Gravity-Assisted Trajectories to Solar-System Targets

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Analytical and numerical results of gravity-assisted trajectory investigations are presented. Targets include Mercury, Jupiter, Saturn, Uranus, Neptune, and the sun. A two-dimensional solar system with circular orbits (excepting Mercury) is assumed. Expressions are derived predicting maximum heliocentric velocity and energy changes from any perturbing mass. Using these expressions, maximum-performance results are presented for all of the planets. Ideal velocity and minimum trip time requirements are analyzed numerically. Mars and Venus are of little or no gravity-assist value to Jupiter missions. Jupiter, on the other hand, is shown to be particularly attractive for gravity assist on Saturn and Uranus fly-by missions. Launch opportunities for many of the gravity-assisted outer-planet missions occur in the late 1970's. New parametric data are presented for the well-known technique of using Jupiter for close solar fly-bys. It is concluded that Jupiter gravity assists are required for solar probes to less than 0.1 a.u. using conventional chemical propulsion. Polar plots of several gravity-assisted trajectories are illustrated.

Introduction

As the aspirations of our space program and its technological capability grow, future mission planning brings into focus more distant yet scientifically justifiable solar-system targets. The problems of diminishing launch-vehicle returns and extremely long flight times are inherited with this expansion. It is the objective of this paper to show that certain gravity-assisted trajectories do improve payload capability and/or reduce flight time to selected planets, with launch opportunities that occur before advanced propulsion systems, e.g., nuclear or low-thrust, will be available.

The phrase "gravity assist" is defined as a significant trajectory perturbation between launch and target intercept due to a close approach (usually less than 25 planet radii) of an intermediate planet. A gravity assist always changes the spacecraft velocity and usually its heliocentric energy as well. Several synonymous phrases for gravity assist seen in literature are swing-by, planet fly-by, and planetary attraction.

The assumptions and ground rules established for this analysis are as follows: 1) a two-dimensional solar system with circular ecliptic orbits for all of the planets except Mercury, which is assumed to have a coplanar orbit but with an eccentricity of 0.2; 2) conic trajectory analysis, i.e., utilization of two-body motion equations; 3) no launch or intercept time constraints (approximate launch opportunities are defined from a match of parametric trajectory data and planet motions); and 4) gravity assist limited to one intermediate planet fly-by between launch and target intercept. Table 1 contains the planetary data used in the analysis. The semimajor axes of the planets' orbits (except Mercury) were selected from the table as their orbit radii. The analysis and equations that follow are a digest of a more detailed report[1] by the author.

Discussion of Method

A simple, direct method for computing gravity-assisted trajectories is possible which is consistent with the assumptions just outlined. The spacecraft hyperbolic excess velocity VHL and injection flight path angle γ are specified as initial conditions at Earth escape (see Fig. 1). Elements of the trajectory leg from Earth (P_1) to the gravity-assist planet (P_2) are computed, and the heliocentric velocity $\mathbf{V}_2$ at P_2 is determined. The near-planet trajectory geometry around P_2 is shown in Fig. 2. To simplify the assist-perturbation analysis, the radius of influence is shrunk to zero, bringing points L and E together at 0. The velocity perturbation due to gravity assist is then equivalent to rotating $\mathbf{V}_3$ through the angle α between the approach and departure asymptotes. This technique ignores the position perturbations during P_2 encounter, which are assumed to be small on a heliocentric scale. The equation for α is

$$\alpha = 2 \tan^{-1}(\mu/BV_3^2) \tag{1}$$

where B, the asymptotic miss distance, is defined as

$$B = \{p^2[1 + (2\mu/pV_3^2)]\}^{1/2} \tag{2}$$

and p is the miss distance at closest approach measured from the center of P_2, V_3 is the hyperbolic approach speed, and μ is the gravitational parameter of P_2.

The heliocentric exit velocity $\mathbf{V}_5$ can now be calculated and the trajectory leg from P_2 to the objective (P_3, Fig. 1

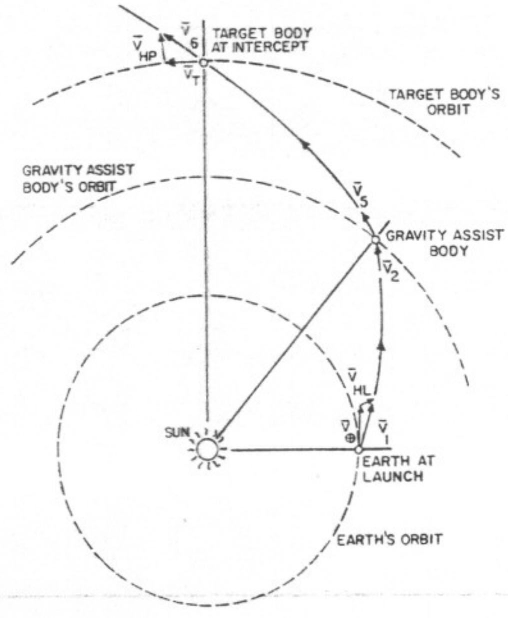

Fig. 1 Heliocentric geometry.

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Table 1 Planet data and maximum heliocentric velocity and energy changes due to planetary gravity assist

Planet	Gravitational parameter,[a] ft³/sec²	Equatorial radius,[b] 10^6 ft	Semimajor orbit axis,[b] a.u.	ΔV_{max}, 10^3 fps	ΔE_{max} Rank	ΔE_{max} 10^8 ft²/sec²
Jupiter	4.474716×10^{18}	229.26	5.2028	139.7	Jupiter	62.8
Saturn	1.339078×10^{18}	188.81	9.5388	84.3	Saturn	28.2
Neptune	2.481219×10^{17}	82.02	30.0577	55.1	Venus	27.5
Uranus	2.049401×10^{17}	83.66	19.1820	49.5	Earth	25.8
Earth	1.407645×10^{16}	20.93	1.0000	25.9	Mercury	18.7
Venus	1.146906×10^{16}	20.34	0.7233	23.6	Uranus	11.6
Pluto	1.171693×10^{16}	...	39.5177	22.6	Mars	10.3
Mars	1.517738×10^{15}	10.86	1.5237	11.8	Neptune	9.9
Mercury	7.658127×10^{14}	8.20	0.3871	9.8	Pluto	4.5

[a] From Ref. 9.
[b] From Ref. 10.

again) determined. The method is completed with the calculation of the hyperbolic approach velocity VHP at P_3.

Expressions for maximum heliocentric velocity and energy changes, consistent with the assumptions stated earlier, were derived to obtain some indication of gravity-assist performance. The equation for maximum velocity change is

$$V_{max} = V_3 = (\mu/p)^{1/2} \quad (3)$$

In other words, when the change in heliocentric velocity due to gravity assist is equal to the hyperbolic approach speed V_3 at the assisting planet, the condition of maximum velocity change exists. This condition is also, of course, a function of the miss distance p. Even at a fixed p, the condition is not unique, however, since, from Fig. 2, it is readily apparent that many values of $\mathbf{V}_2$ yield the same V_3 (with different entrance points E to the circle of influence).

The equation for maximum energy change is

$$\Delta E_{max} = V_p V_3 = V_p(\mu/p)^{1/2} \quad (4)$$

with the constraints that

$$\beta_1 = 120° \qquad \beta_2 = 60° \text{ (energy addition)} \quad (5a)$$

or

$$\beta_1 = 60° \qquad \beta_2 = 120° \text{ (energy subtraction)} \quad (5b)$$

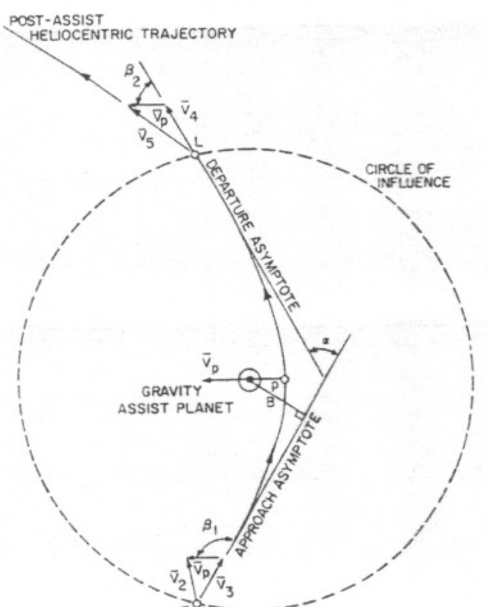

Fig. 2 Gravity-assist geometry.

where V_p is the heliocentric velocity of P_2 and β_1 and β_2 are illustrated in Fig. 2. From Eqs. (3–5), it can be seen that the conditions of maximum energy change form a special case of maximum velocity change. In particular, the net change in velocity must lie in the same or opposite direction of P_2's velocity vector $\mathbf{V}_p$. These conditions dictate only two possible preassist heliocentric trajectories for maximum energy change at a fixed p. One possibility yields maximum energy addition (this case illustrated in Fig. 2), and the other yields maximum energy subtraction.

Equations (3) and (4) were applied to each of the nine planets of the solar system. Results indicate, as expected, that Jupiter with its large mass is the most effective planet from the standpoint of performance for gravity assist. The theoretical maximum velocity change from a Jupiter assist is 139,700 fps. The maximum energy change is 62.8×10^8 ft²/sec². Table 1 contains ranked lists of the planets with the absolute maximum velocity and energy changes (i.e., p minimized to the radius of the planet) available from them. Figure 3 shows the variation of maximum velocity and energy change with p at Jupiter.

Mission Results

A numerical program for the IBM 7094 digital computer was constructed to investigate gravity-assisted trajectory parameters, e.g., total trip time, ideal velocity V_I,† miss dis-

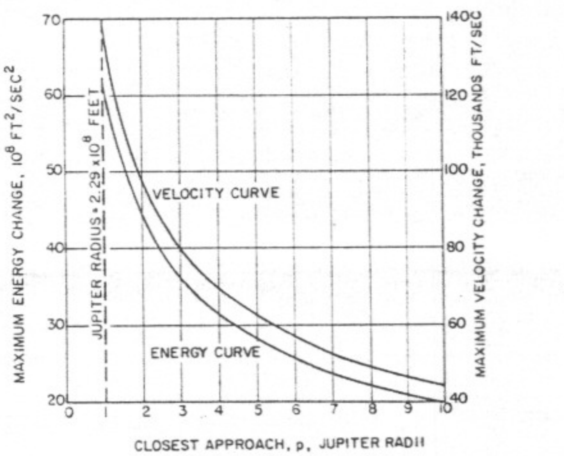

Fig. 3 Maximum velocity and energy changes for Jupiter.

† Ideal velocity is the ideal launch vehicle velocity required (in feet per second) to achieve a given hyperbolic excess velocity VHL beyond Earth escape from a 100-naut-mile parking orbit, assuming that all losses from launch to escape are equivalent to 4000 fps. The equation for ideal velocity is $V_I = [(36,178)^2 + (VHL)^2]^{1/2} + 4000$. V_I is also equal to characteristic velocity plus 4000 fps.

tance p at assist, and target hyperbolic approach velocity VHP, using the method previously discussed. The seven missions discussed in the following paragraphs were analyzed. From the numerical data, it was observed that minimum total trip times existed at fixed V_I's as a function of p at the gravity-assist planet. Using cross-plotting techniques, V_I and VHP comparisons between these minimum-time gravity-assisted trajectories and direct flight are presented. As a rule, slightly longer than minimum-time trajectories are usually required because of geometrical constraints dictated by the planetary positions during any given gravity-assisted mission launch opportunity. The notable exception to this loss in performance is the Earth/Jupiter/solar-probe mission.

Earth/Venus/Mercury

A review of Earth/Venus/Mercury trajectories was considered to check the numerical approach against a more detailed three-dimensional analysis of this mission by Minovitch.[2] A V_I comparison between direct and gravity-assisted trajectories is shown in Fig. 4. Venus-assisted flights are better than direct flights to Mercury when V_I is less than 46,250 fps, i.e., fixing either V_I or trip time reduces the other with the gravity-assist technique. For example, payload for an Atlas-Centaur launch vehicle is increased from 450 to 1050 lb for the same 115-day trip to Mercury (at 0.47 a.u.) with gravity assist. The variation of V_I and trip time across an actual launch window (7/25/70 to 9/13/70) is added to Fig. 4. The latter curve (taken from Minovitch[2]) represents the best launch window between 1965 and 1973. There is less than 1% difference in V_I between it and the Venus-assist curve at their closest point. A polar trajectory plot from this launch window is illustrated in Fig. 5.

Several observations can be made in summarizing missions to Mercury. If very short flight times are required at the cost of higher V_I's, direct missions are favored. Gravity-assist missions to Mercury are definitely desirable with longer trip times and smaller launch vehicles. Guidance requirements have been shown[3] to be modest for the 1970 opportunity: 150 lb of propellant and propulsion hardware are required for midcourse corrections of a 1300-lb spacecraft. Even though this is more than would be required for a direct flight to Mercury, its effect on spacecraft weight is small when compared to the payload growth realized by the reduction in V_I with Venus-assisted flights.

Earth/Venus/Jupiter

The use of a Venus assist on a flight to Jupiter is very inefficient. Over the range of V_I's between 48,000 and 54,000 fps, for Venus miss distances from 1 to 10 Venus radii and all angles of γ (0° to −180°; see Fig. 1), the furthest point reached by all post-Venus trajectories was less than 3 a.u. from the sun. This is contrasted by an aphelion of greater than 10 a.u. for a direct Earth departure trajectory (perihelion of 1 a.u.) with a V_I of 54,000 fps. After all of the results were tabulated, it was concluded that the Earth/Venus/Jupiter mission was the least favorable combination considered in the study.

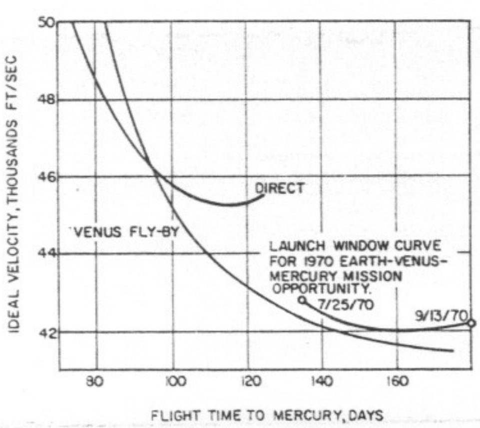

Fig. 4 Ideal velocity comparison: Mercury (0.47 a.u.).

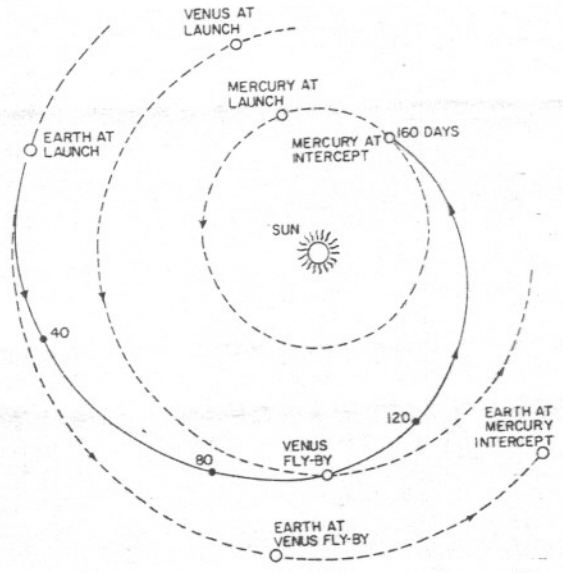

Fig. 5 Earth/Venus/Mercury trajectory illustration. Trajectory data: launch date = August 1970; ideal velocity = 41,950 fps; Venus miss distance = 1.66 Venus radii; total trip time = 160 days; Mercury approach velocity = 30,200 fps; equivalent ΔV at Venus = 18,000 fps.

Earth/Mars/Jupiter

Compared to direct flight to Jupiter, a Mars assist shows little improvement in either ideal velocity or trip time except at the lower end of the curves in Fig. 6, i.e., V_I <51,000 fps. A cursory look at launch opportunities revealed that the next Earth/Mars/Jupiter launch period occurs in 1984. In addition to opportunity shortages, the approach velocity to Mars is more than twice as large as a direct Mars fly-by mission, e.g., 48,900 fps for a 700-day flight to Jupiter, making it difficult to accomplish Mars scientific objectives during fly-by. Considering the added complexity in the mission profile of the Jupiter mission when a Mars assist is included and the moderate returns in trajectory performance, as with Venus there is reason to favor direct rather than gravity-assisted trajectories to Jupiter.

Earth/Jupiter/Saturn

The use of Jupiter for gravity assist on missions to the outer planets is desirable. The V_I comparison for a Saturn mission is shown in Fig. 7. V_I's as low as 50,200 fps are suffi-

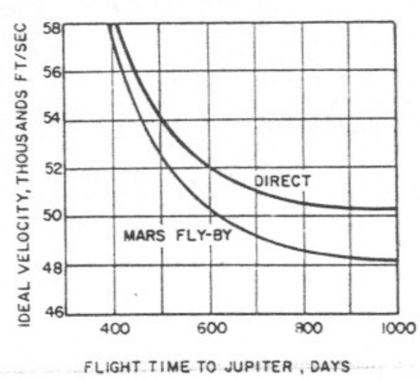

Fig. 6 Ideal velocity comparison: Jupiter (5.2 a.u.).

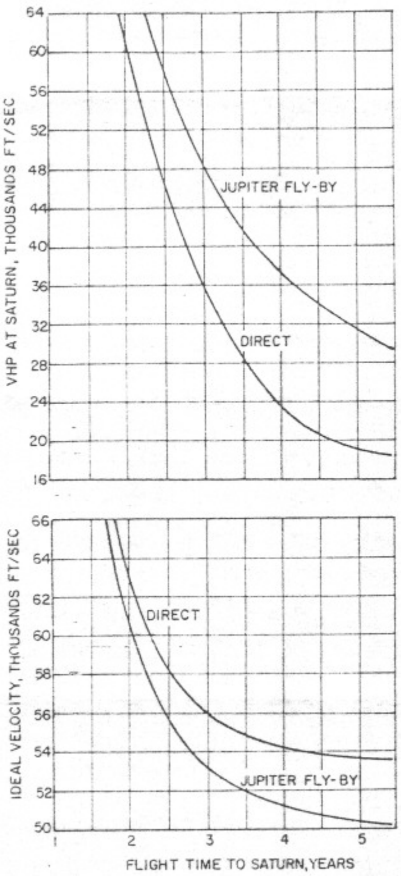

Fig. 7 Ideal velocity and *VHP* comparisons: Saturn (9.5 a.u.).

cient with a Jupiter assist, compared to the 53,400 fps minimum for a direct flight. For outer-planet missions, however, decreasing flight time may be more important than improving energy requirements. It takes four years for a Saturn 1B-Centaur launched 1250-lb. spacecraft to reach Saturn. Obtaining a Jupiter assist on the way reduces the flight time to $2\frac{3}{4}$ years for the same spacecraft/launch-vehicle combination. Curves of *VHP* at Saturn are also shown in Fig. 7 for direct and Jupiter-assisted trajectories. As a result of energy addition at Jupiter, the *VHP* for a fixed flight time is always higher with gravity assist, which limits its attractiveness to fly-by missions. For rendezvous/orbiter missions, more payload can be placed in Saturn orbit using a direct flight mode. Thus, it is better to expend energy during launch than to store propellant on board for a large impulse at the target planet.

A particular Earth/Jupiter/Saturn trajectory is illustrated as a polar plot in Fig. 8 for a 1977 launch opportunity. Opportunities for similar missions occur yearly from 1976 to 1979. The figure is self-explanatory, but it is worth noting that the equivalent ΔV at Jupiter due to gravity assist is 61,350 fps. For the same p, i.e., 4 Jupiter radii, the maximum available equivalent ΔV is 69,850 fps (see Fig. 3). It can be concluded that this mission makes good use of Jupiter's gravitational field for velocity transfer.

The launch dates are yearly, when they occur, and they occur far enough in the future to plan a fly-by investigation program for the planet Saturn. The V_I requirements are low enough to permit payloads greater than 1000 lb to be launched with a Saturn 1B-Centaur class launch vehicle with trip times on the order of 3 years. This appears to be one of the better gravity-assist missions and certainly warrants future consideration.

Table 2 Synodic periods and periods of yearly launch opportunities for multiple outer-planet missions

Mission	Synodic period,[a] years	Next launch period	Start of following launch period
Earth/Jupiter/Saturn	19.86	1976–1979	1996
Earth/Jupiter/Uranus	13.81	1978–1980	1992
Earth/Saturn/Uranus	45.36	1979–1985	2025
Earth/Jupiter/Neptune	12.78	1979–1981	1992
Earth/Saturn/Neptune	35.87	1979–1985	2015

[a] Synodic period between the last two planets within each mission combination.

Earth/Jupiter/Uranus and Earth/Saturn/Uranus

The use of either Jupiter's or Saturn's gravitational field on Uranus missions was considered (Fig. 9). At a fixed V_I, the trip time is best with a Jupiter assist. A 500-lb precursor spacecraft[4] launched with a Saturn 1B-Centaur will reach Uranus in $9\frac{1}{2}$ years on a direct flight. With a Saturn assist, the flight time is reduced to about $6\frac{1}{2}$ years. A Jupiter assist further reduces the flight time to $4\frac{3}{4}$ years. As with the Earth/Jupiter/Saturn mission, gravity assists from Jupiter or Saturn increase the *VHP* at Uranus. The same restriction on orbiter payloads also applies. For Jupiter-assisted flights, yearly opportunities exist from 1978 to 1980; for Saturn assists, opportunities occur yearly from 1979 to 1985.

Earth/Jupiter/Neptune and Earth/Saturn/Neptune

Flight-time improvements are even more dramatic for gravity-assisted flights to Neptune (Fig. 10) than they are for Saturn and Uranus. Unfortunately, even with a Jupiter assist, trip time remains very long. The same spacecraft/launch-vehicle combination that can reach Uranus in $4\frac{3}{4}$ years with a Jupiter assist takes 7 years to reach Neptune. The exploration of Neptune (and Pluto) may well have to wait

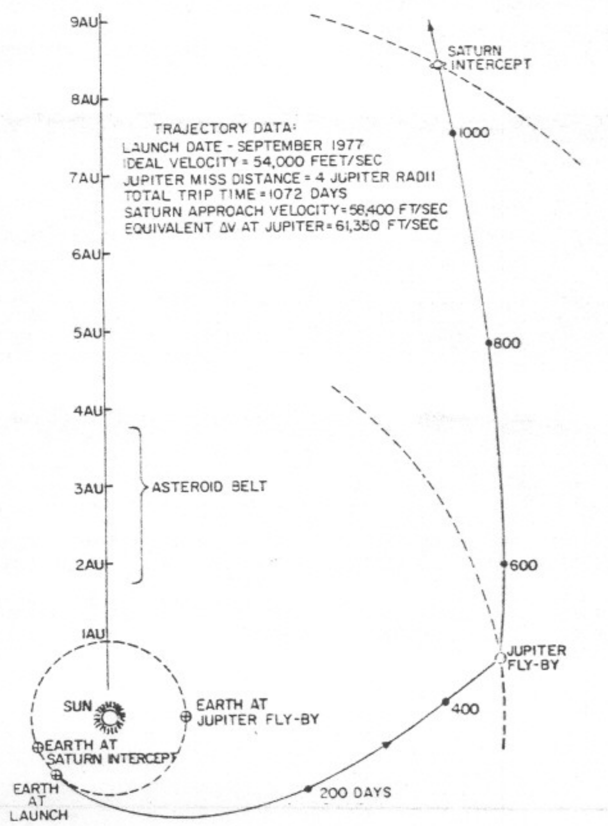

Fig. 8 Earth/Jupiter/Saturn trajectory illustration.

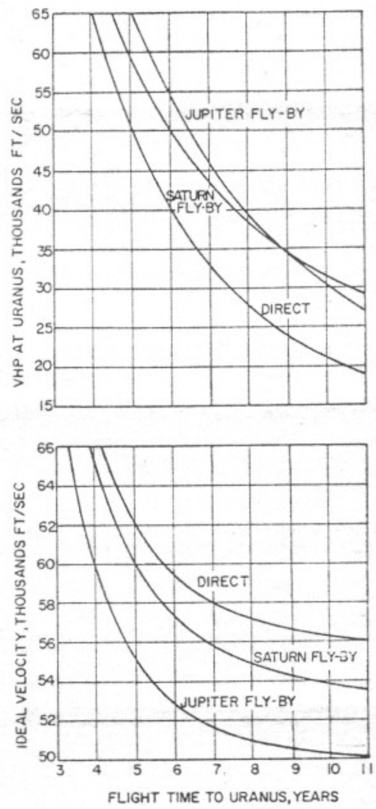

Fig. 9 Ideal velocity and *VHP* comparisons: Uranus (19.2 a.u.).

for more suitable propulsion systems (i.e., low-thrust or nuclear), but gravity assist can also be used with these systems to reduce flight times.

Yearly launch opportunities for gravity-assisted outer-planet missions have been cited for periods of from 3 to 5 years during which the planet positions for these missions is favorable. The waiting time between consecutive launch periods is dictated by the synodic period of the outer two planets of each mission combination. Table 2 lists these synodic periods and the next two launch periods for the outer planet missions discussed. Launch windows of as long as 30 days[5] should be available for each opportunity within a launch period. However, because the planets' positions are constantly changing with respect to one another, only opportunities during the first two-thirds of a launch period provide the shorter trip times noted earlier, whereas the last opportunities of a launch period approach direct flights to the target planets.

All of the next launch opportunities for Jupiter- and Saturn-assisted missions to Saturn, Uranus, and Neptune occur in the 1977–1985 time period. The favorable phasing of the

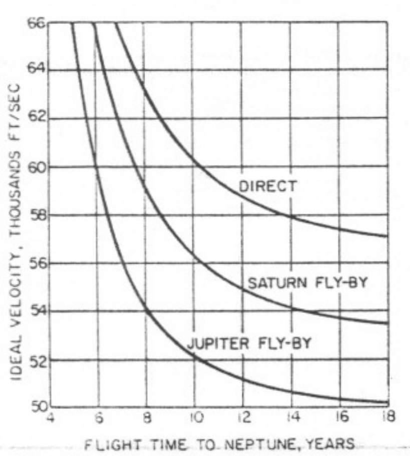

Fig. 10 Ideal velocity comparison: Neptune (30.1 a.u.).

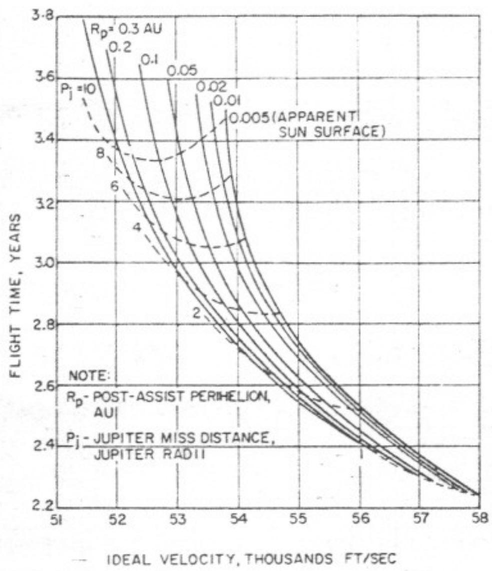

Fig. 11 Jupiter-assisted solar-probe trajectory performance.

outer planets (except Pluto) suggested by this coincidence led Flandro[5] to suspect that they could all be combined into a "grand-tour" gravity-assist mission. From a detailed analysis, he has concluded that an Earth/Jupiter/Saturn/Uranus/Neptune mission is possible in 1978. The next opportunity after 1978 to attempt this mission will not occur for at least 171 years, i.e., the synodic period between Uranus and Neptune.

Earth/Jupiter/Solar Probe

Hunter[6] has shown that drastic reductions in V_I for close solar-probe missions can be achieved with a Jupiter swing-by. Admittedly, long trip times, on the order of 3 years, decrease spacecraft reliability, and the double traversal of the asteroid belt involves hazards. Nevertheless, when missions to less than 0.1 a.u. are desired, it is apparent that the only available route with existing chemical propulsion systems is via a Jupiter fly-by. Equally important is the fact that a V_I of 55,000 fps will take the spacecraft anywhere from 0.1 a.u. to a solar impact simply by varying the miss distance at Jupiter.

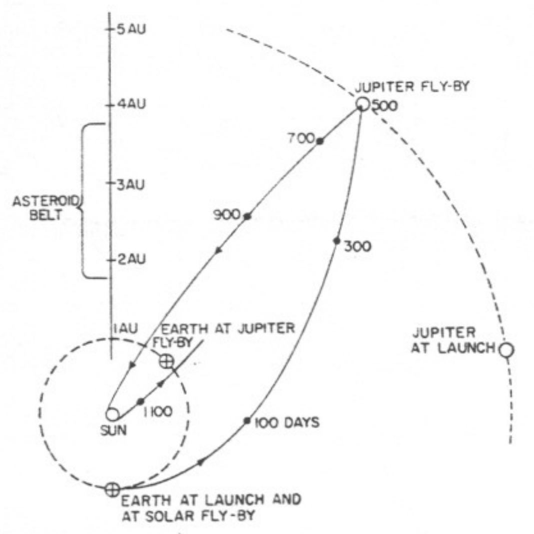

Fig. 12 Earth/Jupiter/solar-probe trajectory illustration. Trajectory data: launch opportunities = 1/year for 1970–1980; ideal velocity = 54,000 fps; Jupiter miss distance = 5.3 Jupiter radii; final perihelion = 0.02 a.u.; flight time to 0.02 a.u. = 3 years; equivalent ΔV at Jupiter = 57,400 fps.

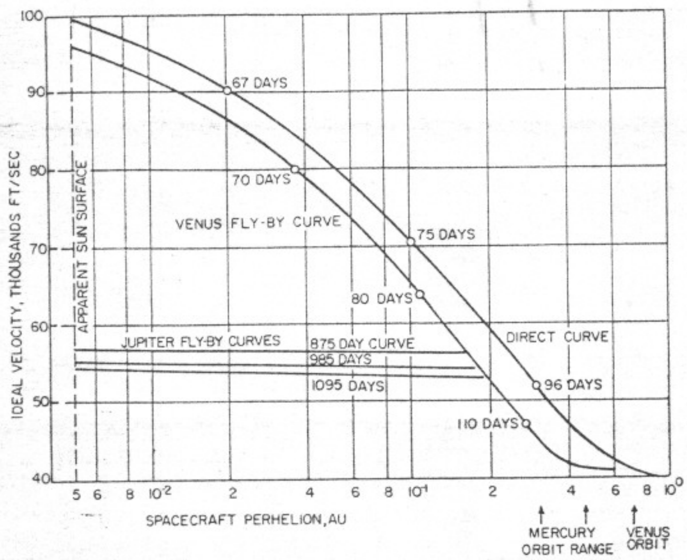

Fig. 13 Solar-probe energy requirements.

A direct flight to 0.1 a.u. requires 70,000 fps; to impact the sun directly requires almost 100,000 fps.

Figure 11 is a plot of total trip time vs V_I for Jupiter-assisted solar-probe missions. Curves of constant post-assist perihelion and curves of constant p at Jupiter are shown. Each point within the grid represents a valid solar-probe trajectory via Jupiter. A sample trajectory is illustrated in Fig. 12. About 400 days of the trip is spent in the asteroid belt. From one point of view, this is a hazard, and yet on the other hand, asteroid-belt experiments could be conducted during the flight as a secondary objective of the mission. Another added objective would be the examination of Jupiter during fly-by. Reference 7 indicates that instruments designed to measure particles and fields around Jupiter are of the same type and sensitivity as those that might be used on a solar probe. Launch opportunities are not a problem; they occur once every 13 months, i.e., once every Jupiter opportunity. Figure 13 presents a comparison of V_I's required to reach perihelia near the sun with direct, Venus-assisted and constant-time, Jupiter-assisted trajectories.

Conclusions

The attractiveness of a Venus assist to Mercury, analyzed by Minovitch,[2] is supported. It is concluded that neither Venus nor Mars provides particularly useful assists for missions to Jupiter. Jupiter, on the other hand, is the most influential gravity-assist planet in the solar system. Jupiter swing-bys provide important reductions in flight time for Saturn and Uranus fly-by missions. Launch opportunities for these missions occur in the late 1970's and not again until the 1990's. Planning for the coming opportunities should begin soon so that a reliable spacecraft configuration is available to take advantage of them.

Solar probes within 0.1 a.u. of the sun require a Jupiter assist with present chemical propulsion. Launch opportunities occur yearly for the Earth/Jupiter/solar-probe mission. Both Hunter[6] and Minovitch[8] have also shown the usefulness of Jupiter's gravitational field for 90° out-of-the-ecliptic flights. In fact, it has become apparent that post-Jupiter objectives are almost limitless, extending from Earth-return trajectories to solar-system escape perpendicular to the ecliptic plane. Although perhaps intuitive, it must be emphasized that the mission designer cannot afford to ignore the exploration bonus available to his spacecraft after a Jupiter encounter.

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