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Table 1. Vidicon coverage expected at various altitudes for a nominal *Ranger* RA-3 trajectory

Picture No.	Altitude, km	Approximate resolution, km	Approximate area covered on lunar surface,* km^2
0	4131	0.638	1850
10	3896	0.604	1680
20	3659	0.570	1400
30	3419	0.536	1300
40	3175	0.501	1090
50	2926	0.466	961
60	2679	0.430	784
70	2425	0.394	625
80	2166	0.357	529
90	1902	0.320	400
100	1631	0.282	289
110	1354	0.243	196
120	1069	0.204	121
130	773	0.163	64
140	465	0.121	25
150	141	0.078	2.25

*Each lunar degree is 30.335 km.

Determination of the impact point by this method will probably be more a backup to the tracking data than an improvement in knowledge of the impact location obtained from the tracking data.

In order to check the process described, a simulated set of pictures is being constructed with the aid of the IBM 1620 computer. These pictures, when completed, will be analyzed in the manner described. The analyst will have no prior knowledge of the input conditions used. The success of this operation will, of course, depend on how well the simulated pictures correspond to an actual set of pictures.

B. Trajectory Analysis

V. C. Clarke and W. E. Kirhofer

1. 24-Hour Circumlunar Satellite Injection Orbit

In *Research Summary 36-11* a method of analysis was presented for obtaining first estimates of circumlunar trajectories in 3 dimensions. An example of a circumlunar trajectory was given for which the probe–Earth distance remains large for a long period of time. A summary of its key elements is given in Table 2. These elements, which are taken about 5 days after passing the Moon, are similar to those of the Moon except for the inclination. There appears little doubt that another trajectory could be obtained for which all elements were identical (except inclination) to those of the Moon. Such a trajectory could be called an Earth–Moon–Moon trajectory and could be made to impact or closely approach the Moon on the second pass (which would occur after a half revolution around the Earth). A trajectory of this type may have little practical value, but it serves to indicate the realm of possibilities for using the Moon to grossly alter or shape a trajectory in Earth–Moon space. For example, the Moon could be used to create a circular Earth satellite orbit at lunar distance. Such an orbit may have significant practical value.

Another useful circumlunar orbit is an alternate method of injecting a 24-hr Earth satellite into an equatorial orbit. The normal method of injecting a spacecraft into a 24-hr satellite orbit consists of: (1) injecting into a 100-nm parking orbit, (2) coasting in this orbit until a chosen crossing of the equator, (3) applying an impulse at equatorial crossing to transfer into an ellipse which has an apogee distance equal to the radius (42165 km) of a 24-hr orbit, and (4) applying an impulse at apogee to both circularize the orbit and perform a plane change into the equatorial plane.

The alternate method of injecting into a 24-hr satellite orbit, however, is to circumnavigate the Moon to attain the proper radius (42165 km), planar orientation (equatorial), and longitude when the probe returns to Earth. The method is as follows: (1) inject into a 100-nm parking orbit, (2) apply an impulse at the proper point in the parking orbit to transfer into an Earth–Moon trajectory of about 80-hr duration, (3) circumnavigate the Moon in a chosen manner so that after leaving the influence of the Moon, the trajectory lies in the equatorial plane, has a perigee equal to 42165 km, and the geographic longitude of perigee is the desired longitude of the 24-hr satellite, and (4) then apply a retro impulse at perigee to circularize into a 24-hr orbit.

Table 2. Elements of sample circumlunar trajectory

Element	Value
Semi-major axis, km	420952
Inclination, deg	57.42
Period, days	31.27
Eccentricity	0.0523

Using this method the Moon must lie in, or near, the equator of the Earth when the probe circumnavigates the Moon. Such a requirement restricts launchings to a period of 1 or 2 days every 2 weeks. Several trajectories using this method have been obtained using the JPL precision lunar Trajectory Program. Characteristics of these trajectories are summarized in Table 3. In this table the final energy, C_{3F}, is calculated at perigee of the geocentric orbit *after* passing the Moon. It is a measure of the amount of retro-impulse required to circularize the orbit.

Table 3. Characteristics of 24-hr circumlunar satellite injection trajectories

Longitude of 24-hr satellite, deg	Earth–Moon flight time, hr	Total flight time			Lunar closest approach distance,[a] km	Injection energy,[b] C_{3I} (km²/sec²)	Final energy,[b] C_{3F} (km²/sec²)
		d	h	m			
91.4	84.21	6	14	14	4094	−1.957	−1.478
121.7	82.73	6	12	14	3833	−1.944	−1.465
178.5	79.81	6	8	23	3358	−1.915	−1.439
238.9	77.51	6	4	15	2877	−1.887	−1.377
268.3	76.09	6	2	3	2667	−1.866	−1.357

[a]To Moon's center.
[b]Twice the total energy per unit mass.

An interesting observation may be made by comparing the sums of the velocity increments (required to achieve a final 24-hr circular orbit) for each method. The sum for each method is about the *same*. On the surface, this would indicate that it costs no more payload to achieve a 24-hr orbit by going around the Moon than going directly. However, different vehicle configurations, that would undoubtedly be used in each case, might alter the equivalence. An obvious disadvantage to this method is the long (> 6 days) total flight time required to achieve the orbit. On the other hand, an advantage is the ease of obtaining the desired satellite longitude by simply selecting the proper flight time.

If placed under close scrutiny, the circumlunar method of injecting a 24-hr satellite would likely be undesirable. However, it provides an illustration of how the Moon may be used to significantly shape a trajectory in Earth–Moon space. In this case, the Moon caused the plane change that would ordinarily be accomplished with a propulsive impulse. In general, the Moon can be used to add or subtract energy from orbits and to change their planar orientation.

C. Measuring Flight Time Variation[s] for Lunar and Interplanetary Trajectories

T. H. Thornton

1. Definition of Time of Flight

This discussion compares three methods used to defi[ne] time of flight variation for lunar and interplanetary tr[a]jectories and investigates the linearity of each. The thr[ee] methods considered are the variations in (1) time t[o] closest approach for a *massy* target, (2) linearized tim[e] of flight, and (3) time to closest approach for a massle[ss] target.

We shall define the standard time of flight to a targ[et] to be the difference between injection time, t_0, and t[he] time that the probe would pass through the center [of] the target, t_2.

$$t_{standard} = t_2 - t_0 \quad (1)$$

When defined in this manner, the standard trajecto[ry] impacts vertically, and t_2 is the time to closest approac[h]. When the standard trajectory is perturbed, the first-ord[er] change in flight time is

$$\Delta t = t_{perturbed} - t_{standard} = \sum_{i=1}^{n} \frac{\partial t}{\partial q_i} \Delta q_i \quad (2)$$

where Δq_i is a perturbation in one of the six injecti[on] coordinates. The problem is to find a satisfactory way [to] define time of flight for nonstandard trajectories, a[nd] thus Δt, so that the differential corrections $\partial t / \partial q_i$ have physical meaning and are not a function of the magnitu[de] of Δq_i within the range of the expected perturbations.

From a physical point of view, time to closest approa[ch] would have the most meaning; that is

$$\frac{\partial t}{\partial q_i} = \frac{\Delta t_{CA}}{\Delta q_i} \quad (3)$$

where Δt_{CA} is the difference between the standard a[nd] perturbed time required to reach the point of clos[est] approach. Trajectory analysis indicates, however, t[hat] time to closest approach does not vary linearly w[ith] injection conditions. C. R. Gates, W. Kizner, and J. Maloy investigated this nonlinear problem (Ref and introduced a correction term to lineari[ze] variations in time to closest approach. The result

$$\frac{\partial t}{\partial q_i} = \frac{\Delta t_{CA}}{\Delta q_i} - \frac{K}{\Delta q_i V_H^3} \ln e$$