TECHNICAL TRANSLATION

F-44

THE ATTAINABILITY OF HEAVENLY BODIES

By Walter Hohmann

Translation of "Die Erreichbarkeit der Himmelskörper."
R. Oldenbourg (Munich - Berlin), 1925.

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PREFACE

The present work will contribute to the recognition that space travel is to be taken seriously and that the final successful solution of the problem cannot be doubted, if existing technical possibilities are purposefully perfected as shown by conservative mathematical treatment.

In the original work 10 years ago, the author believed that 2,000 m/sec was the highest gas velocity attainable for the foreseeable future by our technical means. Calculations originally were only carried out for this value. But in the meantime three works concerning the rocket problem were published, which make it apparent that far higher exhaust-gas velocity can be reached with a suitable array:

Robert H. Goddard: "A Method of Reaching Extreme Altitudes" (mainly on the basis of practical experiments);

Herman Oberth: "The Rocket Into Interspace" (especially valuable for detailed suggestions on the basis of theory);

Max Valier: "The Advance Into Space" (a simple presentation of the problem).

Therefore and for direct comparison with Oberth's results, calculations were extended to higher gas velocity (2,500, 3,000, 4,000, and 5,000 m/sec), leaving the original 2,000 m/sec now as the lower limit. This makes conditions much more favorable; however, the following observations apply:

The use of low gas velocities required avoidance of any deadweight. This led to arranging the fuel to be carried with the rocket in the shape of a cylinder of solid fuel, whose combustion would automatically result in the escape of the gas at the required velocity. This arrangement represents the ideal solution, because no deadweight is involved, but is conceivable only at low gas velocities. According to Oberth the higher velocity can be achieved only by means of gases escaping through narrowed jets during combustion; carrying the jets and the container for the liquid fuel means a more or less large deadweight, which is easier to carry at higher gas velocity.

The weights used in the last two sections do not include these unavoidable deadweights, because estimation without practical experiments with jets and containers is hardly possible. Quoted weights to be lifted G_o therefore represent the lowest value while using an ideal fuel.

Taking higher velocities into consideration as well as later supplements - particularly investigations concerning possibility of landing without deceleration ellipse at the end of section II, and concerning intersecting ellipses at the end of section V, also considerations concerning heating during landing - are due to suggestions by Mr. Valier and Professor Oberth.

Since the writer is an engineer, not a mathematician, clumsy approximations in place of mathematical formulas occasionally appear in the calculations; this should not affect the results.

Essen, October 1925 W. Hohmann

$$v_a^2 = \frac{\mu}{r_a};$$

the orbit in this case is a circle.

14. The period for an elliptic orbit is given by the area theorem (39):

$$\frac{dF}{dt} = \text{constant} = \frac{v_a r_a}{2} :$$

$$F = \frac{v_a r_a}{2} \cdot t = ab\pi ;$$

thus

$$t = \frac{2ab\pi}{v_a r_a}; \qquad (47)$$

and if, according to (46), we substitute the value

$$b = v_a r_a \sqrt{\frac{a}{\mu}}$$

then we get:

$$t = 2a\pi \sqrt{\frac{a}{\mu}} = 2\pi \sqrt{\frac{a^3}{\mu}} . \qquad (48)$$

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CIRCUMNAVIGATION OF OTHER HEAVENLY BODIES

A circumnavigation of the Moon, f.i., to find the nature of its unknown side, will not substantially differ from free space travel, as long as one does not approach it close enough, so that its attraction as well as that of the Earth (which at the same distance is 80 times as effective) becomes significant. Since during the 30 days of the journey the Moon will also orbit the Earth once, we cannot speak of a circumnavigation, but rather a crossing of paths, which may take the form shown in Figure 24, where E, M and F are Earth, Moon and rocket respectively, while the numbers indicate simultaneous positions. The largest Moon perigee is therefore about half of the largest Earth apogee, the relative largest attraction by the Moon therefore about $\frac{4}{80} = \frac{1}{20}$ of the simultaneous Earth attraction. Its influence will not further be investigated here.

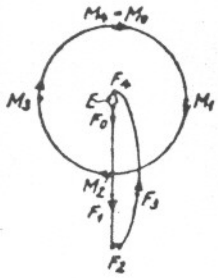

Figure 24.

In our consideration only the Earth attraction was considered, while that of the Sun was ignored, because the vehicle follows the 30 km/sec, which the Earth covers in its orbit about the Sun. This is only strictly true at the instance, when the vehicle is at rest relative to Earth, i.e., at its highest point r_3, and even then only if this point has the same distance from the Sun as the Earth. Assuming that the vehicle leaves the Earth tangentially to the Earth orbit, then its velocity, if 10 km/sec relative to Earth is with respect to the Sun 30 + 10 = 40 or 30 - 10 = 20 km/sec, according to a positive or negative direction. In the latter case, its instantaneous trajectory has a higher curvature, in the first case, less curvature than the Earth orbit, because of the solar attraction. Since the vehicle's velocity relative to Earth however quickly decreases because of the Earth's gravitational effect, and the total time of the ascent is only 15 days, i.e., 1/24 of an Earth orbit, the trajectory in the range considered will not perceptably deviate from the Earth orbit. If on the other hand the ascent is radial with respect to the Earth orbit, then at the apogee r_3 the velocity of the vehicle relative to the Sun equals that of the Earth, but the distance from the Sun is larger or smaller than the Earth-Sun distance, depending on the ascent being from or away from the Sun. In the former case, the trajectory again has a higher curvature, in the latter a lower curvature. But since an apogee of 800,000 km is negligible compared to the distance of 150,000,000 km, the deviation here too is hardly noticeable. The direction of ascent is therefore so far arbitrary. It is advisable however to direct it toward the Sun, so that the Earth can be seen in its entirety and brightly, which is necessary for the distance and velocity measurements. The distance r_3 = 800,000 km to be attained will therefore always be assumed in that direction during our further considerations, even if r_3 is ignored with respect to the distance from the Sun.

If at that point the tangential velocity v_3 is chosen 3 km/sec instead of 0.09 km/sec (see Figure 14) as in section III, then under the influence of the Earth's attraction alone the trajectory is now a flat hyperbola rather than an ellipse, since

$$\frac{2\mu}{r_3} - v_3^2 = \frac{2 \cdot 400{,}000}{800{,}000} - 3^2 = -8$$

and the vehicle will pursue a path directed away from the effective range of the Earth gravitation with nearly uniform velocity, until it finally -- so to say as an independent comet -- it is subject only to the attraction of the Sun. Initially the tangential velocity relative to the Sun is $v_I = 29.7 \pm 3.0 = 32.7$ or 26.7 km/sec, according to whether v_3 is along or against the Earth velocity of 29.7 km/sec. In either case the vehicle describes an ellipse about the Sun in the first case outside, in the second inside of the Earth orbit.

If the vehicle's path is to touch at a distance r_{II} from the Sun, the orbit of a planet other than the Earth, distant r_I from the Sun (see Figure 25), then the major axis of the ellipse is

$$a = \frac{r_I + r_{II}}{2};$$

and by (45)

$$a = \frac{\mu}{\frac{2\mu}{r_I} - v_I^2};$$

thus

$$\frac{2\mu}{r_I} - v_I^2 = \frac{2\mu}{r_I + r_{II}};$$

from this

$$v_I^2 = \frac{2\mu}{r_I + r_{II}} \cdot \frac{r_{II}}{r_I};$$

or

$$v_I = \sqrt{\frac{2\mu}{r_I + r_{II}} \cdot \frac{r_{II}}{r_I}}. \qquad (49)$$

The Earth's mean distance from the Sun is $r_I = 149{,}000{,}000$ km, that of Venus f.i. $r_{II} = 108{,}000{,}000$ km. Since further by (37) $\mu = 132{,}000{,}000{,}000 \text{ km}^3/\text{sec}^2$, we have for a journey close to Venus

$$v_I = \sqrt{\frac{264{,}000}{257} \cdot \frac{108}{149}} = 27.3 \text{ km/sec.}$$

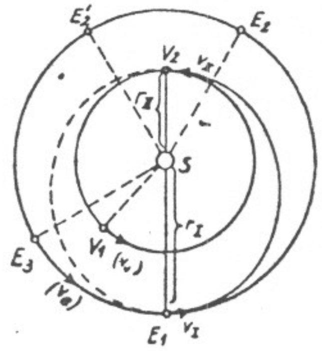

Figure 25.

Now the Earth velocity is $v_e = 29.7$ km/sec and accordingly the velocity to be imparted to the vehicle, after it reaches its apogee, has to be

$$\Delta v_I = v_I - v_e = 27.3 - 29.7 = -2.4 \text{ km/sec}$$

and could be the result of a tangential shot of mass

$$\Delta m = m \cdot \frac{\Delta v_I}{c} ,$$

where m is the mass of the vehicle before the shot and c is the velocity of the projectile. In this case, one can no longer use the value of c = 1 km/sec, assumed in section III, and also a single shot of the required strength would endanger the vehicle and its passengers. A system of continuous mass radiation as in section I must be used with a min. velocity of c = 2 km/sec. We have now for the ratio of total mass before and after the action by (32)

$$\frac{m_o}{m_1} = e^{\left(\frac{\Delta v}{c}\right)} .$$

But since during the initial parallel paths of planet and vehicle orbit interference is unavoidable, an additional safety factor [see Note], say $\nu = 1.1$, must be added, which necessitates:

$$\left(\frac{m_o}{m_1}\right)_I = \nu \cdot e^{\frac{\Delta v_I}{c}} = 1.1 \cdot e^{\frac{2.4}{2.0}} = 1.1 \cdot e^{1.20} = 3.65,$$

where the mass has to be thrown forward in the direction of the Earth's motion.

([Note] Such interferences may be obviated by radiating the mass $\frac{dm}{dt} = - am$ (see (1c)), directed against the disturbing planet and equal to the gravitational effect g, so that at a distance x from the planet by (1a) and (2)

$$\frac{dv}{dt} = c\alpha = g = g_o \frac{r_o^2}{x^2} \text{ and } \frac{m_o}{m} = e^{\alpha t}.$$

At the assumed initial point x = 800,000 km from the Earth with g_o = 9.8 m/sec^2 and r_o = 6,380 km:

$$c\alpha = 9.8 \cdot \frac{6{,}380^2}{800{,}000^2} = \frac{1}{16{,}000} \text{ m/sec}^2$$

and after one day = 86,400 sec, when c = 2,000 m/sec:

$$\alpha t = \frac{c\alpha}{c} \cdot t = \frac{86{,}400}{16{,}000 \cdot 2{,}000} = 0.0270;$$

at the distance x = 800,000 km from Venus with g_o = 8.7 and r_o = 6,090;

$$c\alpha = 8.7 \cdot \frac{6{,}090^2}{800{,}000^2} = \frac{1}{20{,}000} \text{ m/sec}^2$$

and

$$\alpha t = \frac{86{,}400}{20{,}000 \cdot 2{,}000} = 0.0216;$$

at a distance x = 800,000 km from Mars with g_o = 3.7 and r_o = 3,392:

$$c\alpha = 3.7 \cdot \frac{3{,}393^2}{800{,}000^2} = \frac{1}{150{,}000}$$

and

$$\alpha t = \frac{86{,}400}{150{,}000 \cdot 2{,}000} = 0.00288.$$

With each succeeding day, x becomes larger, i.e., the daily increase αt smaller. By plotting planet and vehicle positions, one gets for the first 5 days the following distances x and the corresponding daily values αt:

	Earth		Venus		Mars	
Days	x km	αt	x km	αt	x km	αt
0	800,000	0.0270	800,000	0.0216	800,000	0.0029
1	850,000	0.0240	850,000	0.0191	900,000	0.0023

Days	Earth x km	Earth αt	Venus x km	Venus αt	Mars x km	Mars αt
2	900,000	0.0213	900,000	0.0170	1,000,000	0.0018
3	1,000,000	0.0173	1,000,000	0.0138	1,200,000	0.0013
4	1,100,000	0.0143	1,200,000	0.0096	1,400,000	0.0009
5	1,200,000	0.0120	1,400,000	0.0070	1,700,000	0.0006
Sum	$\Sigma\alpha t = 0.1159$		$\Sigma\alpha t = 0.0881$		$\Sigma\alpha t = 0.0098$	

Accordingly after the first 5 days $\nu = \frac{m_0}{m} = e^{\Sigma\alpha t}$:

for Earth: $\nu = e^{0.116} = 1.123$; for Venus: $\nu = e^{0.088} = 1.093$;

for Mars: $\nu = e^{0.01} = 1.01$.

The above safety factor $\nu = 1.1$ is thus only a rough mean, which must be corrected for each planet. The interference correction need not be done in one step, it will be sufficient to do it daily once or several times with corresponding intensity. [End of note]) The time required to cover half the ellipse by (48), using $a = \frac{r_I + r_{II}}{2} = 128{,}500{,}000$ km:

$$T_I = \pi\sqrt{\frac{a^3}{\mu}} = \pi\sqrt{\frac{128{,}500{,}000^3}{132{,}000{,}000{,}000}} = 12{,}600{,}000 \text{ sec} = 146 \text{ days.}$$

The Earth moves in its orbit with an angular velocity of $\frac{360^\circ}{365 \text{ days}} = 0.987^\circ/\text{day}$, Venus with $\frac{360^\circ}{224 \text{ days}} = 1.607^\circ/\text{day}$. During the time of 146 days, the Earth thus covers an arc of $146 \cdot 0.987 = 144^\circ$, Venus an arc of $146 \cdot 1.607 = 234.5^\circ$. In order to have the vehicle approach Venus in fact (say at a distance of 800,000 km from the center of Venus and on the side closest to the Sun), the launching has to take place at a time when Venus is $234.5 - 180 = 55.5^\circ$ behind Earth in the sense of the motion of the planets (points V_1 and E_1 in Figure 25). If the vehicle would continue its journey unchanged, then it would return after a further 146 days to its initial point in space via the dotted half of the ellipse, the Earth however would be retarded with respect to the vehicle by a further 36° or a total of 72° (point E_3 in Figure 25). To make it possible for the orbits to intersect, the time for the return trip must somehow be increased. Two possibilities present themselves:

1st possibility (see Figure 25). If the dotted branch of the ellipse is to lead back to Earth, then the Earth at the time of departure at V_2 would have to be 36° in front of, rather than behind, Venus, i.e., at E_2', not E_2. The vehicle would have to be kept near Venus until the correct position of the two planets occurrs, i.e., until Venus has almost caught up again with Earth, except for the 36°. Because of its faster travel, Venus gains a daily angle of $1.607 - 0.987 = 0.62^\circ$ and so it would require 464 Earth days for it to advance from its initial 36° advantage over Earth, the remaining 288° to arrive 36° behind the Earth. During this time, the vehicle can be made to circle Venus arbitrarily often. To achieve this, it must first of all be suitably decelerated, say by Δv_{II} and thus subjected to the permanent influence of the gravitational field of that planet, just as it was previously taken out of the Earth's gravitational field by Δv_I. The Venus near point V_2 (Figure 25) is attained with a velocity

$$v_{II} = v_I \cdot \frac{r_I}{r_{II}} = 27.3 \cdot \frac{149}{108} = 37.6 \text{ km/sec},$$

while the orbital velocity of Venus is

$$v_V = \frac{2 \cdot 108{,}000{,}000 \cdot \pi}{224 \cdot 86{,}400} = 35.1 \text{ km/sec}.$$

To attain velocity zero with respect to Venus, a decrease of $37.6 - 35.1 = 2.5$ km/sec must be made. If the orbit about Venus is to be a circle of radius a, then the period by (48) is $t = 2\pi\sqrt{\frac{a^3}{\mu}}$. With regard to the correct vehicle position with respect to the later departure, the following has to be observed concerning the choice of t: During the 464 Earth days, Venus covers its orbit $\frac{464}{224} = 2.07 = 2 + 0.07$ times, i.e., when the orbitting has to stop, Venus will be further in its orbit by 0.07 rotations about the Sun than at the beginning (see Figure 25a). Since the vehicle's velocity, when entering the field of attraction of Venus (v_{II}), as well as at the exit from this field (v_{II}'), must be directed at right angles to the Sun-Venus radius vector, there are, according to Figure 25a, at the moment, when the vehicle leaves the orbit, 0.07 parts of an orbit missing. The total number of orbits may therefore be 3.93 or 4.93 or 5.93 and so on, so that f.i. for 5.93:

$$t = \frac{464}{5.93} = 78.2 \text{ days} = 6{,}750{,}000 \text{ sec}.$$

If mass conditions pertaining to Earth are assumed to apply, for simplicity's sake, also in the case of Venus, which is of the same size or nearly so (exact observations of the trajectory deviations of comets indicate, that Venus has a mass only of 0.82 that of Earth), then we may again put μ = 400,000 km^3/sec^2 and this leads for a to:

$$a = \sqrt[3]{\mu\left(\frac{t}{2\pi}\right)^2} = \sqrt[3]{400,000\ \left(\frac{6,750,000}{2\pi}\right)^2} = 773,000 \text{ km},$$

and for a trajectory velocities during the orbits of

$$v_3 = \frac{2a\pi}{t} = \frac{2 \cdot 773,000 \cdot \pi}{6,750,000} = 0.72 \text{ km/sec}.$$

The desired orbit will automatically occur, if at the time of transit at V_2 (Figure 22) the relative velocity is not zero, but 0.72 km/sec, i.e., the retardation not equal to 2.5, but

$$\Delta v_{II} = 37.6 - 35.1 - 0.72 = \sim 1.8 \text{ km/sec}.$$

This again requires a radiation of mass of

$$\left(\frac{m_o}{m_1}\right)_{II} = \nu \cdot e^{\left(\frac{\Delta v_{II}}{c}\right)} = 1.1 \cdot e^{\frac{1.8}{2.0}} = 1.1 \cdot e^{0.9} = 2.65$$

in the direction of the motion, i.e., toward the front.

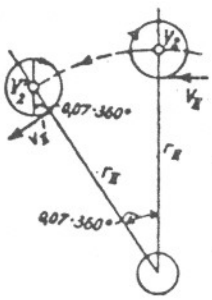

Figure 25a.

After the 464 days, necessary for the 5.93 orbits, an equal radiation of $\left(\frac{m_o}{m_1}\right)_{II}$ = 2.65 must be used to withdraw the vehicle in opposite direction from the gravitational field of Venus and to return it into its own elliptic orbit, in which after a further 146 days it returns to the neighborhood of Earth. At the instant of the crossing, again assumed to occur r_3 = 800,000 km from the Earth center, the relative velocity with respect to Earth has to be reduced by further mass radiation to the value of v_3 = 0.09 km/sec, established in section II. Since at this instant the velocity of the vehicle is v_I = 27.3 km/sec and the speed of the Earth in its trajectory is v_e = 29.7 km/sec, the necessary increase in velocity is

$$\Delta v_I' = 29.7 - 27.3 - 0.09 = \sim 2.3 \text{ km/sec}$$

and must now be effected, so as to accelerate the vehicle, i.e., toward the rear:

$$\left(\frac{m_o}{m_1}\right)_I' = \nu \cdot e^{\frac{2.3}{2.0}} = 1.1 \cdot e^{1.15} = 3.47.$$

The entire journey in this case lasts -- including the 30 days required for ascent and launching:

$$30 + 146 + 464 + 146 = 786 \text{ days} = 2.15 \text{ years}.$$

If m_1 denotes the mass of the returning vehicle, m_o the total mass at the beginning of the ascent, including fuel, then -- not taking into account the change of mass due to supplies being used up -- approximately:

$$\frac{m_o}{m_1} = 933 \cdot 3.65 \cdot 2.65^2 \cdot 3.47 = 83{,}000.$$

2nd possibility (see Figure 26): From the point V_2 the vehicle is to return to Earth E_4 not directly, but by detour via F_3. The coincidence with Earth can happen at best 1.5 Earth years after separation at E_1. The Sun distance r_{III} of point F_3 is to be chosen therefore such, that the entire travel time from E_1 via V_2 and F_3 to E_4 takes 547.5 days. The total time T is composed out of the times T_1 and T_2 and T_3, required to cover the 3 half ellipses I, II and III with the major semi-axes

$$a_1 = \frac{r_I + r_{II}}{2} = 128{,}500{,}000 \text{ km};$$

$$a_2 = \frac{r_{II} + r_{III}}{2}; \; a_3 = \frac{r_{III} + r_I}{2}.$$

From these two expressions:

$$a_3 - a_2 = \frac{r_I - r_{II}}{2} = \frac{149{,}000{,}000 - 108{,}000{,}000}{2} = 20{,}500{,}000 \text{ km}.$$

Further

$$T_3 + T_2 = T - T_1 = 547.5 - 146 = 401.5 \text{ days},$$

or by (48) -- for half an elliptic orbit --

$$\pi\sqrt{\frac{a_3^3}{\mu}} + \pi\sqrt{\frac{a_2^3}{\mu}} = 401.5 \text{ days} = 34{,}700{,}000 \text{ sec},$$

or

$$\sqrt{a_3^3} + \sqrt{a_2^3} = \frac{34,700,000}{\pi} \cdot \sqrt{\mu} = \frac{34,700,000}{\pi} \sqrt{132,000,000,000};$$

therefore

$$\left.\begin{aligned} \sqrt{a_3^3} + \sqrt{a_2^3} &= 4,010,000,000,000, \\ a_3 - a_2 &= 20,500,000. \end{aligned}\right\}$$

and

These two equations are satisfied by:

$$a_3 = 169,000,000 \text{ km and } a_2 = 148,500,000 \text{ km.}$$

Therefore from $a_2 = \frac{r_{II} + r_{III}}{2}$:

$$r_{III} = 2a_2 - r_{II} = 297,000,000 - 108,000,000 = 189,000,000 \text{ km.}$$

Departure at E_1 occurred with a velocity $v_I = 27.3$ km/sec and arrival at V_2 with a velocity:

$$v_{II} = v_I \cdot \frac{r_I}{r_{II}} = 27.3 \cdot \frac{149}{108} = 37.6 \text{ km/sec.}$$

The velocity required at V_2, to reach F_3, is by (49):

$$v_{II}' = \sqrt{\frac{2\mu}{r_{II} + r_{III}} \cdot \frac{r_{III}}{r_{II}}} = \sqrt{\frac{264,000}{297} \cdot \frac{189}{108}} = 39.4 \text{ km/sec;}$$

which determines the velocity of arrival at F_3:

$$v_{III} = v_{II}' \cdot \frac{r_{II}}{r_{III}} = 39.4 \cdot \frac{108}{189} = 22.5 \text{ km/sec.}$$

The departure velocity necessary at F_3, to attain E_4, is

$$v_{III}' = \sqrt{\frac{2\mu}{r_{III} + r_I} \cdot \frac{r_I}{r_{III}}} = \sqrt{\frac{264,000}{338} \cdot \frac{149}{189}} = 24.8 \text{ km/sec,}$$

and finally the velocity, arriving at E_4

$$v_{IV} = v_{III}' \cdot \frac{r_{III}}{r_I} = 24.8 \cdot \frac{189}{149} = 31.5 \text{ km/sec}$$

compared to the Earth velocity

$$v_e = 29.7 \text{ km/sec.}$$

Accordingly the following velocity changes are necessary:

at departure E_1: $\Delta v_I = 27.3 - 29.7 = -2.4$ km/sec,

at passing V_2: $\Delta v_{II} = 39.4 - 37.6 = +1.8$ km/sec,

at passing F_3: $\Delta v_{III} = 24.8 - 22.5 = +2.3$ km/sec,

at arrival E_4: $\Delta v_{IV} = 29.7 - 31.5 + 0.09 = -1.7$ km/sec

(initiating landing).

The masses necessary to attain these velocity changes, using c = 2.0 km/sec, are given in sequence by

$$\left.\begin{aligned}
\left(\frac{m_o}{m_1}\right)_I &= \nu \cdot e^{\frac{2.4}{2.0}} = 1.1 \cdot e^{1.20} = 3.65 \\
\left(\frac{m_o}{m_1}\right)_{II} &= \nu \cdot e^{\frac{1.8}{2.0}} = 1.1 \cdot e^{0.90} = 2.71 \\
\left(\frac{m_o}{m_1}\right)_{III} &= \nu \cdot e^{\frac{2.3}{2.0}} = 1.1 \cdot e^{1.15} = 3.47 \\
\left(\frac{m_o}{m_1}\right)_{IV} &= \nu \cdot e^{\frac{1.7}{2.0}} = 1.1 \cdot e^{0.85} = 2.57
\end{aligned}\right\}$$

these are to be directed forward at E_1 and E_4 and backwards at V_2 and F_3

With the same notation as before

$$\frac{m_o}{m_1} = 933 \cdot 3.65 \cdot 2.71 \cdot 3.47 \cdot 2.57 = 82,000.$$

The entire journey takes

$$30.5 + 547.5 = 578 \text{ days} = 1.58 \text{ years},$$

including landing and ascent.

Of these possibilities, the second one for the same fuel consumption has the advantage of a shorter travel time, while the first permits a longer stay in the neighborhood of Venus.

A visit to Mars would take a similar form. Here however its position at the instant of proximity would have to be more accurately calculated, since its orbit is more eccentric than that of Earth or Venus, its Sun distance varying between 248,000,000 km and 205,000,000 km. Now the detour via F_3, according to Figure 26, at its apogee r_{III} equals 189,000,000 km, is nearly equal to the smallest distance of Mars to the Sun, leaving only 16,000,000 km. With a suitable choice of the time of ascent during a mutual constellation of Earth, Venus and Mars and with a suitable adjustment of r_{II} and r_{III}, a passage at relatively small distance (about $\frac{16}{2}$ = 8 million km each time) of Mars as well as Venus can be achieved in a single journey of about 1 1/2 years' duration.

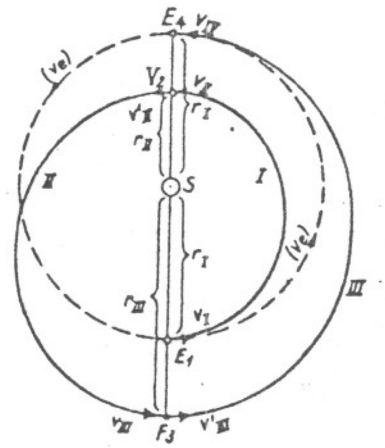

Figure 26.

This 580 day journey would not quite take 20 times as much as the 30 day journey into space, described in section III. For a rough estimate of the vehicle mass now required, we multiply by 20 the parts of the weight, which depend on the duration and are denoted on pages 67-68 under b), c), d), e), while those independent of time, i.e., listed under a), f), g) and i), are left as is. With regard to the larger weight h), required by the larger storage space, we allow 3 times its previous weight. Since with the storage space also the heat-transferring surface increases, we imply here a better insulation. These assumptions lead to a weight less fuel

$$(240 + 60 + 200 + 140) \cdot 20 = 12{,}800 \text{ kg}$$

$$+ 200 + 200 + 240 + 200 + 740 = 1{,}580 \text{ kg}$$

$$+ 780 \cdot 3 = \underline{2{,}340 \text{ kg}}$$

total of 16,720 kg = 16.72 t.

Between E_1 and V_2, T_1 = 146 days pass, between V_1 and F_3

$$T_2 = T_1 \cdot \sqrt{\frac{a_2^3}{a_1^3}} = 146 \sqrt{\frac{148.5^3}{128.5^3}} = 181 \text{ days;}$$

between F_3 and E_4

$$T_3 = T_1 \cdot \sqrt{\frac{a_3^3}{a_1^3}} = 146 \sqrt{\frac{169.0^3}{128.5^3}} = 220 \text{ days.}$$

Of the 12.8 tons of supplies, consumed were:

during the 15 days ascent up to E_1: $12.8 \cdot \frac{15}{578} = 0.33$ t,

between E_1 and V_2: $12.8 \cdot \frac{146}{578} = 3.20$ t,

between V_2 and F_3: $12.8 \cdot \frac{181}{578} = 3.95$ t,

between F_3 and E_4: $12.8 \cdot \frac{220}{578} = 4.80$ t,

between departure and E_4 therefore 12.28 tons.

After arrival at E_4, the weight of the vehicle remains 16.72 - 12.28 = 4.44 tons.

Immediately before arrival at E_4, the total mass is	$4.44 \cdot 2.57 =$	11.40 t
after arrival at F_3	$11.40 + 4.80 =$	16.20 t
immediately before arrival at F_3	$16.20 \cdot 3.47 =$	56.30 t
after arrival at V_2	$56.30 + 3.95 =$	60.25 t
immediately before arrival at V_2	$60.25 \cdot 2.71 =$	163.00 t
after arrival at E_1	$163.00 + 3.20 =$	166.20 t
immediately before arrival at E_1	$166.20 \cdot 3.65 =$	606.67 t
after acceleration has ended	$606.67 + 0.33 =$	607 t
at departure $G_0 =$	$607 \cdot 933 =$ 567,000	t

or abbreviated:

$$G_0 = [\{[(4.44 \cdot 2.57 + 4.8) \cdot 3.47 + 3.95] \cdot 2.71 + 3.2\} \cdot 3.65 + 0.33] \cdot 933 = 567{,}000 \text{ t.}$$

The main part of the ammunition, to be taken along, is taken up by the fuel required for the initial acceleration, but such fuel is also necessary (say 607 - 17 = 590 tons), to change the velocity during the journey and such a mass will present difficulties as regards maneuverability. How much G_o depends upon the velocity, with which the mass is radiated, c, is clarified by the following list of values for G_o, resulting for different values of c for a constant acceleration of $\alpha c = 30\ m/sec^2$:

$$c = 2\ km/sec:\ G_o = [\{[(4.44 \cdot 2.57 + 4.8) \cdot 3.47 + 3.95] \cdot 2.71 + 3.2\} \cdot 3.65 + 0.33] \cdot 933 = 567{,}000\ t$$

$$c = 2.5\ km/sec:\ G_o = [\{[(4.44 \cdot 2.17 + 4.8) \cdot 2.77 + 3.95] \cdot 2.27 + 3.2\} \cdot 2.87 + 0.33] \cdot 235 = 69{,}500\ t$$

$$c = 3\ km/sec:\ G_o = [\{[(4.44 \cdot 1.95 + 4.8) \cdot 2.38 + 3.95] \cdot 2.00 + 3.2\} \cdot 2.45 + 0.33] \cdot 95 = 17{,}600\ t$$

$$c = 4\ km/sec:\ G_o = [\{[(4.44 \cdot 1.69 + 4.8) \cdot 1.98 + 3.95] \cdot 1.73 + 3.2\} \cdot 2.00 + 0.33] \cdot 30 = 3{,}150\ t$$

$$c = 5\ km/sec:\ G_o = [\{[(4.44 \cdot 1.55 + 4.8) \cdot 1.75 + 3.95] \cdot 1.57 + 3.2\} \cdot 1.78 + 0.33] \cdot 15 = 1{,}130\ t.$$

V

LANDING ON OTHER CELESTIAL OBJECTS

Of the planets, Venus seems to be particularly suited for a landing, because, presumably, it has an atmosphere similar to that of Earth. This and the further assumption of similar gravitation conditions would accordingly permit a landing exactly as described in sections II and III for Earth. It could begin by imparting to the vehicle at the distance r_3 = 800,000 km from the center of Venus a tangential velocity v_3 = 0.09 km/sec (see Figure 14). (Compare what has been said about Mars and Venus on pages 88-89. Since furthermore the atmosphere of Venus is very high and dense, landing should be simpler than on Earth.) The preceding journey proceeds exactly as determined for the segment E_1 - V_2, following Figure 25, i.e., V_2 is passed with a velocity v_{II} = 37.6 km/sec, compared with a velocity of Venus v_v = 35.1 km/sec; the relative velocity at that instant being 37.6 - 35.1 = 2.5 km/sec. To reduce it to 0.09 km/sec, a reduction by approximately Δv_{II} = 2.4 km/sec is necessary, requiring a mass