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Fast Reconnaissance Missions to the Outer Solar System Utilizing Energy Derived from the Gravitational Field of Jupiter

By

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With 16 Figures

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Abstract

Fast Reconnaissance Missions to the Outer Solar System Utilizing Energy Derived from the Gravitational Field of Jupiter. Contrary to popular belief, indirect ballistic trajectories involving close approach to one or more intermediate planets need not require longer flight duration than is characteristic of direct transfer orbits. In fact, significant reduction of both required flight time and launch energy results if efficient use is made of the energy which can be gained during a midcourse planetary encounter. From the point of view of a passing space vehicle, the intermediate planet appears as a field of force moving relative to the inertial heliocentric coordinate system. Thus, work is done on the spacecraft, and its heliocentric energy may be increased or decreased depending upon the geometric details of the encounter. This paper describes the application of energy derived in this fashion, utilizing gravitational perturbations from Jupiter, for reduction of required launch energy and flight duration for exploratory missions to all of the outer planets of the solar system. The latter half of the next decade abounds in interesting multiple planet opportunities due to the similar heliocentric longitudes of the major planets during this time period. Trajectories to Saturn, Uranus, Neptune, and Pluto using the midcourse energy boost from Jupiter are best initiated in the years 1978, 1979, 1979, and 1977 respectively. Flight time reduction ranges from one half the required direct trajectory duration for Earth-Jupiter-Saturn Missions to as much as 85% of the direct transfer time for Pluto flights via Jupiter. Many multiple-target trajectories are also possible. Of particular interest is the 1978 Earth-Jupiter-Saturn-Uranus-Neptune "grand tour" opportunity which would make possible close-up observation of all planets of the outer solar system (with the exception of Pluto) in a single flight.

Résumé

Missions de reconnaissance rapides aux confins du système solaire utilisant l'énergie du champ gravitationnel de Jupiter. Contrairement à une opinion répandue, les trajectoires balistiques indirectes avec approche d'une ou de plusieurs planètes intermédiaires n'im-
pliquent pas une durée supérieure à celle des orbites de transfert direct. En fait, à condition d'utiliser de façon efficiente l'énergie qui peut être gagnée par une approche planétaire intermédiaire et la durée de vol et l'énergie nécessaire au lancement peuvent être réduites de façon marquée. Du point de vue du véhicule spatial, la planète intermédiaire se présente comme un champ de force en mouvement relatif dans un système de coordonnées héliocentriques. Le travail est effectué sur le véhicule et son énergie héliocentrique est accrue ou réduite d'après la configuration géométrique de l'approche. L'article décrit l'utilisation de l'énergie qui peut être soumise de cette façon de la planète Jupiter afin de réduire l'énergie de lancement et la durée de vol pour les missions d'exploration vers les planètes extérieures du système solaire. La deuxième moitié de la prochaine décennie offre plusieurs possibilités intéressantes de visite de multiples planètes à cause d'une conjoncture favorable des longitudes héliocentriques des planètes principales pendant cette période. Des trajectoires vers Saturne, Uranus, Neptune, Pluton utilisant une énergie de mi-
course s'orienteront à Jupiter peuvent débuter avantagé-
ment dans les années 1978, 1979, 1979 et 1977 respecti-
vement. Les réductions de durée de vol s'échelonnent
la moitié de la durée d'une trajectoire directe dans le
cas des missions Terre—Jupiter—Saturne jusqu'à 85% de
la durée du transfert direct dans le cas des missions
vers Pluton via Jupiter. Plusieurs trajectoires à objectifs
multiples sont aussi possibles. En particulier le "Grand
Tour" Terre—Jupiter—Saturne—Uranus—Neptune en
1978 permettrait une observation à faible distance
de toutes les planètes extérieures du système solaire (à
l'exception de Pluton) en un seul vol.

Zusammenfassung

Eine schnelle Aufklärungsmission zum äußeren Sonnensystem, wobei auch aus dem Schwerefeld des Jupiter abgeleitete Energie benutzt wird. Im Gegensatz zur allgemein herrschenden Meinung müssen indirekte ballistische Flugbahnen, die eine enge Annäherung an einen oder mehrere Zwischenplaneten einschließen, keine längere Flugdauer benötigen, als auf direkten Übergangsbahnen üblich ist. Tatsächlich ergeben sich bedeutsame Verringerungen, sowohl der benötigten Flugzeit als auch der Startenergie, wenn die Energie, die während des Vorbeifluges bei einer Planetenbegle-

1 This paper presents the results of one phase of research carried out at the Jet Propulsion Laboratory, California Institute of Technology, under contract No. NAS 7-100, sponsored by the National Aeronautics and Space Administration.

1. Introduction

Of crucial importance in the study of the origin, evolution, and structure of the solar system is the acquisition of close-up scientific data from the major planets (Jupiter, Saturn, Uranus, and Neptune) and Pluto. However, as indicated in Table 1, direct trajectories to these bodies are characterized by high launch energy and very long flight duration. At least the latter of these two factors must be reduced if practical exploration of the outer solar system is to be accomplished. A very attractive source of energy which can be tapped to bring about this reduction is the gravitational perturbation of an intermediate planet. The gravitational perturbation technique for trajectory shaping has been under intensive study recently [1–5], however, due to the nature of the missions investigated, a widely held misconception has arisen to the effect that indirect multiple-planet trajectories in general require greater flight time than direct transfers to the same target bodies with the same launch energy. It will be shown here that significant reduction in flight duration results if efficient use is made of the energy which can be gained during a midcourse planetary encounter.

The latter half of the next decade abounds in interesting multiple-planet missions utilizing massive gravitational perturbations from Jupiter. Mnesivos [2] has studied deep-space, out-of-ecliptic, and close solar probe trajectories via Jupiter. The goal of the present study was the determination of optimum launch opportunities and corresponding trajectory characteristics for flights to the outer planets of the solar system using energy gained during close approach to Jupiter. Three-dimensional conic computer programs were employed, this procedure having been verified by comparison of conic and integrated trajectory results [1, 3]. The most interesting mission possibilities and the corresponding launch years are summarized in Table 2. More detailed descriptions of these missions are presented later.

2. Heliocentric Energy Change Due to a Midcourse Planetary Encounter

The mechanism of heliocentric energy change during encounter of a space vehicle with an intermediate planet is a simple one. The moving gravitational force field of the planet does work on the spacecraft and, depending upon the geometry of the encounter, an energy gain or loss relative to the inertial heliocentric coordinate system results. The principle of energy conservation is of course not violated in this process; the energy gained or lost by the probe is lost or gained, respectively, by the planet. The percentage energy change, being inversely dependent on the mass of the body in question, is truly infinitesimal for the planet, but may be very large for the spacecraft.

Fig. 1 illustrates the encounter process. The specific heliocentric energy prior to encounter is

$$E_i = \frac{V_i}{2}$$

while that after encounter is

$$E_o = \frac{V_o}{2}$$

where $V_i$ and $V_o$ are the pre-encounter and post-

Table 1. Characteristics of Direct Minimum Energy Trajectories to the Outer Planets

<table>
<thead>
<tr>
<th>Mission</th>
<th>Minimum Launch Energy, $C_1$ (km/sec$^2$)</th>
<th>Flight Duration, $T$ (Years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earth–Jupiter</td>
<td>86.5</td>
<td>2.3</td>
</tr>
<tr>
<td>Earth–Saturn</td>
<td>108.8</td>
<td>6.1</td>
</tr>
<tr>
<td>Earth–Uranus</td>
<td>126.1</td>
<td>16.0</td>
</tr>
<tr>
<td>Earth–Neptune</td>
<td>135.0</td>
<td>30.7</td>
</tr>
<tr>
<td>Earth–Pluto</td>
<td>135.3</td>
<td>45.7</td>
</tr>
</tbody>
</table>

Table 2. Multiple-Planet Trajectories to the Outer Solar System

<table>
<thead>
<tr>
<th>Mission</th>
<th>Launch Years</th>
</tr>
</thead>
</table>

1 Optimum launch year.
encounter velocity vectors respectively. Thus the resultant energy change is

\[ \Delta E = \frac{1}{2} [V_0 \cdot V_0 - V_i \cdot V_i] \]

where \( V_p \) is the heliocentric velocity vector of the planet, assumed constant during the residence of the probe in the activity sphere of the planet. Assuming constant spacecraft mass and no propulsive maneuvers in the activity sphere, energy is conserved relative to the planet. Thus,

\[ |V_s'| = |V_s'| = v_s, \]

and the energy increment can be written

\[ \Delta E = V_s \cdot (V_0' - V_i'). \]  \hspace{1cm} (1)

This result is approximate, since \( V_s \) may change considerably during encounter with one of the outer planets due to the large activity sphere radii characteristic of these bodies.

Fig. 1. Encounter hyperbola

Let \( V_i' \) and \( V_0' \) be the incoming and outgoing asymptotic velocity vectors relative to the planet,

and note the relation of these to the corresponding heliocentric vectors,

Fig. 2. Characteristic energy vs. hyperbolic excess speed of spacecraft

The geometrical properties of the energy equation can be clarified by setting

\[ V_s = v_s \hat{p}, \]

\[ V_i' = v_s \hat{t}, \]

\[ V_0' = v_s \hat{\theta}, \]

where \( \hat{p}, \hat{t}, \) and \( \hat{\theta} \) are unit vectors in the directions of the planet's velocity vector, incoming asymptote, and outgoing asymptote, respectively. In terms of these quantities the energy equation becomes

\[ \Delta E = v_s v_s [\hat{p} \cdot (\hat{\theta} - \hat{t})], \]  \hspace{1cm} (2)

Fig. 3. Maximum deflection angle vs. hyperbolic excess speed of spacecraft
where \( v_p \) is the speed of the planet and \( v_a \) is the hyperbolic excess speed of the spacecraft relative to the planet. It is evident that, for a given incoming asymptote direction, the maximum energy gain and energy loss correspond respectively to the outgoing asymptote pointing along or opposite to the planet's velocity vector. Since the vector \( \vec{\Omega} - \vec{I} \) defines the direction of the axis of the encounter hyperbola, it can be seen that the largest possible energy change corresponding to a given flight path deflection angle

\[
\Psi = \cos^{-1} \left( \vec{I} \cdot \vec{\Omega} \right)
\]

results if the planet's velocity vector is coincident with this axis.

It is convenient to write the energy increment equation in the form

\[
\Delta E = f E^* \tag{4}
\]

where the characteristic energy

\[
E^* \equiv 2 v_p v_a \tag{5}
\]

represents the maximum energy increase for a given planet and spacecraft approach velocity. This increment is, of course, unobtainable in practice, since it would require a deflection angle of 180°, which would in turn require passage of the vehicle through the planet. Plots of \( E^* \) vs. \( v_a \), based on mean values of \( v_p \), are shown in Fig. 2 for all planets of the solar system. As expected Mercury exhibits the highest value of \( E^* \) for a given \( v_a \) due to its close proximity to the Sun and corresponding high heliocentric velocity. The coefficient

\[
f = \frac{\vec{F} \cdot (\vec{\Omega} - \vec{I})}{2}
\]

is a number between \(-1\) and \(1\) which determines the magnitude and sign of the energy change actually
achieved in the encounter, and will be referred to in what follows as the energy change index. The

constant of the planet, the hyperbolic excess speed, and distance of closest approach to the surface as

\[ \Psi = 2 \sin^{-1} \left[ 1 + \frac{v^2}{\mu} \left( d + r_p \right) \right]^{-1}, \]  

where \( r_p \) is the radius of the planet (assumed spherical). For trajectory calculations it is necessary to define a forbidden sphere, concentric with the planet, and with radius

\[ R_f = r_p + \delta \alpha + \delta \beta, \]

where \( r_p \) is in this case the equatorial radius of the planet, \( \delta \alpha \) is the maximum depth of sensible atmosphere at the planet's equator, and \( \delta \beta \) is a guidance
error contingency. The maximum allowable deflection angle corresponds to a flight path grazing this forbidden sphere. Thus,

\[ \Psi_{\text{max}} = 2 \sin^{-1} \left( \frac{\mu}{v_a^2 R_f} \right) \]  

Fig. 3 shows plots of \( \Psi_{\text{max}} \) versus \( v_a \) for all planets of the solar system except Pluto, whose gravitational constant is not precisely known. The calculations were made for \( R_f = r_p \). As expected, Jupiter is capable of the largest flight path deflection for a given asymptotic speed due to its great mass.

Using (8), the maximum possible value of the energy change index corresponding to a given incoming asymptote direction can be computed. The direction of the asymptote is conveniently referenced to the planet's velocity vector by the angle

\[ |\xi| = \cos^{-1} (-\hat{r} \cdot \hat{P}). \]  

Fig. 7. Arrival date vs. launch date contours for 1978 Earth—Jupiter—Saturn Mission.

Fig. 8. Typical planetary configuration for 1978 Earth—Jupiter—Uranus Mission.
\( \xi \) is measured positive counter clockwise from \( \hat{P} \) to \(-\hat{f}\) as viewed from the northern hemisphere of the celestial sphere. The maximum energy gain which can be achieved corresponds to

\[
I_{\text{max}} = \begin{cases} \frac{\cos \frac{\xi}{2} + 1}{2}, & \xi \geq (\pi - \psi_{\text{max}}) \\ \frac{\cos \frac{\xi}{2} - \cos \left(\psi_{\text{max}} + \frac{\pi}{2}\right)}{2}, & \xi < (\pi - \psi_{\text{max}}). \end{cases}
\]

Values of \( f \) for maximum energy loss relative to the Sun for a given incoming asymptote direction are

\[
I_{\text{max}} = \begin{cases} \frac{\cos \frac{\xi}{2} - 1}{2}, & \xi \leq \psi_{\text{max}} \\ \frac{\cos \frac{\xi}{2} - \cos \left(\psi_{\text{max}} - \frac{\pi}{2}\right)}{2}, & \xi > \psi_{\text{max}}. \end{cases}
\]

These relations are plotted in Fig. 4.

![Figure 4](image)

**Fig. 4. Typical planetary configuration for 1979 Earth—Jupiter—Neptune Mission**

It is of interest to assess the abilities of the various planets to influence the trajectories of passing space vehicles. This can be accomplished by comparing the maximum available energy increments at given values of approach speed \( v_a \). It is often assumed that Jupiter, because of its great mass, is capable of producing the largest energy change under all conditions. This is not the case however; e.g., for low hyperbolic excess speeds, say \( v_a < 3 \) km/sec, all of the much less massive inner planets are capable of larger energy boosts. This effect results from the higher heliocentric speed of planets which are closer to the Sun. Thus \( E^* \) dominates the energy increment at low approach speeds. At higher speeds, the ability of a more massive planet to deflect the trajectory can be exploited in the form of higher values of \( f \), and thus larger, energy changes result. Fig. 5 shows the maximum possible energy increments which can be produced by encounters with the planets. Only in exceptional cases would it be possible to utilize the maximum available energy boost because of the geometrical constraints imposed by the required approach and departure asymptotic velocity vectors. The trajectory designer must endeavor to find those
flight paths linking the launch, midcourse, and terminal planets which can utilize as large a percentage of the available energy as possible in a way which reduces the over-all launch energy and flight time requirements of the mission. A measure of his success is the figure of merit

$$F = \frac{f}{f_{\text{max energy change}}}$$

(12)

where $f$ is the energy change index actually achieved in the encounter, and $f_{\text{max energy change}}$ is the theoretical maximum energy gain or loss as given by Eqs. (10) and (11). Eq. (10) is used if $f$ is positive; Eq. (11) is used if an energy loss is involved.

3. Trajectory Results

Three-dimensional conic trajectory programs were utilized in studying multiple-planet opportunities for the latter half of the next decade with Jupiter as the primary intermediate planet. Vehicles with launch energies in the range $100 < C_1 < 200 \text{ km/s}^2$ were assumed. It was discovered that, for a given mission, several consecutive Earth—Jupiter launch opportunities (which occur about every 13 months) resulted in acceptable trajectories to the outer planets. Detailed trajectory information is given for the better launch years. The data is presented in the form of arrival date vs. launch date plots with launch energy, $C_2$, as parameter. The solid curves represent contours of constant $C_2$; the superimposed dashed lines represent contours of constant flight duration.

$C_1$ is the square of the hyperbolic excess velocity of the probe at orbital injection measured relative to the Sun. Thus it represents twice the launch energy per unit spacecraft mass.

3.1 Earth—Jupiter—Saturn Trajectory Characteristics

Missions to Saturn via a close encounter with Jupiter are best initiated in 1978, with the 1977 launch opportunity requiring only slightly longer flight duration for a given launch energy. The 1978 trajectories are also acceptable, but require about 10% more flight time than the 1978 launches. Fig. 6 shows a typical 1978 Earth—Jupiter—Saturn flight path projected on the plane of the ecliptic. It will be noticed that the trajectory after Saturn flyby is depicted as escaping from the solar system. The actual post-Saturn flight path would depend on the targeting conditions selected by the trajectory designer, but solar system escape energy is available. This is true of all trajectories considered in this article.

Typical arrival dates vs. launch date plots for the 1978 Earth—Jupiter—Saturn mission are shown in Fig. 7 for the optimum 30-day launch period. High launch energies are accommodated by these trajectories, with corresponding distance of closest approach at Jupiter of about 6 Jupiter radii. Comparable 1977 trajectories require much larger flight path deflections at Jupiter, and this results in closest approach distances of about 2 Jupiter radii. Typical encounter data and other trajectory characteristics are summarized in Table 3. Earth—Jupiter—Saturn launch opportunities recur with a period of about 20 years.

3.2 Earth—Jupiter—Uranus Trajectory Characteristics

Launch opportunities to Uranus utilizing the gravitational energy boost at Jupiter are available in 1977, 1978, 1979, and 1980. The 1978 flights are superior with regard to required flight time for a given launch energy. This results from near optimum
## Table 3. Typical Trajectory Characteristics

<table>
<thead>
<tr>
<th>Mission</th>
<th>Launch Energy ( C_3 ) (km²/sec²)</th>
<th>Best Launch Date</th>
<th>Time of Flight (Days)</th>
<th>Distance of Closest Approach to Jupiter (Jupiter Radii)</th>
<th>Deflection Angle ( \psi ) (Deg)</th>
<th>Hyperbolic Excess Velocity at Jupiter ( V_e ) (km/sec)</th>
<th>Energy Gain at Jupiter ( \Delta E ) (km²/sec²)</th>
<th>Energy Change Index ( f )</th>
<th>Figure of merit ( F )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earth—Jupiter—Saturn</td>
<td>130</td>
<td>11 Oct 1978</td>
<td>838</td>
<td>6.37</td>
<td>56.8</td>
<td>16.42</td>
<td>192</td>
<td>.46</td>
<td>.70</td>
</tr>
<tr>
<td>Earth—Jupiter—Uranus</td>
<td>130</td>
<td>11 Oct 1978</td>
<td>1937</td>
<td>0.17</td>
<td>127.2</td>
<td>14.26</td>
<td>227</td>
<td>.63</td>
<td>.95</td>
</tr>
<tr>
<td>Earth—Jupiter—Neptune</td>
<td>150</td>
<td>12 Nov 1979</td>
<td>2525</td>
<td>2.50</td>
<td>80.7</td>
<td>16.68</td>
<td>241</td>
<td>.58</td>
<td>.88</td>
</tr>
<tr>
<td>Earth—Jupiter—Pluto</td>
<td>150</td>
<td>8 Sept 1977</td>
<td>2530</td>
<td>1.93</td>
<td>88.7</td>
<td>16.23</td>
<td>251</td>
<td>.60</td>
<td>.91</td>
</tr>
</tbody>
</table>

Usage of Jovian energy boost. The figure of merit for these trajectories is typically \( F = 0.96 \), but very close approach distances to Jupiter are required as indicated in Table 3. Fig. 8 shows the planetary constellation typical of these missions, and Fig. 9 shows the constant launch energy contours for the 1978 opportunity. The cross-hatching on Fig. 9 for launch energies higher than \( C_3 = 130 \) indicates that these trajectories are not physically realizable due to the very large flight path deflections required. Uranus opportunities via Jupiter are available about every 14 years.

#### 3.3 Earth—Jupiter—Neptune Trajectory Characteristics

Optimum launch opportunities for flights to Neptune occur in the fall of 1979, but acceptable trajectories are also available in 1977, 1978, 1980, and 1981. Fig. 10 shows the typical flight path configuration for the mission. Arrival date vs. launch date plots are shown in Fig. 11. A typical figure of merit for these trajectories is \( F = 0.88 \), indicating good usage of the Jupiter encounter energy boost. Earth—Jupiter—Neptune opportunities occur every 13 years.

#### 3.4 Earth—Jupiter—Pluto Trajectory Characteristics

The optimum launch year for Earth—Jupiter—Pluto missions was found to be 1977. Fig. 12 illustrates the planetary configuration typical of the mission, and arrival date vs. launch date plots for the 1977 opportunity are shown in Fig. 13. A high-energy transfer orbit \( (C_3 = 150) \) with a flight time of about 7 years would require the probe to pass within less than 2 radii of Jupiter's surface. The short flight times to Pluto are the result of efficient usage of the available energy increment \( (F = 0.91) \), coupled with the fact that Pluto is near perihelion at closest approach of the spacecraft. Although launch opportunities are available every 12.5 years, the optimum conditions characteristic of the 1977 opportunity will not be repeated until the year 2224. The 1990 launch opportunities should also be favorable, however.

#### 3.5 Earth—Jupiter—Saturn—Uranus—Neptune "grand tour" Trajectory Characteristics

Many interesting multiple-target trajectories to the major planets are available during the latter half of the next decade due to the similar heliocentric longitudes of these bodies during that time period. Typical of these are the Earth—Jupiter—Saturn—Uranus—Neptune opportunities. Fig. 14 illustrates the planetary configuration typical of this mission. Figs. 7, 15, and 16 show the arrival date at each target planet vs. launch date from Earth for the best 30-day launch period in 1978. The cross-hatching in Fig. 15 and 16 indicates that trajectories with launch energies higher than \( C_3 = 130 \) are not attain-
able due to the very close approach distances at Saturn required for putting the spacecraft onto the subsequent legs of the trajectory. 1977 launches are also limited to low \((90 < C_2 < 120 \text{ km}^2/\text{sec}^2)\) launch energies due to the large flight path deflections required at Saturn. The required flight time from Earth to the terminal planet Neptune with launch energy of \(C_2 = 130 \text{ km}^2/\text{sec}^2\) is about 8.5 years, which compares favorably with the 7.7 years flight duration for an Earth-Jupiter-Neptune mission with the same launch energy. The longer flight time for the four-planet mission is a result of the less-than-optimum usage of the jovian energy boost in order to achieve the optimum Jupiter-Neptune flight path. Earth-Jupiter-Saturn-Uranus-Neptune mission opportunities occur only every 175 years due to the constraints imposed by the required positions of Uranus and Neptune.

4. Conclusions

The 1975—1980 time period is characterized by an abundance of interesting multiple planet trajectories which efficiently utilize energy derived from a close approach to the planet Jupiter. The trajectories discussed here are characterized by very short flight times in comparison to those for direct flights from Earth to the corresponding target planets. Although higher launch energies are suggested for some of the multiple-planet flights, the additional expense of this energy might be offset by the great savings afforded by the short flight times. This is due to the expense of providing adequate vehicle reliability for the extended flight duration characteristic of direct trajectories, and to the high costs involved in maintaining tracking, orbit determination, and other flight related activities for protracted periods.

The great communications distances involved in outer solar system flights give rise to some difficulties with regard to antenna size and positioning, transmitter power requirements and so on. Signal propagation times are of course very long (about 4 hours one way from Neptune), and real-time control of the spacecraft, especially during the critical encounter sequences, would have to be relegated to automatic onboard control devices. An interesting discussion of long-distance communications problems is given by Kirsten [6].

The very important problem of guidance was not considered in the present study, but it is expected that development of planetary approach guidance techniques [7] coupled with improved Earth-based radio guidance should make the missions discussed herein entirely feasible. The large boost vehicles which should be available in highly developed form by that time should enable the spacecraft to accommodate the large supply of fuel required for necessary midcourse trajectory corrections, as well as a significant instrument payload.

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