to the moon and earth are also about 3000 and 1000 miles, respectively. Because of the higher initial velocity, the time of closest approach to the moon is reduced to 2.5 days. Since the vehicle's energy is higher, the moon does not perturb the trajectory so drastically as in the previous case. Perigee occurs at a time of 9.9 days. The trajectory is continued to a time of 12 days, to indicate the ellipse which the vehicle will follow.

The positions of the points of closest approach to the moon are shown in Fig. 9.35. Trajectories defined by relatively low initial velocities have positions of closest approach located nearly diametrically opposite the earth. As the initial velocity is increased, this location moves forward, in the direction of the moon's east limb. A portion of a

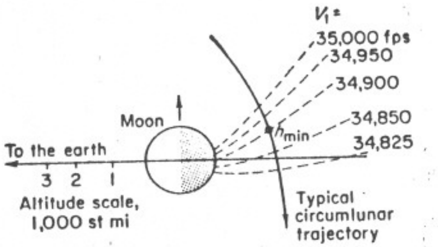


Fig. 9.35 Loci of minimum altitudes at the moon.

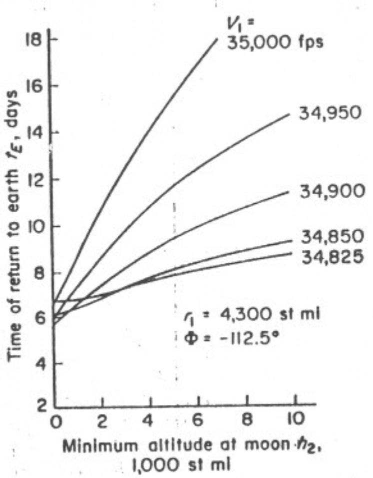


Fig. 9.36 Time of return to the earth: $r_1$ = 4300 st mi; $\Phi$ = 112.5°.

typical circumlunar trajectory, defined by a velocity of 34,900 fps, is shown as an example.

The time of return to the earth from a circumlunar trajectory is given in Fig. 9.36, as a function of minimum altitude at the moon for several values of the initial velocity. The round-trip time is seen to have a minimum value of about 5.5 days for trajectories which skim the surface of the moon, while the total return time is about one month for trajectories which reach maximum distances of twice the moon's orbital radius before returning to the earth. The curves for the two lowest velocities, which correspond to figure-eight type trajectories, are relatively insensitive to variations in the distance of closest approach to the moon. The return times for trajectories defined by the higher velocities are quite sensitive to this approach distance, which determines the resulting apogee radius, and, therefore, the time spent in the geocentric ellipse after passing the moon.

Additional discussion of two-dimensional circumlunar trajectories is contained in [23]. Reference [24], which describes the trajectories of the Russian Luniks, includes a discussion of three-dimensional circumlunar trajectories.

## 9.23 Interplanetary Flight

*By* H. O. Ruppe

### 9.231 Introduction

Only recently has a more than academic interest been shown in interplanetary transfers. Much simple feasibility work has been done already by the pioneers—to mention a few, Hohmann [25], Oberth [33], and von Pirquet [26]. Later studies (e.g., by Ehricke [29] and Moeckel [7]) have deepened our understanding, and the fundamentals of *precision trajectory work* in contrast to the *feasibility work* have been developed—e.g., by Herrick et al. [37–41]. Problems of optimization have been recognized, some of which have been solved (e.g., Lawden [27]).

Low-acceleration trajectories were thought of early (Oberth), but realistic trajectories were achieved only after World War II (see Sec. 9.1).



### 9.232 Astronomical and Technical Model

Similar to the problem of flight in the earth-moon system, a model is assumed to describe the structure of the solar system. Again, a simple model has the advantage that fundamental conclusions can be reached more easily, after employing analytical methods. On the other hand, a complex model can describe reality better, and certainly must be utilized to investigate real interplanetary flights, but results of a general nature may be hidden. Because of its complexity, the precision-type analysis must be left to the large electronic computer, and even here the development of an efficient and capable program is no simple matter [28,51].

To get results which are generally good enough for conceptual or even preliminary design, a simple model and procedure will be described in this section:

Assumptions Concerning the Astronomical Model. All planets move in circular coplanar orbits around the sun. Satellites move in circular orbits around planets; these orbits are not necessarily coplanar with the plane of planetary motion. The gravitational field of celestial bodies will be as though these bodies were mass points.

Assumptions Concerning the Vehicle. The plane of motion of the vehicle is practically identical with the plane of planetary motion unless stated otherwise.

The vehicle will be considered to move always as in a two-body problem, where the central body is either a planet (if the vehicle is within its sphere of influence) or the sun. This means that all perturbations are neglected. The series of two-body problem solutions have to be generated so that they fit together. The two-body problem (or really the one-body problem to which it can be reduced) has been treated in Secs. 4.2 and 4.3, and low-acceleration vehicles in Sec. 9.1. High-acceleration vehicles operating in space will be treated as though their burning times were infinitely short in comparison to the free-flight time. This is approximately true for initial accelerations higher than 5 m/sec². It is only during the launch phase from the surface of a celestial body that the approximation of impulsive thrusting does not hold without further corrections (gravity loss, drag loss, etc.). For details, see Chap. 6 and Sec. 25.2.

Generally speaking, low-acceleration systems result in high payload fractions at long flight times, whereas high-acceleration systems can lead to a low payload fraction and short flight duration. This indicates that mixed systems may get a medium payload fraction together with a medium flight time.*

### 9.233 Types of Transfer Trajectories (Fig. 9.37)

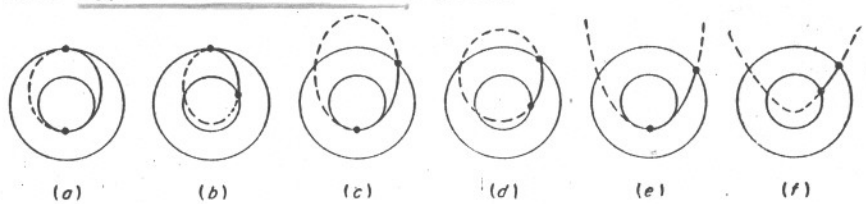


Fig. 9.37 Typical interplanetary trajectories: (*a*) Hohmann trajectory; (*b,c,d*) elliptic trajectories, (*e,f*) parabolic and hyperbolic trajectories.

There are a great number of possibilities for interplanetary transfer trajectories. As all involve nothing more than the simple two-body problem, they will not be discussed in any detail here. Some computational details are given in Sec. 25.2. The classical Hohmann trajectories [25] are ellipses, which are tangential both to the launch and to the arrival orbit. The energy for transfer from the launch orbit to the target orbit is a minimum for this trajectory,† but the transfer time is usually quite long. The time can be easily found from Kepler's third law. Three main groups

* T. N. Edelbaum, United Aircraft Corporation Research Laboratories, drew the author's attention to this possibility.

† This is true with some restrictions (see Sec. 25.2).

of transfer trajectories can be evolved from the Hohmann case: staying tangential to the larger orbit but intersecting the smaller one, or intersecting the larger orbit and being tangential to the smaller one, or intersecting both orbits. Whenever intersection occurs, there is an option for a shorter and a longer connection. Parabolic or hyperbolic transfer trajectories intersect the larger orbit, and may be, but do not have to be, tangential to the smaller one. Counting all subcases, there are 12 possibilities, and, since any trajectory going to a planet can be combined with itself or with any other, to perform the return flight, this results in 144 *mission profiles*, where the mission is to transfer from one planetary orbit to another, and then back to the first orbit (or to still another one). Besides the 144 profiles, the complete picture is even more complex, as intersecting trajectories are not uniquely defined. Therefore, the complete spectrum of mission possibilities is extremely wide, considering just the transfer phase only. At launch, or at arrival, orbital technique may or may not be used, and braking may be by rocket or aerodynamic (lift or ballistic) action:* A complete survey throughout all possibilities apparently has not been made so far, but excellent studies are presented in [29] and [7]. For a Hohmann mission profile which minimizes energy demand, e.g., for an earth-Mars-earth mission, the total flight time is long, good launch constellations are rare, and some minor disadvantages exist. Therefore, better mission profiles may, and in some cases do, exist, and must be found and developed.

### 9.234 Round Trips (Fig. 9.38)

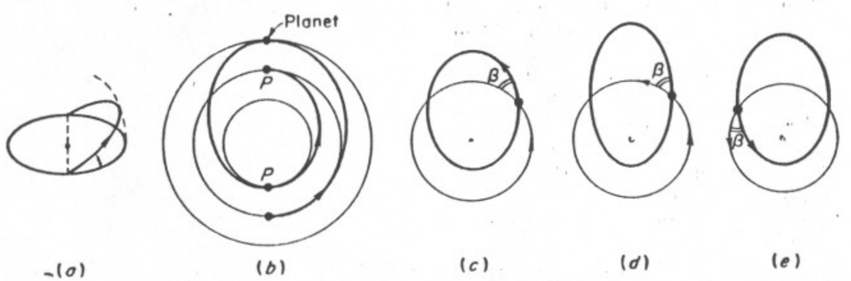


FIG. 9.38 Typical planetary round trips. (The circles indicate powered maneuvers.)

Round trips have the feature that no waiting time near a target is required in order to return to earth. There are several families of such trips:

*a*. Out of ecliptic
*b*. Hohmann [25]
*c*. Crocco [30]
*d*. Resonance or generalized Crocco
*e*. Straight-line trip

The out-of-ecliptic case *a* permits the probing of outer-space environment far beyond the ecliptic plane, and the probe vehicle meets the earth after six months for the transfer of the data.

The Hohmann round trip *b* is timed in such a way that the space vehicle returns to earth after 1½ years after touching the orbits of two planets. A minimum of three power maneuvers is required.

The Crocco round trip *c* lasts one year and is laid out in such a way that the space vehicle meets the earth at the same place in its elliptic flight path. $\beta$ cannot be zero; the major axis of the space-vehicle orbit is equal to the major axis of the earth orbit.

The resonance—or generalized Crocco—round trip *d* is characterized by the fact that $n$ vehicle periods are equal to $m$ earth periods, where $m$ and $n$ are integers. $\beta$ may but does not have to be zero.

* Aerodynamic braking to assist in establishing a satellite orbit around the target appears to be quite promising.



The straight-line solution to the round-trip problem *e* has a duration of less than 1½ year. $\beta$ may but does not have to be zero. At some time $T$ after launch, the following conditions must be met: (1) vehicle is at one of the apsides, (2) vehicle, planet, and sun are on one straight line. Another time $T$ later, the vehicle returns to the planet, e.g., after a total mission time of $2T$.

These trajectories have large potential significance for system testing, crew training, planetary exploration, etc. Some of the missions—e.g., Crocco's trip—may be of interest for early manned interplanetary missions.

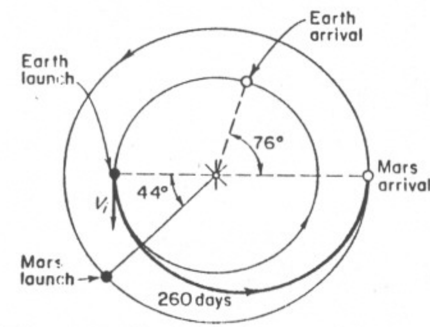


FIG. 9.39 Typical Hohmann mission profile to Mars.

### 9.235 Examples of Mars Trajectories

Only two examples will be looked at.

HOHMANN MISSION PROFILE (Fig. 9.39) [32]. Since $V_i = 32.83$ km/sec, and earth orbital speed is 29.80 km/sec, an excess of 3.03 km/sec is required. With earth escape speed being 11.1862 km/sec, a total injection speed from earth of

$$\sqrt{11.1862^2 + 3.03^2} = 11.5893 \text{ km/sec}$$

results. This assumes that, in one burning period, the total energy required is transferred to the vehicle.

A different flight technique is as follows: In a first propulsive period, parabolic speed is reached. The vehicle leaves earth, and, when it is far enough away, the excess of 3.03 km/sec is added to the vehicle speed. This latter method has been used in Hohmann's early publications. It is simple to understand and may have some guidance advantage, but the total speed requirement is

$$11.1862 + 3.03 = 14.2162 \text{ km/sec}$$

markedly higher than in the case of combined kicks. The general advantage of combining propulsive periods has been discovered and discussed by Oberth [33].

About 260 days after launch, the vehicle reaches the Mars orbit. Mars moves, during 260 days, through a central angle of 136°. This means that, at launch, Mars must lead earth by 44°, as seen from the sun; then, arrival of the vehicle at the Martian orbit coincides with Mars being at the arrival spot.

At arrival, the vehicle speed is 2.55 km/sec below the orbital speed of Mars moving around the sun. As escape speed from the Martian surface is 5.04 km/sec, the unbraked impact speed of the vehicle is $\sqrt{5.04^2 + 2.55^2} = 5.64$ km/sec.

Earth is now 76° ahead of Mars. To initiate a Hohmann return flight, terminating at earth, Mars must be ahead of earth by 76°. This can be accomplished by waiting near Mars for 449 days. The total mission duration is 970 days. The total velocity requirement is:

| | km/sec |
|---|---|
| Launch from earth | 11.59 |
| Gain due to earth's rotation | −0.3 |
| Gravity loss | 1.42 |
| Drag loss | 0.16 |
| Maneuvering reserve | 0.05 |
| Maneuvering during transfer | 0.4 |
| At target: maneuvering, otherwise aerodynamic braking | 0.1 |
| Launch from target | 5.64 |
| Rotational gain | −0.2 |
| Gravity loss | 0.3 |
| Drag loss | 0.15 |
| Maneuvering reserve | 0.05 |
| Maneuvering during transfer | 0.4 |
| At earth: maneuvering, otherwise aerodynamic braking | 0.05 |
| Total | 19.81 |

**Table 9.3. Space-probe Velocity Requirements**
(In kilometers per second)

| Maneuver | No recovery | | | | Plus return | | | |
|---|---|---|---|---|---|---|---|---|
| | Simple earth escape | Martian probe | Venusian probe | Mercury probe | Hohmann probe | Crocco probe | Simple solar system escape | Solar probe |
| Earth launch: | | | | | | | | |
| Ideal minimum launch | 11.2 | 11.589 | 11.441 | 13.504 | 11.441 | 13.948 | 16.658 | 17.0–31.8 |
| Rotational gain | 0.4 | 0.3 | 0.3 | 0.3 | 0.3 | 0.3 | 0.4 | 0.4 |
| *g* loss | 1.42 | 1.42 | 1.42 | 1.45 | 1.42 | 1.45 | 1.46 | 1.46 |
| Drag loss | 0.16 | 0.16 | 0.16 | 0.17 | 0.16 | 0.17 | 0.18 | 0.18 |
| Maneuvering | 0.05 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 |
| At target arrival: | | | | | | | | |
| Maneuvering | ..... | 0.2 | 0.2 | 0.2 | 0.6+ 1.8 + 2.3 | 0.6 | ...... | 0.2 |
| Earth-landing maneuver | ..... | ...... | ...... | ...... | 0.1 | 0.1 | | |
| Total | 12.45 | 13.15 | 13.0 | 15.1 | 17.60 | 16.05 | 18.0 | 18.55–33.35 |

**Table 9.4. Planetary-mission Velocity Requirements**
(In kilometers per second)

| Maneuver | No recovery | | | | Plus return | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | Mars satellite | Venus satellite | Mars soft landing | Venus soft landing | Mars satellite | Venus satellite | Mars soft landing | Venus soft landing | Planetoid in Mars orbit, soft landing |
| Earth launch: | | | | | | | | | |
| Ideal minimum launch | 11.589 | 11.441 | 11.589 | 11.441 | 11.589 | 11.441 | 11.589 | 11.441 | 11.589 |
| Rotational gain | 0.3 | 0.3 | 0.3 | 0.3 | 0.3 | 0.3 | 0.3 | 0.3 | 0.3 |
| *g* loss | 1.42 | 1.42 | 1.42 | 1.42 | 1.42 | 1.42 | 1.42 | 1.42 | 1.42 |
| Drag loss | 0.16 | 0.16 | 0.16 | 0.16 | 0.16 | 0.16 | 0.16 | 0.16 | 0.16 |
| Maneuvering | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| Maneuvering transfer | 0.4 | 0.5 | 0.4 | 0.5 | 0.4 | 0.5 | 0.4 | 0.05 | 0.4 |
| At target arrival: | | | | | | | | | |
| Ideal minimum | 2.01 | 3.104 | ...... | ...... | 2.01 | 3.104 | ...... | ...... | 2.55 |
| *g* loss | 0.1 | 0.15 | ...... | ...... | 0.1 | 0.15 | | | |
| Maneuvering | 0.05 | 0.05 | 0.01 | 0.01 | 0.05 | 0.05 | 0.01 | 0.01 | 0.05 |
| Target launch: | | | | | | | | | |
| Ideal minimum launch | ...... | ...... | ...... | ...... | 2.01 | 3.104 | 5.64 | 10.5 | 2.55 |
| Rotational gain | ...... | ...... | ...... | ...... | ...... | ...... | 0.2 | | |
| *g* loss | ...... | ...... | ...... | ...... | 0.05 | 0.1 | 0.3 | 1.5 | |
| Drag loss | ...... | ...... | ...... | ...... | ...... | ...... | 0.15 | 0.2 | |
| Maneuvering | ...... | ...... | ...... | ...... | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| Maneuvering transfer | ...... | ...... | ...... | ...... | 0.04 | 0.05 | 0.04 | 0.05 | 0.04 |
| Earth-landing maneuver | ...... | ...... | ...... | ...... | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 |
| Total | 15.50 | 16.60 | 13.40 | 13.35 | 18.05 | 20.40 | 19.80 | 26.15 | 18.95 |

Velocity requirements for a number of typical missions are listed in the same manner in Tables 9.3 and 9.4 [32]. For planetary-satellite missions, circular orbits in 1000 km altitude and rocket braking have been assumed. If aerodynamic braking is used instead, 1.5 km/sec for Mars and 2.6 km/sec for Venus can be saved.

In all cases, the flight might start from an earth orbit instead of from the earth's surface; and at the target, instead of landing directly, the exploring party might go through an orbit, and only part of the group would land, while the others would stay



in the orbit. There is a great flexibility here to pattern the detail procedure according to the established practice, available means, etc.

PARABOLIC FLIGHT (Fig. 9.40). The flight time to Mars is 70 days; during this time, Mars moves through 37°, meaning that, at launch, Mars has to lead earth by 72° − 37° = 35°.

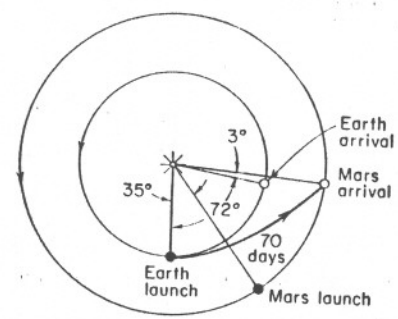


FIG. 9.40 Typical parabolic flight profile to Mars.

After 12 days stay time near Mars, conditions for the return flight are right; the mission duration is only 152 days. The required speed is shown in the following table.

| | km/sec |
|---|---|
| Launch from earth: $\sqrt{11.186^2 + 12.3^2}$ | 16.63 |
| Gain due to earth rotation | −0.3 |
| Gravity loss | 1.42 |
| Drag loss | 0.16 |
| Maneuvering | 0.05 |
| Transfer maneuvering | 0.4 |
| Arrival maneuvering | 0.1 |
| Launch from Mars: $\sqrt{5.04^2 + 20.1^2}$ | 20.72 |
| Rotational gain | −0.2 |
| Gravity loss | 0.3 |
| Drag loss | 0.15 |
| Maneuvering | 0.05 |
| Transfer maneuvering | 0.4 |
| Earth-return maneuvering | 0.05 |
| Total | 39.95 |

By increasing the speed requirement 102 per cent over that for the Hohmann case, mission duration has been reduced to about 15.7 per cent.

## 9.236 Considerations for Optimization

Many considerations are in order for optimizing certain features of an interplanetary trajectory:

ENERGY [27,50]. Hohmann transfer orbits are practical minimum-energy transfers. For noncoplanar orbits, a difficulty is introduced if a strict two-impulse maneuver is required. If, on the other hand, a mid-course maneuver is allowed to take care of the plane change, the reasoning as derived from the coplanar model remains valid. As such a mid-course maneuver is required anyway [34], no large additional difficulty is introduced.

MINIMUM LAUNCH WEIGHT. The weight of transmitters, power supply, life support, etc., is dependent on mission duration and certain mission characteristics, e.g., the distance between earth and target during the time the vehicle is at the target. Therefore, in spite of a higher energy requirement, a somewhat faster (or otherwise different from Hohmann) trajectory can result in lower lift-off weight. This fact was discussed as early as 1928 [26].

Simplicity. To some degree, simplicity has to do with mission time and the number of engine starts. The Crocco round trip [30], for example, appears to be quite favorable in these respects.

Guidance [35]. For certain injection conditions, the sensitivity in injection angle or in injection speed is lower than for other injection conditions. In some cases, this may be an important consideration.

Communication [36]. For purposes of communication, it is advantageous to have earth near by when the vehicle is near the target.

Timing. The aspect of timing enters into several considerations:

1. Flight duration
2. Waiting time
3. Mission duration
4. Optimum launch time
5. Penalty for nonoptimum launch time, e.g., necessity for emergency operations

### 9.237 High-precision Trajectories [28,37–41]

High-precision trajectory work is required before actual mission accomplishment. It is only with modern computers that large-scale experimental work in precision trajectory analysis becomes practical. Even so, there are difficulties which have to do with:

1. Computational accuracy (round-off vs. truncation errors)
2. Efficient computational procedures [41]
3. Lack of knowledge of astronomical constants [44]
4. Complexity of a correct astronomical model
5. Lack of knowledge of environmental data (e.g., interplanetary dust and/or gas, electric and magnetic fields)
6. Vehicle dependence on some forces (e.g., radiation pressure)

### 9.238 Guidance Aspects [34,35,42]

There are trajectories which are especially insensitive to injection errors in speed or in injection angle. This may not be of practical importance, because of the capability of including mid-course guidance and corrections.

For guidance intelligence, inertial systems governed by some means of long-time stability (e.g., star trackers, perhaps radio guidance from earth) look promising. Computations could be made via a communication link on earth, but the trend appears to be to perform them also by vehicle-borne computers.

Great improvement is to be expected in reliability, ruggedness, lifetime, weight reduction, power consumption, precision, etc.

### 9.239 Typical Results of Interplanetary Trajectory Analyses

Some typical results pertaining mainly to earth-Mars transfers will be mentioned here:

1. Velocity requirements for some typical space missions are presented in Tables 9.3 and 9.4 [31,32].

2. Ehricke has investigated several mission profiles for earth-Mars-earth flights. Figure 9.41 shows results for a promising profile, which is number 12 in Ehricke's nomenclature [29]. The symbols mean:



$\tau$ = stay time at Mars
$T$ = total mission duration
$\Delta t_{\mathrm{II}P_2,\mathrm{III}}$ = flight time from point II via $P_2$ to point III
$\Delta V_{P_1,\mathrm{I}}$ = speed requirement to go from $P_1$ (300-n-mi circular orbit) to point I (1000-n-mi circular orbit)
$\Delta V_{\mathrm{II}P_2,\mathrm{III}}$ = speed requirement from point II (1000-n-mi circular orbit) via $P_2$ to point III (optimum single-impulse-entry circular orbit)

$$\Delta V_{\mathrm{tot}} = \Delta V_{P_1,\mathrm{I}} + \Delta V_{\mathrm{II}P_2,\mathrm{III}}$$

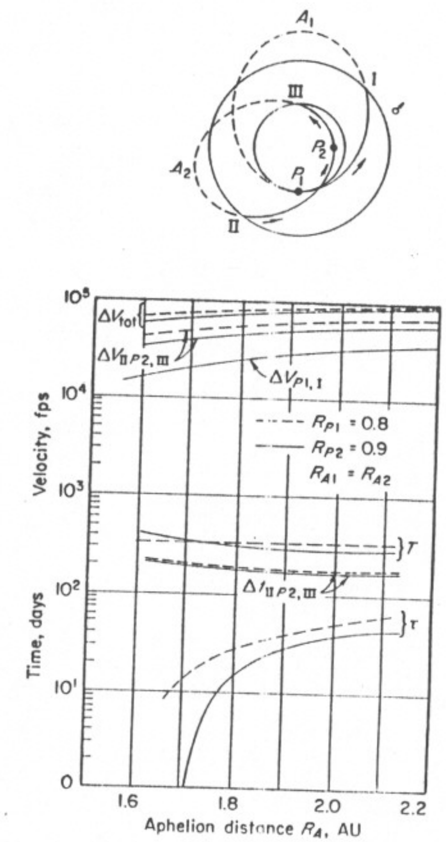


Fig. 9.41 Mission profile 12 (Ehricke) for earth-Mars mission [29].

For comparison, some results will be given:

| Parameter | Hohmann transfer | From Fig. 9.42 (for $R_P$ = 0.9) |
|---|---|---|
| $T$ | 980 | 320 days |
| $\tau$ | 460 | 30 days |
| $\Delta V_{\mathrm{tot}}$ | 31,000 | 76,000 fps |

This is a good example of a fast mission.

3. Injection speed (or hyperbolic excess speed = residual speed of vehicle with respect to earth, when the vehicle-earth distance is very large) is given in Fig. 9.42, which is taken from [45]. No mid-course inclination change was assumed. Dates are given in Julian days, where, for example, 2134.5 equals April 15, 1963.

4. Another way of showing the influence of launch speed (or hyperbolic excess speed) and launch date on transfer time is shown in Fig. 9.43. Again, no mid-course inclination change has been assumed. The figure has been taken from [8].

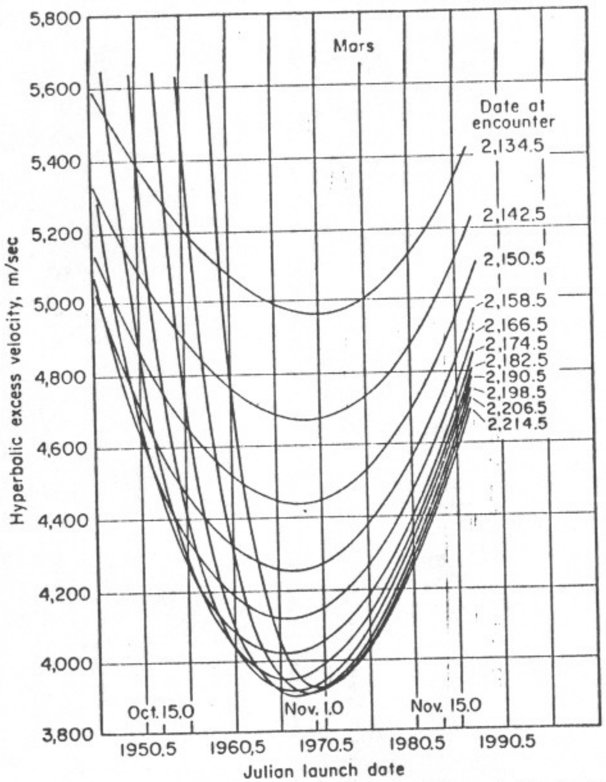


FIG. 9.42 Hyperbolic excess velocity requirements for Mars mission in 1962–1963 [45].

5. Figure 9.44 shows impulsive speed requirements to go from earth surface to Mars surface, assuming a two-impulse transfer and neglecting aerodynamic effects [46].

6. From [35], some injection-guidance sensitivity results are shown in Figs. 9.45 and 9.46. Only the two-body problem, sun and vehicle, has been considered. The symbols mean:

$\lambda$ = injection vehicle speed ÷ circular speed at one astronomical unit distance from sun
$\beta$ = injection angle, measured against local vertical
$r$ = radial distance from sun

$$A = \frac{1}{r_{\text{Mars}}}\frac{\partial b}{\partial \lambda} \quad \text{where } b = \text{miss distance due to } \lambda \text{ error}$$

$$B = \frac{1}{r_{\text{Mars}}}\frac{\partial b}{\partial \beta}$$



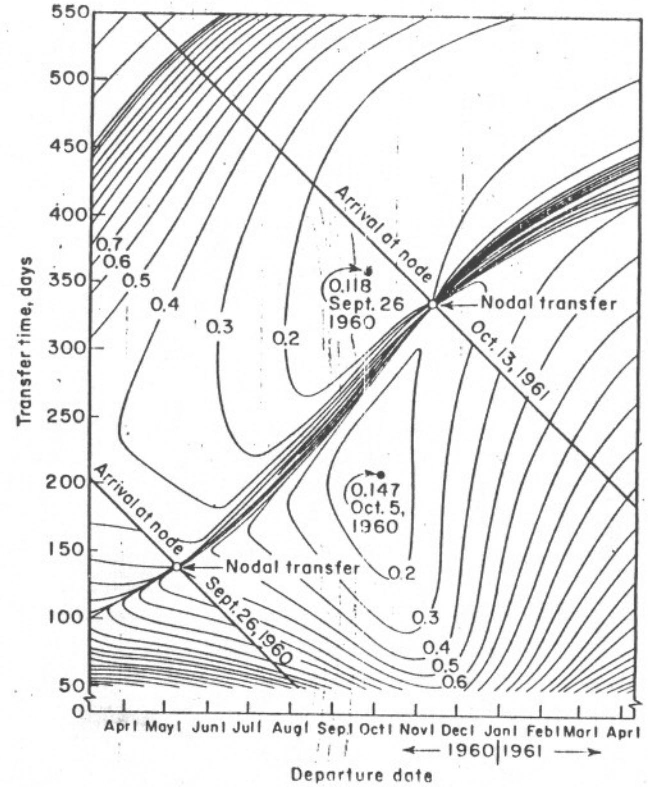


FIG. 9.43 Earth-to-Mars hyperbolic excess departure speeds normalized with respect to earth's mean orbital speed [8].

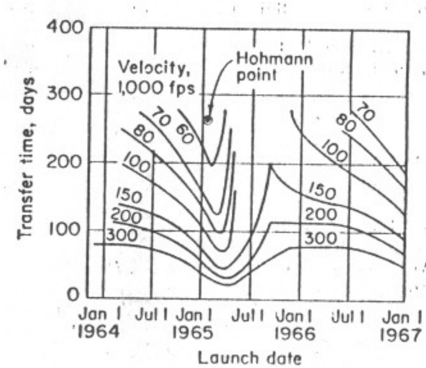


FIG. 9.44 Impulsive velocity requirement for Earth-to-Mars transfer and Martian landing [46].

7. In [47–49], general curves are contained, assisting in the computation of the characteristics of interplanetary transfer trajectories. These curves do not represent any optimization studies.

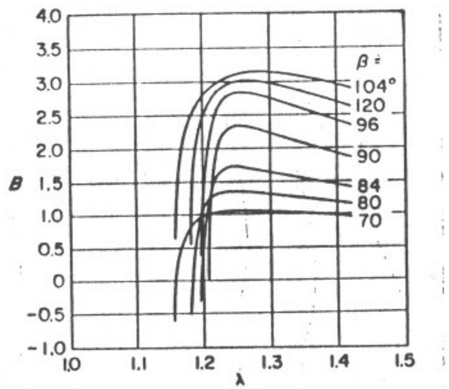


Fig. 9.45 Burnout angle miss coefficient $\delta b/\delta\beta$ versus $\lambda$ and $\beta$ for Mars ($r_i = 2.27 \times 10^{13}$ cm for Mars) [35].

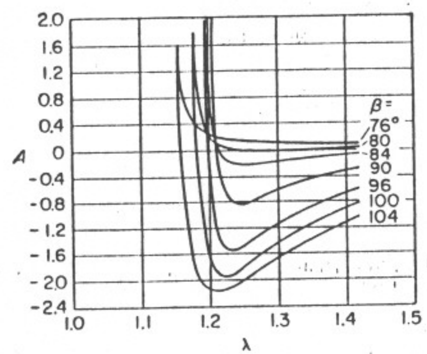


Fig. 9.46 A miss coefficient versus $\lambda$ and $\beta$ for Mars [35].

8. For mission planning, not only trajectories but also the over-all system, timing, cost, and many other aspects have to be optimized. As examples, see [43,46,53].

## 9.24 Bibliography


1. Goldstein, A. A., and C. Fröberg: A Collision Path from the Earth to the Moon in the Restricted Problem of Three Bodies, *Kgl. Fysiograf. Sällskap. Lund Forh.*, vol. 22, no. 14, 1952.
2. Clement, G. H.: The Moon Rocket, in Earth Satellites as Research Vehicles, *Franklin Inst. Penna. Monograph* 2, 1956.
3. Buchheim, R. W.: Artificial Satellites of the Moon, *Proc. 8th Intern. Astronaut. Federation Congr.*, Rome, 1956, Springer-Verlag, Vienna.
4. Egorov, V. A.: Certain Problems of Moon Flight Dynamics, in "The Russian Literature of Satellites," Part 1, International Physical Index, Inc., New York, 1958.
5. Vertregt, M.: Interplanetary Orbits, *J. Brit. Interplanet. Soc.*, vol. 16, no. 6, pp. 326–354, March–April, 1958.
6. Magness, T. A., J. B. McGuire, and O. K. Smith: Accuracy Requirements for Interplanetary Ballistic Trajectories, *Proc. 9th Intern. Astronaut. Federation Congr.*, Amsterdam, 1958, Springer-Verlag, Vienna.
7. Moeckel, W. E.: Interplanetary Trajectories with Excess Energy, *Proc. 9th Intern. Congr.*, Amsterdam, 1958, Springer-Verlag, Vienna,





8. Breakwell, J. V., R. W. Gillespie, and S. Ross: Researches in Interplanetary Transfer, *ARS Preprint* 954-59, presented at Fourteenth Annual Meeting, Washington, D.C., November, 1959.
9. Mickelwaite, A. B., E. H. Tompkins, Jr., and R. A. Parks: Three Dimensional Interplanetary Trajectories, *IRE Trans. Mil. Electronics*, vol. MIL-3, no. 4, pp. 149–159, October, 1959.
10. Battin, R. H.: The Determination of Round-trip Planetary Reconnaissance Trajectories, *J. Aero/Space Sci.*, vol. 26, no. 9, pp. 545–567, September, 1959.
11. Buchheim, R. W.: Motion of a Small Body in Earth-Moon Space, *Rand Corp. Research Mem.* RM-1726, June 4, 1956.
12. Michael, Wm. H., Jr., and R. H. Tolson: Effect of Eccentricity of the Lunar Orbit, Oblateness of the Earth, and Solar Gravitational Field on Lunar Trajectories, *NASA Tech. Note* D-277, June, 1960.
13. Ehricke, K. A., and G. Gamow: A Rocket around the Moon, *Sci. American*, vol. 196, no. 6, pp. 47–53, June, 1957.
14. Mickelwaite, A. B.: Lunar Trajectories, *J. ARS*, vol. 29, no. 12, pp. 905–914, December, 1959.
15. Liu, A.: Normal Lunar Impact Analysis, *J. Astronaut. Sci.*, vol. 7, no. 2, pp. 35–39, summer, 1960.
16. Goldbaum, G. C., and R. J. Gunkel: Comparison of Two-dimensional and Three-dimensional Analyses of Earth-Moon Flight, *Proc.* AAS Western Regional Meeting, Palo Alto, Calif., August, 1959.
17. Hoelker, R. F., and N. J. Braud: Lunar Probe Trajectories: Characteristics and Tolerances for Coplanar Flights, *ABMA Rept.* DA-TR-7-59, July 18, 1959.
18. Lieske, H. A.: Lunar Instrument Carrier: Trajectory Studies, *Rand Corp. Research Mem.* RM-1728, June 4, 1956.
19. Lieske, H. A., Lunar Trajectory Studies, *Rand Corp. Paper* P-1293, Feb. 26, 1958.
20. Michael, Wm. H., Jr., and R. H. Tolson: Three-dimensional Lunar Mission Studies, *NASA Mem.* 6-29-59L, June, 1959.
21. Reference [14].
22. Lieske, H. A.: Circumlunar Trajectory Studies, *Rand Corp. Paper* P-1441, June 25, 1958.
23. Reference [4].
24. Sedov, L. I.: Orbits of Cosmic Rockets to the Moon, *J. ARS*, vol. 30, no. 1, pp. 14–21, January, 1960.
25. Hohmann, W.: "Die Erreichbarkeit der Himmelskoerper," R. Oldenbourg-Verlag, Munich, 1925.
26. von Pirquet, Guido: "Die Rakete," Germany, June, 1925.
27. Lawden D. F.: Interplanetary Rocket Trajectories, in F. Ordway (ed.), "Advances in Space Science," vol. 1, Academic Press, Inc., New York, 1959.
28. High Precision Orbit Determination, *Aeronutronic Co. Rept.* U-583, 1959.
29. Ehricke, K. A.: Interplanetary Operations, in H. S. Seifert (ed.), "Space Technology," John Wiley & Sons, Inc., New York, 1959.
30. Crocco, G. A.: One Year Exploration Trip Earth-Mars-Venus-Earth, *Proc. 7th Intern. Astronaut. Federation Congr.*, Rome, 1956.
31. Ruppe, H. O.: "Introduction to Astronautics," Addison-Wesley Publishing Company, Reading, Mass., 1962.
32. Ruppe, H. O.: Minimum Energy Requirements for Space Travel, *Proc. 10th Intern. Astronaut. Federation Cong.*, London, 1959, Springer-Verlag, Vienna, 1960.
33. Oberth, H.: Grundprobleme der Raumschiffahrt, in W. Ley (ed.), "Die Moeglichkeit der Weltraumfahrt," Hachmeister und Thal, Leipzig, Germany, 1928.
34. Unger, J. H. W.: On the Mid-course Navigation for Manned Interplanetary Space Flight, *ABMA Rept.* DSP-TR-2-58, 1958.
35. Magness, T. A., et al.: Accuracy Requirements for Interplanetary Ballistic Trajectories, *Proc. 9th Intern. Astronaut. Federation Congr.*, Amsterdam, 1958, Springer-Verlag, Vienna, 1959.
36. Unger, J. H. W.: Radio Links for Space Flight, *ABMA Rept.* DSP-TN-4-59, Huntsville, Ala., 1959.
37. Baker, R. M. L., Jr., and Maud W. Makemson: "Introduction to Astrodynamics," Academic Press, Inc., New York, 1960,
38. Herrick, Samuel: "Astrodynamics and Rocket Navigation," D. Van Nostrand Company, Inc., Princeton, N.J., 1962.
39. Herrick, Samuel, and R. M. L. Baker, Jr.: Recent Advances in Astrodynamics, *Jet Propulsion*, vol. 28, October, 1958.
40. Herrick, Samuel, and R. M. L. Baker, Jr.: State of the Art: Astrodynamics, *Astronautics*, November, 1959.

41. Baker, R. M. L., Jr.; G. B. Westrom, C. G. Hilton, J. Arsenault, R. Gersten, and E. Browne: Efficient Precision Orbit Computation Techniques, *ARS Preprint* 869-59, 1959.
42. Ehricke, K. A.: Error Analysis of Single and Two-force Field Spacecraft Orbits, *J. Franklin Inst. Monograph* 6, December, 1958.
43. von Braun, Wernher: "Das Mars Projekt," Umschau-Verlag, Frankfurt on Main, Germany, 1952.
44. Herrick, Samuel, R. M. L. Baker, Jr., and C. G. Hilton: Gravitational and Related Constants for Accurate Space Navigation, *Proc. 8th Intern. Astronaut. Federation Congr.*, Barcelona, 1957, pp. 147-235, Springer-Verlag, Vienna, 1958; *Univ. Calif. Los Angeles Astron. Papers*, vol. 1, pp. 297-338; *ARS Preprint* 497-57, 1957.
45. *Jet Propulsion Lab. Research Summary* 36-4, vol. 1, Aug. 15, 1960.
46. Hunter, M. W., et al.: The Potential of Nuclear Space-transport Systems, *Douglas Aircraft Co. Rept.* SM-37427, March, 1960.
47. Swanson, C. D., and J. W. Russell: Departure Velocities for Interplanetary Probes, *ABMA Rept.* DSP-TN-2-59, 1959.
48. Russell, J. W.: Transit Times for Interplanetary Probes, *ABMA Rept.* DSP-TN-7-59, 1959.
49. Perry, W. R.: Departure Velocities for Interplanetary Probes with Inclined Orbits, *ABMA Rept.* DSP-TN-11-59, 1959.
50. Burns, R. E.: The Optimization of Interplanetary Trajectories, *ABMA Rept.* DSP-TN-20-59, 1959.
51. Szebehely, V. G.: Reliability of Interplanetary and Lunar Orbit Computation, *ARS Preprint* 874-59, 1959.
52. Ley, W., and Werner von Braun: "The Exploration of Mars," The Viking Press, Inc., New York, 1956.
53. Dugan, J. F., Jr.: Analysis of Trajectory Parameters for Probe and Roundtrip Missions to Mars, *NASA Tech. Note* TN D-281, June, 1960.

# *Chapter* 10

# ATMOSPHERIC ENTRY

The atmospheric flattening may be treated as a second-order effect by the introduction of the perigee latitude through the ratio $\rho_\phi/\rho_0$. Alternatively, if the ephemeris is integrated on a revolution-by-revolution basis, the perigee height above the oblate spheroid may be calculated and used to compute $\rho_\phi/\rho_0$.

### 8.35 Three-body Perturbations

The perturbations occasioned by a third body, e.g., the sun or moon, on a satellite orbit are customarily [28] treated by defining the disturbing function $R$, which is analogous to the potential function $\Phi$. For the lunar perturbations on a close earth satellite,

$$R \equiv k_e^2 \frac{m_☾}{r_☾}\left(\frac{r_☾}{|\mathbf{r}_☾ - \mathbf{r}|} - \frac{r}{r_☾}S\right)$$

$$= k_e^2 \frac{m_☾}{r_☾}\left[\left(1 - 2\frac{r}{r_☾}S + \frac{r^2}{r_☾^2}\right)^{-1/2} - \frac{r}{r_☾}S\right] \tag{8.150}$$

$$\text{where } S \equiv \frac{\mathbf{r}\cdot\mathbf{r}_☾}{rr_☾} = \mathbf{U}\cdot\mathbf{U}_☾ = \cos(\mathbf{r},\mathbf{r}_☾) \tag{8.151}$$

$m_☾$ = mass of moon in terms of earth masses
$\mathbf{r}_☾$ = geocentric radius vector of moon
$\mathbf{r}$ = geocentric radius vector of satellite
$k_e$ = geocentric gravitational constant
$\mathbf{U}$ and $\mathbf{U}_☾$ = unit vectors along $\mathbf{r}$ and $\mathbf{r}_☾$, respectively

When a close-earth satellite of negligible mass is considered, the quantity $r/r_☾$ is small and the perturbative function $R$ can be expressed in terms of a power series of $r/r_☾$. Thus,

$$R = k_e^2 \frac{m_☾}{r_☾}\left[1 + \left(\frac{r}{r_☾}\right)^2 (3/2 S^2 - 1/2) + \left(\frac{r}{r_☾}\right)^3 (5/2 S^3 - 3/2 S) + \cdots\right] \tag{8.152}$$

The expansion of $S$ follows from its definition [Eq. (8.151)] and the components

$$\begin{aligned} U_x &= \cos u \cos \Omega - \sin u \sin \Omega \cos i \\ U_y &= \cos u \sin \Omega + \sin u \cos \Omega \cos i \\ U_z &= \sin u \sin i \end{aligned} \tag{8.153}$$

There are similar expressions for the components of $\mathbf{U}_☾$.

Thus,

$$\begin{aligned} S = {} & 1/4(1 - \cos i)(1 - \cos i_☾)\cos(\Omega - \Omega_☾ - u + u_☾) \\ & + 1/4(1 + \cos i)(1 + \cos i_☾)\cos(\Omega - \Omega_☾ + u - u_☾) \\ & + 1/4(1 + \cos i)(1 - \cos i_☾)\cos(\Omega - \Omega_☾ + u + u_☾) \\ & + 1/4(1 - \cos i)(1 + \cos i_☾)\cos(\Omega - \Omega_☾ - u - u_☾) \\ & + 1/2 \sin i \sin i_☾[\cos(u - u_☾) - \cos(u + u_☾)] \end{aligned} \tag{8.154}$$

As was done for the geopotential and atmospheric-drag perturbations, the quantities

$$\dot{r} = \frac{\partial R}{\partial r}$$

$$r^2\dot{l} = r^2\dot{v} = \frac{\partial R}{\partial v} = \frac{\partial R}{\partial l}$$

$$r^2\dot{b} = \frac{\partial R}{\partial b}$$



can now be evaluated and substituted into the expressions for the variations in the elements [Eqs. (8.102) through (8.110)]. It is found that the semimajor axis, eccentricity, and inclination have only periodic variations. The secular variations in the longitude of the ascending node, argument of perigee, and mean anomaly are:

*Longitude of Ascending Node*

$$\frac{d\Omega}{dt} = -\frac{3}{4}\frac{n_☾^2}{n} m_☾ \frac{\cos i}{\sqrt{1 - e^2}}(1 + 3/2 e^2)(1 - 3/2 \sin^2 i_☾)$$

*Argument of Perigee*

$$\frac{d\omega}{dt} = \frac{3}{4}\frac{n_☾^2}{n} m_☾ \frac{1}{\sqrt{1 - e^2}}(2 - 5/2 \sin^2 i + 1/2 e^2)(1 - 3/2 \sin^2 i_☾)$$

*Mean Anomaly*

$$\frac{dM}{dt} = -\frac{1}{4}\frac{n_☾^2}{n} m_☾(7 + 3e^2)(1 - 3/2 \sin^2 i)(1 - 3/2 \sin^2 i_☾)$$

It should be pointed out that, for close-earth satellites, lunar and solar perturbations are of several magnitudes smaller than perturbations due to the asphericity of the earth. For highly eccentric geocentric satellite orbits the periodic solar and lunar perturbations have been found to be quite influential.

### 8.36 Solar Radiation Pressure Perturbations

Solar radiation pressure can produce significant changes in the orbits of satellites having large area-to-mass ratios. The analytical expressions for the first-order perturbations are easily established.

By neglecting the solar parallax, the equatorial components of the force $\mathbf{F}_\odot$ acting on a satellite due to the radiation pressure of the sun can be expressed as:

$$\begin{aligned} F_{x\odot} &= -AP_\odot\gamma\nu \cos l_\odot \\ F_{y\odot} &= -AP_\odot\gamma\nu \cos \epsilon \sin l_\odot \\ F_{z\odot} &= -AP_\odot\gamma\nu \sin \epsilon \sin l_\odot \end{aligned}$$

where $A$ = effective cross-sectional area of satellite
$P_\odot$ = solar radiation pressure in vicinity of earth, $4.5 \times 10^{-5}$ dynes/cm²
$\gamma$ = factor depending on satellite's reflecting characteristics
$\nu$ = eclipse factor
$\epsilon$ = obliquity of ecliptic, 23°.4444
$l_\odot$ = true longitude of sun

The true longitude of the sun can be found at any time $t$ from

$$l_\odot = (L_\odot)_0 + n(t - t_0) + 2e \sin M + 5/4 e^2 \sin 2M$$

where $M = (L_\odot)_0 + n(t - t_0) - \Pi$

$(L_\odot)_0$ = sun's mean longitude at some epoch $t_0$
$n$ = 0°.98565/day
$e$ = 0.016725
$\Pi$ = 282°.26971

The radial, transverse, and normal components of the perturbing acceleration are

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