Periodic Swing-By Orbits between Earth and Venus

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The general problem of finding fly-by dates for multiple swing-by missions involves an iterative search in a space with dimension equal to the number of swingbys assuming the launch date and final arrival date are specified. A double swing-by trajectory that visits Mars and Venus requires a search in two dimensions. Periodic orbits connecting Earth and Venus require a search in a space of dimension between 10 and 15. This paper reports the results of a study of the latter class of orbits. The computing procedures are generally applicable to any multiple swing-by problem. A summary is given of the computational experience gained. Trajectory data are presented for those periodic orbits which were computed.

Introduction

THE use of a multiple swingby as part of an interplanetary mission was considered as early as 1925 by Hohmann[1] and 1956 by Crocco.[2] They each proposed interplanetary fly-by missions that would take a vehicle past both Mars and Venus before returning to Earth. Several investigators[3-6] have subsequently studied this class of mission in more detail. It was Minovitch,[6] however, who first recognized the fundamental role which the planetary flyby can play in trajectory design. He saw the planets as sources of free thrust which could be utilized to project a vehicle from one planet to another without the use of fuel. In Ref. 7 he described, for example, a round trip mission leaving and arriving Earth with six intermediate flybys at Venus, Mars, Earth, Mars, Earth, and Venus. He further proposed an interplanetary transportation network,[8] using multiple fly-by trajectories that would continue indefinitely. In the interest of finding such trajectories it has been proposed that the natural periodicity of the solar system be used to develop periodic orbits.[9]

For the orbit to be periodic, the spacecraft must recurrently flyby a sequence of planets. The first and last flyby of the sequence must therefore occur at the same planet with identical spacecraft velocities and absolute planet orientations. For this reason the duration of one period of an acceptable orbit is restricted to integral multiples of the time required for the encountered planets to repeat their absolute orientation. Ideally a perfectly established periodic orbit would result in no subsequent thrust requirements. In practical application injection errors and small perturbations from the periodic orbit will inevitably be present. Hickman[10] has shown, however, that the guidance requirements for nominal periodic orbits connecting Earth and Venus are quite reasonable.

Although periodic orbits are perhaps the most difficult of the multiple swing-by problems to analyze, their large number of swingbys make them particularly interesting. The computing procedures are generally applicable to any multiple swing-by problem. Rall,[11] for example, has used the techniques reported here to find periodic orbits connecting Earth and Mars.

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Multiple Swing-By Orbits

All the results in this report are based on patched conic analysis.[12] The interplanetary trajectory is completely defined by the dates at each planet. Assume that only the initial and final dates of a multiple flyby are given. Then the number of dates to be selected is equal to the number of swingbys. For the case of a double swingby, all the possible combinations of dates are represented by the points on a plane. The locus of dates that produce equal magnitude inbound and outbound hyperbolic velocity at one planet is a line in the plane. The locus of dates that produce equal magnitude inbound and outbound hyperbolic velocity at the other planet is another line in the plane. The intersection of the two lines represents a pair of dates that satisfies the first necessary condition for a multiple swingby. Figure 1 shows these loci for a double swingby of Venus and Mars. This example has six intersections. The problem of finding the intersections is equivalent to finding the zeros of a function defined as the sum of the squares of the differences in relative velocity magnitudes at the two flybys. At each intersection the value of the function vanishes and thereby achieves a global minimum. The function represents a surface in three dimensions. The value of the function determines the height of the surface above the date-at-Mars-date-at-Venus plane. Contours are sketched around one of the better behaved intersections. A study of Fig. 1 shows how complex the surface can be even with only two swing-by dates to consider. Successful solution by iterative methods is contingent upon the initial approximation of the independent variables. A poor initial choice may cause the iterative procedure to find a local minimum rather than the desired solution. It is also possible for the search process to bog down in a ravine of the surface. Since the problem is compounded for higher-dimensional spaces, the need for a good initial guess is apparent.

The feasibility of a swing-by orbit also rests on each planet's capability to turn the inbound relative velocity into the outbound relative velocity on a hyperbolic path that does not pass below the planet surface. If this is possible, the dates of the planet encounters and corresponding planet locations are the solution to the problem. In this report all flybys occurring beyond 1.1 planet radii are considered acceptable. Not all the intersections in Fig. 1 satisfy the second condition. Each point needs to be found and then tested to ascertain that the hyperbolic path does not go below the surface.

Direct Return Orbits

A direct return orbit is a sun-centered ellipse that returns a spacecraft to the same planet from which it was launched

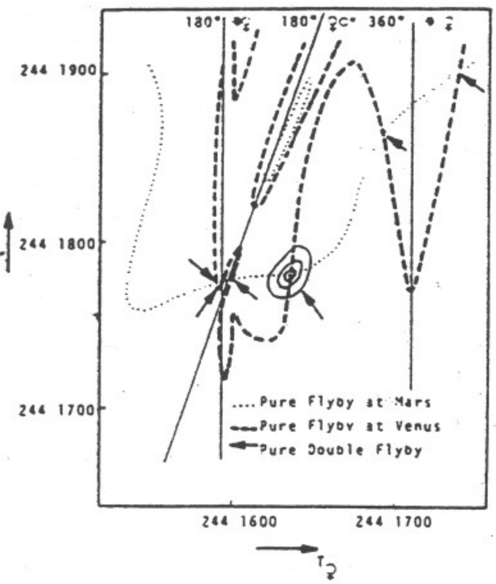

Fig. 1 Flyby dates for double swingby.

without interruption by encounters with other planets. These orbits find useful application when the elliptical trajectory between the planets results in large excess hyperbolic velocities. Large excess hyperbolic velocities reduce the maximum allowable turn angle at a given planet and hence the chances for making acceptable flybys above the planet's surface. Large values of the excess hyperbolic velocity can often be reduced by delaying the interplanetary flight with direct returns until the relative planet positions permit interplanetary transfers with lower excess hyperbolic velocities. The long delay is unfortunate, however, the planet and spacecraft remain relatively close to one another during this period, which may be of practical use in the ultimate mission. Two commonly used direct return orbits are the "full-revolution return" and the "symmetric return."

A full-revolution return is a sun-centered elliptical orbit which returns the spacecraft to the launch planet in one launch planet period. Because the spacecraft and planet have equal periods, they must have equal velocity magnitudes relative to the sun. A double infinity of such orbits exists at each encountered planet. When the magnitude of the excess hyperbolic velocity is small with respect to the launch planet's orbital velocity, the two velocities will be nearly perpendicular. Furthermore, the orbital plane of the full-revolution return will be only slightly inclined to the plane of the launch planet's orbit. A "half-revolution return" is a special case of the full-revolution return in which the spacecraft has both the same period and eccentricity as the launch planet's orbit. In these cases the relative velocity vector is nearly perpendicular to the plane of the launch planet's orbit and the spacecraft returns to the launch planet after a half revolution of the sun.

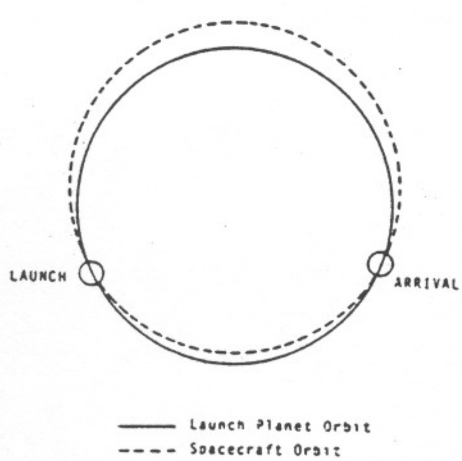

Fig. 2 Symmetric return orbit.

A symmetric return is a sun-centered elliptical orbit which is coplanar with the launch planet's orbit and returns the spacecraft to the launch planet after a time greater than one launch planet period.[4] In the construction of periodic orbits, the symmetric returns of most interest are those with times of flight greater than one, but less than two, launch planet periods. An example of a symmetric return is shown in Fig. 2. In this diagram the spacecraft and planet each pass through the arrival point (independently) and complete one circuit on their respective trajectories before the encounter is made. For a symmetric return orbit, the launch and arrival relative velocity magnitudes will be equal when the launch planet is in circular orbit.

Iterative Solutions

Swing-by orbits are obtained by adjusting approximate dates of the planet flybys until differences in the relative velocity magnitudes at each flyby simultaneously vanish. Since the second flyby of a full-revolution return is constrained to occur one launch planet period after the first, only one fly-by date is an independent variable. Similarly, only one date can be considered independent for direct return orbits consisting of two or more consecutive full-revolution returns. In any event the number of independent dates N equals the total number of interplanetary transfers and symmetric returns in a periodic orbit.

Successful solution by iterative methods is contingent upon the initial approximation of the independent variables. Approximate solutions to periodic orbits are not easily obtained. Hollister[9] has discovered three periodic orbits that connect Earth and Venus. In the circular coplanar case each orbit includes a direct return orbit at Earth, an interplanetary transfer to Venus, two direct return orbits at Venus, and an interplanetary transfer back to Earth. The duration of each orbit is 3.2 yr; they differ in the type of direct return orbits occurring at Earth and Venus. The first orbit has a full-revolution return at Earth and two consecutive full-revolution returns at Venus. The second orbit consists of a

Table 1 Key to orbit descriptions[a]

Periodic orbit number	Direct return orbits at Earth	Direct return orbits at Venus
1 and 1H	5FR	5TFR
2 and 2H	5FR	5FRSY
3 and 3H	5SY	5TFR
4	2FR, SY, 2FR	5TFR
5	FR, 2SY, 2FR	5TFR
6	2FR, 2SY, FR	5TFR
7	FR, 3SY, FR	5TFR
8	FR, 4SY	5TFR
9	5FR	2TFR, FRSY, 2TFR
10	5FR	2TFR, 2FRSY, TFR
11	5FR	FRSY, TFR, 2FRSY, TFR
12	5FR	FRSY, TFR, 3FRSY
13	2FR, SY, 2FR	FRSY, 4TFR
14	2FR, SY, 2FR	FRSY, 2TFR, FRSY, TFR
15	2FR, SY, FR, SY	FRSY, 2TFR, FRSY, TFR

[a] FR = full-revolution return; SY = symmetric return; FRSY = full-revolution return followed by a symmetric return; TFR = two consecutive full-revolution returns. Direct return orbits are listed in the order they occur.

Table 2 Flyby dates, (Julian date −2440000) orbits 1H–3H

Planet	Orbit 1H	Orbit 2H	Orbit 3H
E	441	417	352
E	806	782	852
V	971	914	970
V	1196	1139	1195
V	1421	1470	1420
E	1592	1612	1542
E	1957	1977	2042
V	2125	2086	2142
V	2350	2311	2367
V	2575	2642	2592
E	2797	2763	2697
E	3163	3128	3197
V	3316	3253	3297
V	3541	3478	3522
V	3765	3809	3747
E	3935	3927	3853
E	4300	4293	4353
V	4471	4427	4477
V	4696	4642	4702
V	4921	4953	4927
E	5077	5107	5038
E	5442	5472	5538
V	5664	5591	5644
V	5889	5816	5869
V	6114	6149	6094
E	6285	6261	6196
	Repeating after 16 years		

full-revolution return at Earth and a full-revolution and symmetric return at Venus. In the third orbit are a symmetric return at Earth and two full-revolution returns at Venus.

For the inclined elliptic case, Earth and Venus repeat their absolute orientation to within several degrees accuracy every 16 yr.[13] The error made by assuming exact periodicity of the solar system is of the same order of magnitude as the error made by the patched conic model. By using five circular coplanar orbits in succession (see Table 1) as an initial approximation, Hollister found solutions for each of the three periodic orbits in the inclined elliptic case. To simplify analysis, the duration of symmetric returns was assumed constant and the launch and arrival relative velocity (V_r) magnitudes on the symmetric returns were assumed equal. The duration of the symmetric returns was chosen in accordance with expected values for the semimajor axes of the symmetric return orbits. The solutions for these three orbits are reproduced in Table 2 where they are denoted as orbits 1H, 2H, and 3H, respectively. Since orbit 1H contains no symmetric returns, the solution for this orbit is rigorous in the patched conic sense. Dates of planet flybys in orbits 2H and 3H are used herein as initial approximations for a solution that eliminates the assumptions of constant time of flight and equal relative velocity magnitudes on symmetric returns.

The function to be minimized can be considered an N dimensional surface whose arguments are the dates of the N planet flybys. The gradient of the function is an N dimensional vector in the direction of the greatest rate of change of the function value. In steepest-descent iterations, dates of the N flybys are incremented to correspond with movement along the gradient vector. Reduction of the function value is guaranteed for sufficiently small date increments. Following reduction of the function value, a new gradient vector is calculated and the iteration repeated.

In Newton-Raphson iterations the difference in the velocity magnitudes at each of the N flybys is expanded in a Taylor series about the current values of the N fly-by dates. Only first-order terms are retained in each Taylor series. Date increments are made to cause each of the linearized expressions for velocity difference to simultaneously vanish. Upon reduction of the function value, velocity differences are expanded in Taylor series about the new fly-by dates and the iteration repeated. Convergence is likely only when the initial date approximations are sufficiently accurate and higher derivatives of the velocity difference expressions are excessively large.

In both the steepest-descent and Newton-Raphson iterations, the first partial derivatives of the differences in velocity magnitude are required with respect to the fly-by dates. Approximate values of the partial derivatives are obtained by calculating the change in velocity difference at each flyby which results from making small changes in the N fly by dates one at a time.

First attempts at obtaining rigorous solutions to the periodic orbit problem employed both steepest-descent and Newton-Raphson procedures. Steepest-descent methods were first used to reduce the sum of the absolute differences in relative velocity (V_r) magnitudes to 0.1 EMOS (Earth Mean Orbital Speed Unit). Newton-Raphson methods were then used to reduce the value from 0.1 EMOS to assumed convergence at 0.005 EMOS. Sometimes oscillations in the date increments indicated that a "ravine" problem had been encountered on the N dimensional surface. The Davidon[14] and conjugate gradient[15] methods were employed when this situation developed.

Attempts to obtain a solution to Hollister's third periodic orbit resulted in convergence to a local minimum rather than a solution. Endeavors were made to sequentially modify orbit 1H until an accurate approximation for orbit 3H could be obtained.

Although orbit 1H and 3H have the same type of direct return orbits at Venus, orbit 1H has five full-revolution returns at Earth and orbit 3H has five symmetric returns at Earth. A solution was first attempted for orbit 1H modified to include one symmetric return and four full-revolution returns at the Earth encounters. An approximate solution for this orbit was obtained by merely replacing one of the full-revolution returns at Earth in orbit 1H by a symmetric return of 1.37 yr duration. The symmetric return was inserted so as to equate the times of flight for the interplanetary transfers on either side of the Earth encounter. Rapid convergence to the orbit solution was achieved with the steepest-descent and Newton-Raphson procedures. Following this favorable outcome, the modified orbit was altered by replacing one of the remaining full-revolution returns by a second symmetric return. A solution was easily obtained in this case and also for orbits with three, four, and finally five symmetric returns at the Earth encounters. Solutions for each of these orbits are listed in Table 3 (orbits 3–8). For orbits 4, 7, and 8, convergence was rapid and predictable. In orbits 5 and 6 oscillation was encountered and convergence slowed at low levels of the function value. In these cases the conjugate gradient method was used to further reduce the function value.

Although a solution for each of the modified orbits was required to obtain a rigorous solution to orbit 3H, the modified orbits are more than a means to an end. Each is a unique periodic orbit with characteristics far different from similar orbits. This fact is illustrated by a comparison of orbits 5 and 6. Although these orbits each have two symmetric and three full-revolution returns at Earth, the order in which the returns occur differs. In orbit 5 the minimum pass distance occurs during a flyby of Earth in which the spacecraft comes within 1.16 Earth radii of the planet surface. In orbit 6 the minimum pass distance also occurs during a flyby

Table 3 Solutions for orbits 1–15

Planet	Date[a]	V_r,[b] EMOS	θ,[b] deg	R_{min}[b]	Planet	Date[a]	V_r,[b] EMOS	θ,[b] deg	R_{min}[b]	Planet	Date[a]	V_r,[b] EMOS	θ,[b] deg	R_{min}[b]
		Orbit 1					Orbit 2					Orbit 3		
E	441	0.155	81.1	1.59	E	415	0.137	69.3	2.87	E	382	0.162	58.8	2.78
E	806	0.155	81.1	1.59	E	780	0.137	69.3	2.87	E	877	0.162	34.5	6.42
V	971	0.179	61.3	1.77	V	914	0.231	20.9	4.99	V	1012	0.248	20.7	4.38
V	1196	0.179	61.3	1.77	V	1139	0.231	18.4	5.83	V	1237	0.248	19.7	4.66
V	1421	0.179	61.3	1.77	V	1470	0.230	30.9	3.06	V	1462	0.248	19.7	4.66
E	1591	0.154	81.7	1.57	E	1614	0.145	77.4	2.02	E	1540	0.165	28.7	7.92
E	1957	0.154	81.7	1.57	E	1979	0.145	77.4	2.02	E	2034	0.164	18.4	13.74
V	2125	0.206	34.6	3.30	V	2086	0.257	14.3	6.26	V	2149	0.136	50.1	4.36
V	2350	0.206	34.6	3.30	V	2311	0.257	14.3	6.26	V	2374	0.136	50.1	4.36
V	2575	0.206	34.6	3.30	V	2643	0.258	6.9	13.92	V	2599	0.136	54.3	3.81
E	2798	0.193	42.5	3.36	E	2762	0.161	67.5	2.18	E	2697	0.176	11.2	21.04
E	3163	0.193	42.5	3.36	E	3127	0.161	67.5	2.18	E	3190	0.176	43.6	3.86
V	3316	0.196	60.2	1.53	V	3252	0.246	16.4	5.85	V	3308	0.204	62.7	1.31
V	3541	0.196	60.2	1.53	V	3477	0.246	8.7	11.80	V	3533	0.204	62.7	1.31
V	3766	0.196	60.2	1.53	V	3809	0.246	23.4	3.83	V	3758	0.204	62.7	1.31
E	3935	0.158	84.1	1.40	E	3927	0.146	85.6	1.57	E	3885	0.215	67.1	1.23
E	4300	0.158	84.1	1.40	E	4293	0.146	85.6	1.57	E	4374	0.216	26.7	5.06
V	4471	0.194	59.2	1.61	V	4426	0.245	20.9	4.44	V	4485	0.158	57.2	2.57
V	4696	0.194	59.2	1.61	V	4651	0.245	20.9	4.44	V	4710	0.158	57.2	2.57
V	4921	0.194	59.2	1.61	V	4983	0.245	12.6	8.00	V	4935	0.158	57.2	2.57
E	5076	0.175	45.4	3.66	E	5108	0.143	66.9	2.81	E	5039	0.165	27.3	8.40
E	5442	0.175	45.4	3.66	E	5473	0.143	66.9	2.81	E	5533	0.165	10.7	25.12
V	5664	0.225	32.0	3.06	V	5590	0.257	16.3	5.42	V	5641	0.144	67.9	2.26
V	5889	0.225	32.0	3.06	V	5715	0.257	10.5	8.87	V	5866	0.144	67.9	2.26
V	6114	0.225	32.0	3.06	V	6147	0.256	13.0	7.06	V	6091	0.144	67.9	2.26
E	6285	0.155	81.1	1.59	E	6259	0.137	69.3	2.87	E	6226	0.162	58.8	2.78

Planet	Date[a]	V_r,[b] EMOS	θ,[b] deg	R_{min}[b]	Planet	Date[a]	V_r,[b] EMOS	θ,[b] deg	R_{min}[b]	Planet	Date[a]	V_r,[b] EMOS	θ,[b] deg	R_{min}[b]
		Orbit 4					Orbit 5					Orbit 6		
E	432	0.133	76.8	2.45	E	442	0.133	80.3	2.21	E	444	0.125	85.0	2.18
E	798	0.133	76.8	2.45	E	808	0.133	80.3	2.21	E	809	0.125	85.0	2.18
V	989	0.180	43.4	4.11	V	977	0.165	34.0	5.25	V	995	0.187	41.7	3.06
V	1214	0.180	43.4	4.11	V	1242	0.165	34.0	5.25	V	1220	0.187	41.7	3.06
V	1439	0.180	43.4	4.11	V	1467	0.165	38.7	3.37	V	1445	0.187	41.7	3.06
E	1614	0.188	78.6	1.16	E	1542	0.173	43.9	3.96	E	1620	0.200	75.7	1.12
E	1980	0.188	78.6	1.16	E	2036	0.173	15.5	15.16	E	1986	0.200	75.7	1.12
V	2133	0.282	18.9	3.79	V	2147	0.136	52.0	4.06	V	2135	0.302	17.5	3.60
V	2358	0.282	13.8	5.41	V	2372	0.136	52.0	4.06	V	2585	0.302	11.4	5.85
V	2573	0.282	19.9	3.58	V	2597	0.136	52.9	3.95	V	2810	0.302	18.0	3.48
E	2710	0.203	57.4	1.86	E	2698	0.171	13.2	18.71	E	2716	0.209	64.1	1.43
E	3201	0.202	35.5	3.94	E	3192	0.171	37.7	5.03	E	3206	0.209	11.4	14.62
V	3313	0.208	30.9	3.77	V	3307	0.186	32.9	4.30	V	3299	0.150	62.8	2.43
V	3538	0.208	26.5	4.59	V	3532	0.186	26.0	5.86	V	3524	0.150	62.8	2.43
V	3763	0.208	26.5	4.59	V	3757	0.186	29.4	4.00	V	3749	0.150	62.8	2.43
E	3929	0.159	86.5	1.28	E	3922	0.135	80.0	2.16	E	3870	0.186	48.0	2.98
E	4294	0.159	86.5	1.28	E	4287	0.135	80.0	2.16	E	4363	0.186	32.3	5.18
V	4468	0.185	57.4	1.86	V	4474	0.148	64.4	2.37	V	4474	0.134	54.9	3.87
V	4693	0.185	57.4	1.86	V	4699	0.148	64.4	2.37	V	4699	0.134	45.4	5.26
V	4918	0.185	57.4	1.86	V	4924	0.148	64.4	2.37	V	4924	0.134	45.4	5.26
E	5074	0.163	46.9	4.03	E	5088	0.143	52.3	4.37	E	5093	0.133	56.6	4.41
E	5439	0.163	46.9	4.03	E	5453	0.143	52.3	4.37	E	5458	0.133	56.6	4.41
V	5659	0.204	34.1	3.41	V	5662	0.188	35.6	3.78	V	5660	0.173	38.3	4.02
V	5884	0.204	34.1	3.41	V	5877	0.188	35.6	3.78	V	5885	0.173	38.3	4.02
V	6109	0.204	34.1	3.41	V	6102	0.188	35.6	3.78	V	6110	0.173	38.3	4.02
E	6276	0.133	76.8	2.45	E	6286	0.133	80.3	2.21	E	6288	0.125	85.0	2.18

[a] Julian − 2440000; dates repeating after 16 yr (add 5844 days).
[b] V_r = relative velocity; θ = turn angle; R_{min} = minimum distance to planet center, planet radii.

Table 3 Continued

Orbit 7

Planet	Date[a]	V_r,[b] EMOS	θ,[b] deg	R_{min}[b]
E	452	0.135	79.2	2.21
E	817	0.135	79.2	2.21
V	1009	0.224	21.2	5.24
V	1234	0.224	1.3	99.11
V	1459	0.224	23.6	4.59
E	1539	0.164	26.5	8.90
E	2034	0.163	18.6	13.82
V	2148	0.134	51.1	4.36
V	2373	0.134	51.1	4.36
V	2598	0.134	52.4	4.18
E	2700	0.164	14.3	18.53
E	3195	0.164	22.2	11.05
V	3299	0.148	58.4	2.84
V	3524	0.148	58.4	2.84
V	3749	0.148	58.4	2.84
E	3870	0.185	47.2	3.09
E	4362	0.185	31.8	5.45
V	4473	0.131	56.3	3.83
V	4698	0.131	51.4	4.47
V	4923	0.131	51.4	4.47
E	5100	0.128	60.2	4.27
E	5465	0.128	60.2	4.27
V	5664	0.175	38.3	3.95
V	5889	0.175	38.3	3.95
V	6114	0.175	38.3	3.95
E	6296	0.135	79.2	2.21

Orbit 8

Planet	Date[a]	V_r,[b] EMOS	θ,[b] deg	R_{min}[b]
E	454	0.129	81.1	2.30
E	819	0.129	81.1	2.30
V	1008	0.218	21.8	5.34
V	1233	0.218	2.0	71.47
V	1458	0.218	24.6	4.60
E	1539	0.163	26.1	9.16
E	2034	0.163	18.9	13.62
V	2149	0.136	49.8	4.42
V	2374	0.136	49.8	4.42
V	2599	0.136	54.4	3.81
E	2697	0.176	11.4	20.71
E	3190	0.176	43.6	3.86
V	3308	0.205	62.6	1.30
V	3533	0.205	62.6	1.30
V	3758	0.205	62.6	1.30
E	3889	0.216	67.3	1.22
E	4374	0.216	26.4	5.11
V	4485	0.158	56.9	2.48
V	4710	0.158	56.9	2.48
V	4935	0.158	56.9	2.48
E	5039	0.164	27.9	8.30
E	5534	0.164	13.3	20.16
V	5645	0.158	46.3	3.66
V	4870	0.158	46.3	3.66
V	6095	0.158	46.3	3.66
E	6298	0.129	81.1	2.30

Orbit 9

Planet	Date[a]	V_r,[b] EMOS	θ,[b] deg	R_{min}[b]
E	430	0.133	61.2	3.86
E	795	0.133	61.2	3.86
V	973	0.148	60.6	2.65
V	1198	0.148	60.6	2.65
V	1423	0.148	60.6	2.65
E	1602	0.142	62.5	3.27
E	1967	0.142	62.5	3.27
V	2165	0.194	30.4	4.43
V	2390	0.194	30.4	4.43
V	2615	0.194	30.4	4.43
E	2785	0.146	85.9	1.55
E	3150	0.146	85.9	1.55
V	3254	0.277	11.7	6.80
V	3479	0.277	10.9	7.31
V	3812	0.278	16.3	4.62
E	3918	0.148	62.5	2.99
E	4283	0.148	62.5	2.99
V	4467	0.153	57.1	2.75
V	4692	0.153	57.1	2.75
V	4917	0.153	57.1	2.75
E	5082	0.135	76.8	2.38
E	5447	0.135	76.8	2.38
V	5622	0.194	33.2	3.92
V	5847	0.194	33.2	3.92
V	6072	0.194	33.2	3.92
E	6274	0.133	61.2	3.86

Orbit 10

Planet	Date[a]	V_r,[b] EMOS	θ,[b] deg	R_{min}[b]
E	430	0.129	61.2	4.11
E	795	0.129	61.2	4.11
V	973	0.148	60.4	2.66
V	1198	0.148	60.4	2.66
V	1423	0.148	60.4	2.66
E	1602	0.142	62.7	3.23
E	1967	0.142	62.7	3.23
V	2164	0.192	30.5	4.48
V	2389	0.192	30.5	4.48
V	2614	0.192	30.5	4.48
E	2784	0.144	85.6	1.62
E	3149	0.144	85.6	1.62
V	3255	0.270	12.5	6.67
V	3480	0.270	11.5	7.30
V	3812	0.270	18.5	4.24
E	3926	0.151	84.8	1.49
E	4291	0.151	84.8	1.49
V	4422	0.259	18.5	4.60
V	4647	0.259	18.5	4.60
V	4980	0.259	13.6	6.55
E	5086	0.127	83.2	2.21
E	5451	0.127	83.2	2.21
V	5624	0.188	34.0	4.05
V	5849	0.188	34.0	4.05
V	6074	0.188	34.0	4.05
E	6274	0.129	61.2	4.11

Orbit 11

Planet	Date[a]	V_r,[b] EMOS	θ,[b] deg	R_{min}[b]
E	430	0.127	80.9	2.36
E	795	0.127	80.9	2.36
V	915	0.248	17.6	5.31
V	1140	0.248	17.6	5.31
V	1471	0.247	23.1	3.85
E	1603	0.140	86.0	1.69
E	1968	0.140	86.0	1.69
V	2165	0.191	30.6	4.50
V	2390	0.191	30.6	4.50
V	2615	0.191	30.6	4.50
E	2784	0.144	85.8	1.61
E	3149	0.144	85.8	1.61
V	3255	0.270	12.4	6.68
V	3480	0.270	11.6	7.18
V	3813	0.270	18.3	4.30
E	3925	0.151	84.8	1.49
E	4291	0.151	84.8	1.49
V	4422	0.259	18.6	4.55
V	4647	0.259	18.6	4.55
V	4979	0.258	13.6	6.55
E	5086	0.127	83.3	2.21
E	5451	0.127	83.3	2.21
V	5624	0.188	34.3	4.01
V	5849	0.188	34.3	4.01
V	6074	0.188	34.3	4.01
E	6274	0.127	80.9	2.36

Orbit 12

Planet	Date[a]	V_r,[b] EMOS	θ,[b] deg	R_{min}[b]
E	417	0.138	69.5	2.81
E	782	0.138	69.5	2.81
V	914	0.237	19.9	5.03
V	1139	0.237	17.6	5.82
V	1469	0.237	27.2	3.41
E	1605	0.138	85.0	1.79
E	1970	0.138	85.0	1.79
V	2165	0.188	31.1	4.56
V	2389	0.188	31.1	4.56
V	2614	0.188	31.1	4.56
E	2785	0.143	86.6	1.58
E	3150	0.143	86.6	1.58
V	3255	0.273	12.1	6.74
V	3480	0.273	12.0	6.80
V	3813	0.272	17.1	4.56
E	3923	0.151	86.0	1.45
E	4289	0.151	86.0	1.45
V	4426	0.244	22.1	4.18
V	4651	0.244	22.1	4.18
V	4982	0.244	12.5	8.10
E	5108	0.143	66.8	2.82
E	5473	0.143	66.8	2.82
V	5591	0.255	16.5	5.42
V	5816	0.255	10.4	9.05
V	6147	0.255	13.8	6.64
E	6261	0.138	69.5	2.81

[a] Julian − 2440000: dates repeating after 16 yr (and 5844 days).
[b] V_r = relative velocity; θ = turn angle; R_{min} = minimum distance to planet center, planet radii.

Table 3 Concluded

Planet	Date[a]	V_r,[b] EMOS	θ,[b] deg	R_{min}[b]	Planet	Date[a]	V_r,[b] EMOS	θ,[b] deg	R_{min}[b]	Planet	Date[a]	V_r,[b] EMOS	θ,[b] deg	R_{min}[b]
		Orbit 13					Orbit 14					Orbit 15		
E	437	0.129	74.6	2.99	E	443	0.122	78.5	2.76	E	448	0.121	86.5	2.23
E	802	0.124	74.6	2.99	E	808	0.122	78.5	2.76	E	813	0.121	86.5	2.23
V	916	0.250	17.0	5.47	V	916	0.255	16.2	5.53	V	917	0.261	15.4	5.62
V	1141	0.250	17.0	5.47	V	1141	0.255	16.2	5.53	V	1142	0.261	15.4	5.62
V	1473	0.250	22.9	3.80	V	1473	0.255	21.3	4.01	V	1474	0.261	20.2	5.07
E	1607	0.146	59.4	3.40	E	1605	0.146	60.8	4.25	E	1603	0.146	62.9	3.12
E	1972	0.146	59.4	3.40	E	1970	0.146	60.8	4.25	E	1968	0.146	62.0	3.12
V	2132	0.217	25.1	4.54	V	2130	0.213	25.6	4.56	V	2130	0.211	26.0	4.57
V	2357	0.217	22.5	5.19	V	2355	0.213	24.2	4.89	V	2355	0.211	26.0	4.57
V	2582	0.217	27.3	4.07	V	2580	0.213	28.6	3.96	V	2580	0.211	29.5	3.87
E	2701	0.180	40.1	4.19	E	2699	0.183	36.7	4.59	E	2696	0.189	33.6	4.88
E	3194	0.178	36.4	4.89	E	3192	0.183	44.0	3.53	E	3188	0.189	50.7	2.65
V	3309	0.190	32.7	4.16	V	3310	0.212	28.3	4.07	V	3310	0.233	24.9	3.96
V	3534	0.190	26.6	5.44	V	3535	0.212	21.5	5.73	V	3535	0.233	17.7	5.97
V	3759	0.190	28.7	4.94	V	3760	0.212	25.5	4.66	V	3760	0.233	23.0	4.35
E	3924	0.140	82.2	1.87	E	3921	0.154	51.8	3.86	E	3918	0.167	53.4	3.10
E	4289	0.140	82.2	1.87	E	4287	0.154	51.8	3.86	E	4284	0.167	53.4	3.10
V	4471	0.155	61.0	2.40	V	4423	0.253	20.3	4.32	V	4417	0.281	16.2	4.58
V	4696	0.155	61.0	2.40	V	4648	0.253	20.3	4.32	V	4642	0.281	16.2	4.58
V	4921	0.155	61.0	2.40	V	4980	0.252	13.9	4.74	V	4975	0.281	17.0	4.21
E	5084	0.144	52.1	4.39	E	5092	0.130	74.6	2.72	E	5042	0.173	28.0	7.39
E	5449	0.144	52.1	4.39	E	5457	0.130	74.6	2.72	E	5535	0.173	9.5	26.13
V	5659	0.184	36.6	3.82	V	5659	0.171	38.9	4.05	V	5644	0.154	48.7	3.53
V	5884	0.184	36.6	3.82	V	5884	0.171	38.9	4.05	V	5869	0.154	48.7	3.53
V	6109	0.184	36.6	3.82	V	6109	0.171	38.9	4.05	V	6094	0.154	48.7	3.53
E	6281	0.124	74.6	2.99	E	6287	0.122	78.5	2.76	E	6291	0.121	86.5	2.23

[a] Julian − 2440000; dates repeat after 16 yr (add 5844 days).
[b] V_r = relative velocity; θ = turn angle; R_{min} = minimum distance to planet center, planet radii.

of the Earth. In this case the spacecraft comes within 0.12 Earth radii of the planet surface. Thus despite similarity of the orbits, each combination of direct return orbits requires individual evaluation. Details of the turn angle (θ) calculation are contained in Ref. 16.

The method of sequential modification was also used to obtain solutions to a series of orbits which differ only in the direct return orbits at the Venus encounters. These orbits are numbered 9–12 in Table 3. Solutions to orbits 13–15 demonstrate the effectiveness of sequential modification for periodic orbits with symmetric returns at both Earth and Venus. In all orbits numbered 9–15 no additional convergence problems were encountered.

Summary

Four types of iterative solutions were attempted in solving the periodic orbit problem. The Davidon method was found to be an inappropriate choice due to the excessive amount of time required for each iteration. The steepest-descent method proved to be extremely useful in obtaining initial reduction of the function value resulting from the approximate fly-by dates. Since only calculation of the function value and gradient vector are required, each steepest-descent interaction requires little time. When simplicity of the steepest-descent method slowed convergence at a "ravine," the conjugate gradient method was used successfully to obtain additional reduction of the function value. For periodic orbit problems, the conjugate gradient method is characterized by an appropriate tradeoff between the time required for each iteration and the sophistication required for satisfactory reduction of the function value. The Newton-Raphson method was used with success only after a sufficiently accurate approximation to the fly-by dates had been reached. Although Newton-Raphson iterations suffered from the time required to invert an N-order matrix, large reductions in the function value were obtained from each iteration.

An average of 1.2 min of computer time was required to reach a solution for each periodic orbit in which no local minimums were encountered between the initial approximation and the global minimum. Of the 1.2 min, approximately 0.3 min were required for compilation whereas approximately 0.9 min were required for execution. The longest time required to obtain a solution was approximately 2.5 min (70 iterations) while the shortest time required was approximately 0.4 min (3 iterations).

The technique described herein is in no way restricted to the solution of periodic orbit problems. A periodic orbit is merely a multiple-fly-by problem in which the first and last flyby are constrained to occur at the same planet with equal relative velocity magnitudes on dates separated by an integral multiple of a specified time period. In a general multiple-fly-by problem any one or all of these constraints may be relaxed.

As specifically demonstrated by orbits 5 and 6, the order in which direct return orbits occur drastically affects the characteristics of each orbit. It is important to consider approximating the total number of acceptable periodic orbits which connect Earth and Venus. Each orbit contains five encounters at Earth and five encounters at Venus. At each encounter either one of two direct return orbits is available for selection. For this reason a minimum of 1024 acceptable orbits may exist. Although acceptable orbits containing as many as five symmetric returns have been shown to exist, the times of flight for interplanetary transfers on these orbits are quite small. The inclusion of additional symmetric returns would tend to further reduce the time available for interplanetary transfers. It is reasonable to assume that periodic orbits with six or more symmetric returns would require flybys passing below the surface of a planet. This assumption reduces the total number of periodic orbits with acceptable flybys to 648.

A larger number of periodic orbits are possible if additional variations in the direct return orbits are considered. Those Venus encounters with two consecutive full-revolution returns could be replaced by encounters consisting of a half-revolution return followed by a full-revolution return and a second half-revolution return. This alternative would cause rotation of the relative velocity vector at four rather than three flybys. The additional flyby might reduce the largest turn angle enough to allow more desirable flybys. Still more orbits could be obtained by reversing the order of the full-revolution and symmetric returns which occur in succession at many of the Venus encounters. This variation would change the direction of the inbound and outbound relative velocities at the full-revolution and symmetric returns so that a reduction in the largest turn angle might result. No variations of this type were required for the orbits listed in the Appendix since the flybys at all such Venus encounters occur well above the planet surface.

The periodic orbits require 16 yr to complete each cycle. Earth and Venus repeat their absolute orientation every 8 yr. Therefore, two spacecraft are required to take advantage of all the opportunities for each periodic orbit. When one spacecraft is leaving Earth for Venus, the other is approaching Earth from Venus. The alternate sets of fly-by dates can be obtained by adding 8 yr to each set of fly-by dates listed in the Appendix. The 15 periodic orbits presented here would allow 30 spacecraft to simultaneously make periodic flights between Earth and Venus. The large number of trajectory choices provides considerable flexibility in establishing a particular mission.

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