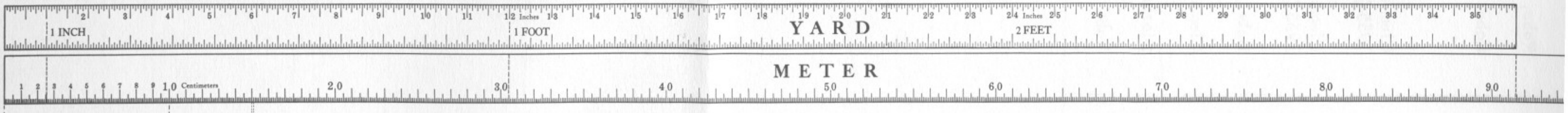
Future plans include the development of third-order planetary theories and the "tracking" of values obtained from them.

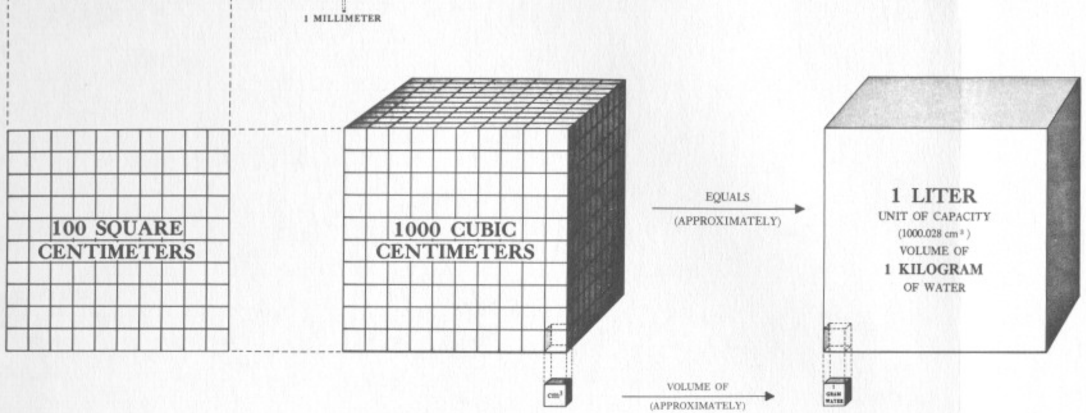
Table 1. Planetary mass ratios and gravitational constants

Planet	Mass ratio M_s/M_{planet}	Gravitational constant GM_p, au^3/day^2	Gravitational constant GM_p, km^3/sec^2	Reference
Sun	1	$2.959122083 \times 10^{-4}$	$1.32715445 \times 10^{11}$	
Mercury	6120000	4.835167×10^{-11}	2.168553×10^{4}	Rabe, 1950 (Ref. 10)
Venus	408645	7.241303×10^{-10}	3.247695×10^{5}	Rabe, 1950 (Ref. 10)
Earth	332951.3	8.887552×10^{-10}	3.986032×10^{5}	
Mars	3088000	9.582649×10^{-11}	4.297780×10^{4}	Clemence, Brouwer, 1955 (Ref. 11)
Jupiter	1047.39	2.825234×10^{-7}	1.267106×10^{8}	Clemence, Brouwer, 1955 (Ref. 11)
Saturn	3500	8.454635×10^{-8}	3.791870×10^{7}	Baker, 1961 (adopted)
Uranus	22869	1.293945×10^{-8}	5.803292×10^{6}	Clemence, Brouwer, 1957
Neptune	18889	1.566585×10^{-8}	7.026072×10^{6}	Biesbroeck, 1957
Pluto	400000	7.397805×10^{-10}	3.317886×10^{5}	Rabe

U.S. DEPARTMENT OF COMMERCE
NATIONAL BUREAU OF STANDARDS

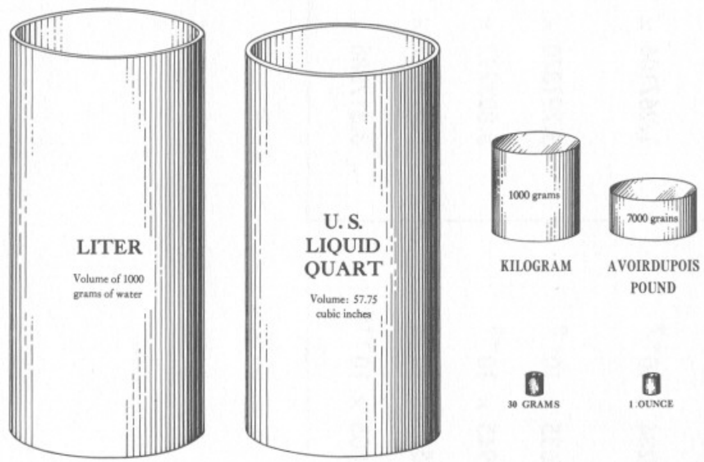
THE METRIC SYSTEM OF MEASUREMENT



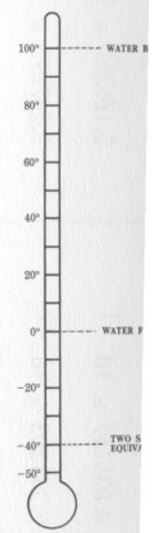


GRAPHIC COMPARISONS

IN THE FOLLOWING ILLUSTRATIONS THE UNITS IN EACH PAIR HAVE EQUAL BASES SO THAT THE HEIGHTS SHOW DIRECTLY THE RELATIVE VALUES OF THE UNITS



TEMPERATU



CONVERSION: TEMPERATURE FAHREN
TEMPERATURE CELSIUS

ABSOLUTE OR KELVIN TEMPERATURE

*Formerly known as Centigrade

THE METRIC SYSTEM

The metric system comprises the meter as the unit of length and the kilogram as the unit of mass. The unit of fluid capacity, the liter, is the volume of one kilogram of water and is approximately equal to 1/1000 of a cubic meter. All other metric units are the decimal subdivisions or multiples of these three basic units, and are named according to the table of prefixes below.

In conjunction with the metric system the Celsius temperature scale is used and the second is taken as the unit of time. The temperature scale is defined by certain fixed points (see table). The second is defined as $\frac{1}{31,556,925.9747}$ of the tropical year 1900 (approximately 1/86,400 of the average day) and is determined by astronomical observation.

Responsibility for the preservation and promotion of the metric system is centered at the International Bureau of Weights and Measures, at Sèvres, France, which was established under the terms of the Treaty of the Meter of 1875. The United States is an adherent to the Treaty of the Meter.

Multiples and Submultiples	Prefixes	Symbols
1 000 000 000 000 = 10^{12}	tera	T
1 000 000 000 = 10^{9}	giga	G
1 000 000 = 10^{6}	mega	M
1 000 = 10^{3}	kilo	k
100 = 10^{2}	hecto	h
10 = 10	deka	dk
0.1 = 10^{-1}	deci	d
0.01 = 10^{-2}	centi	c
0.001 = 10^{-3}	milli	m
0.000 001 = 10^{-6}	micro	μ*
0.000 000 001 = 10^{-9}	nano	n
0.000 000 000 001 = 10^{-12}	pico	p

For example, 1000 meters (or 10^3 meters) is called a kilometer, and one millionth of a gram (or 10^{-6} gram) is called a microgram.

*1 millionth of a meter is called a **micron**, and is abbreviated simply μ.

Most Common Uses of Prefixes

Length

1 000 millimeters mm 100 centimeters cm 10 decimeters dm	= 1 METER (about 40 inches)	m
1 000 meters	= 1 kilometer (about 5/8 mile)	km

Mass

1 000 000 micrograms μg 1 000 milligrams mg	= 1 GRAM (about 15 grains)	g
1 000 grams	= 1 kilogram (about 2 pounds)	kg

Capacity

1 000 milliliters ml	= 1 LITER (about 1 quart)	l
1 000 liters	= 1 kiloliter	kl

EQUIVALENTS

1 YARD = 0.9144 METER
1 AVOIRDUPOIS POUND = 0.453 592 37 KILOGRAM
(See Federal Register, July 1, 1959)

Length

Centimeter	= 0.3937 inch
Meter	= 3.28 feet
Meter	= 1.094 yards
Kilometer	= 0.621 statute mile
Kilometer	= 0.5400 nautical mile
Inch	**= 2.54 centimeters**
Foot	**= 0.3048 meter**
Yard	**= 0.9144 meter**
Statute mile	= 1.61 kilometers
Nautical mile	**= 1.852 kilometers**

Area

Sq centimeter	= 0.155 sq inch
Sq meter	= 10.76 sq feet
Sq meter	= 1.196 sq yards
Hectare	= 2.47 acres
Sq kilometer	= 0.386 sq mile
Sq inch	= 6.45 sq centimeters
Sq foot	= 0.0929 sq meter
Sq yard	= 0.836 sq meter
Acre	= 0.405 hectare
Sq mile	= 2.59 sq kilometers

Volume

Cu centimeter	= 0.0610 cu inch
Cu meter	= 35.3 cu feet
Cu meter	= 1.308 cu yards
Cu inch	= 16.39 cu centimeters
Cu foot	= 0.0283 cu meter
Cu yard	= 0.765 cu meter

Capacity

Milliliter	= 0.0338 U.S. fluid ounce
Liter	= 1.057 U.S. liq quarts
Liter	= 0.908 U.S. dry quart
U.S. fluid ounce	= 29.57 milliliters
U.S. liq quart	= 0.946 liter
U.S. dry quart	= 1.101 liters

Mass or Weight

Gram	= 15.43 grains
Gram	= 0.0353 avdp ounce
Kilogram	= 2.205 avdp pounds
Metric ton	= 1.102 short or net tons
Grain	= 0.0648 gram
Avdp ounce	= 28.35 grams
Avdp pound	= 0.4536 kilogram
Short or net ton	= 0.907 metric ton

Equivalents shown in bold face type are exact; all others are approximate.
For complete conversion tables, see NBS Miscellaneous Publication 233.

U.S. PROTOTYPE KILOGRAM NO. 20 AND PROTOTYPE METER NO. 27

Maintained by
NATIONAL BUREAU OF STANDARDS

The U.S. customary units are derived from the national metric standards by mathematical equivalents. These standards provide the basis for a consistent measurement system in the United States, and are periodically compared with the standards at the International Bureau of Weights and Measures.

The wavelength of orange light emitted by excited atoms of the element krypton 86 has been proposed as the international length standard. The meter would then be defined as 1,650,763.73 wavelengths of this light. This will make the international standard indestructible, independently reproducible, and universally available.

National Bureau of Standar
Issued
(Supersedes Misce

Slight changes in the paper due to

REFERENCES

1. Muhleman, D. O., D. B. Holdridge, and N. Block, *The Astronomical Unit Determined by Radar Reflections from Venus*, Technical Report 32-221, Jet Propulsion Laboratory, Pasadena, California, March 8, 1962.

2. Smith, O. E., *A Reference Atmosphere For Patrick AFB, Florida*, NASA TN D-555, Washington, D. C., July 29, 1960.

3. Smith, O. E., *A Reference Atmosphere For Patrick AFB, Florida*, NASA TN D-595, Washington, D. C., March 1961.

4. Smith, O. E., and H. B. Chenoweth, *Range of Density Variability From Surface to 120 km Altitude*, NASA TN D-612, Washington, D. C., July 1961.

5. Cohen, E. R., K. M. Crowe, and J. W. M. Du Mond, *Fundamental Constants of Physics*, Vol. 1, Interscience Publishers, New York, 1957.

6. *American Ephemeris and Nautical Almanac*, published annually by the U. S. Government Printing Office, Washington, D. C.

7. Makemson, M. W., R. M. L. Baker, Jr., and G. B. Westrom, "Analysis and Standardization of Astrodynamic Constants," *Journal of Astronautical Sciences*, Vol. 8, No. 1, Spring 1961.

8. Kalensher, B. E., *Selenographic Coordinates*, Technical Report 32-41, Jet Propulsion Laboratory, Pasadena, California, February 24, 1961.

9. Woolard, E. W., "Theory of the Rotation of the Earth Around Its Center of Mass," *Astronomical Papers of the American Ephemeris*, Vol. XV, Part 1, U. S. Government Printing Office, Washington, D. C., 1953.

10. Rabe, E. K., "Derivation of Fundamental Astronomical Constants from the Observations of Eros During 1926-1945," *Astronomical Journal*, Vol. 55, 1950, pp. 112-125.

11. Clemence, G. M., and D. Brouwer, "The Accuracy of the Coordinates of the Five Outer Planets and the Invariable Plane," *Astronomical Journal*, May 1955, p. 118.

12. *Astronomical Papers of the American Ephemeris*, Vol. XIV, U. S. Government Printing Office, Washington, D. C., 1953.

REFERENCES (Cont'd)

13. *Astronomical Papers of the American Ephemeris*, Vol. XV, Part 3, U. S. Government Printing Office, Washington, D. C., 1955.

14. *Astronomical Papers of the American Ephemeris*, Vol. XII, U. S. Government Printing Office, Washington, D. C., 1951.

15. *Astronomical Papers of the American Ephemeris*, XIII, Part 5, U. S. Government Printing Office, Washington, D. C., 1954.

16. *Improved Lunar Ephemeris 1952-1959*, U. S. Government Printing Office, Washington, D. C., 1954.