Chapter 10

INTERPLANETARY TRAJECTORIES

1. FUNDAMENTAL DATA

Chapter 1, Sections 7.1 and 7.2 and the tables in the appendices describe the scene of operations in travel between the planets of the Solar System.

Of the planets, Mars and Venus are the most easily reached, according to energy requirements. Mars presents a much simpler landing problem than Venus since not only is its mass less than one seventh that of Venus, resulting in a much weaker gravitational field to overcome, but surface conditions are not nearly so rugged.

Voyages to the other planets, except Mercury, are orders of magnitude more difficult to accomplish.

A number of terms frequently used in describing interplanetary configurations are illustrated in Figure 10-1 in which E is the Earth and S is the Sun. The letters V and J refer respectively to an *inferior* planet (one that orbits inside the Earth's orbit) and to a superior planet (one that orbits outside the Earth's orbit).

A superior planet on the observer's meridian at apparent midnight is said to be in *opposition* (configuration SEJ_1).

A planet whose direction is the same as that of the Sun is said to be in *conjunction* (configurations EV_1S, ESV_3, ESJ_3); an inferior planet can be in *superior conjunction* (configuration ESV_3) or in *inferior conjunction* (configuration EV_1S).

The angle the geocentric radius vector of the planet makes with the Sun's geocentric radius vector is called the planet's *elongation* (for example, configurations SEV_2 or SEJ_4). It is obvious that an inferior planet has zero elongation when it is in conjunction and maximum elongation (less than 90°) when its geocentric radius vector is tangential to its orbit (configuration SEV_2). The elongation of a superior planet can vary from zero (configuration SEJ_3) to 180° (configuration SEJ_1). When its elongation is 90° it is said to be in *quadrature* (configurations SEJ_2 and SEJ_5). These

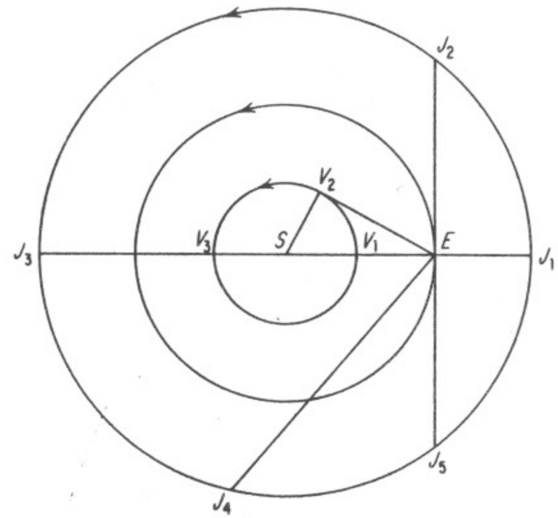

Figure 10-1

quadratures are distinguished by adding *eastern* or *western*; in the diagram the north pole of the ecliptic is directed out of the plane of the paper so that J_5 and J_2 are in eastern and western quadratures respectively.

The diagram has been drawn for coplanar, circular orbits; the actual planetary orbits are ellipses of low eccentricity in planes inclined only a few degrees to each other so that the terms defined above are obviously still applicable.

Another useful concept, the *synodic period* S of a planet, was defined in Section 8.3.7 and may be taken in the present context to be the time between successive similar geometrical configurations of planet, Earth and Sun. If T_P and T_E are the sidereal periods of revolution of planet and Earth about the Sun respectively, then

$$\frac{1}{S} = \frac{1}{T_P} - \frac{1}{T_E},$$

for an inferior planet, while

$$\frac{1}{S} = \frac{1}{T_E} - \frac{1}{T_P},$$

for a superior planet.

These relationships are derived for circular, coplanar orbits and therefore apply only approximately to the Earth and any other planet in the

Solar System. The mean synodic periods for the planets are given in Appendix III.

2. THE SOLAR SYSTEM AS A CENTRAL FORCE FIELD

The dominant gravitational field of the Sun, due to its mass being over one thousand times that of the most massive planet, means that in space a few million kilometers away from any planet, a vehicle moves in a gravitational field closely resembling that of a simple central force field, namely, the Sun's, in which the intensity falls off as the square of the distance from the Sun. The formulas and conclusions of Chapter 4 and those sections in Chapter 8 devoted to transfer in a single force field may therefore be used with a high degree of confidence in the study of interplanetary transfer operations.

Near the planets, at distances from them given approximately by the sphere of influence argument, there exist regions where the force fields of both planet and Sun are present in comparable intensities, and for precision studies, the special perturbation methods of Chapter 6 must be used, though in many feasibility studies, the approximate methods sketched in Chapter 8 can be applied with confidence. That this is so may be seen by studying Tables 10.1 and 10.2 and also Figure 8-13.

In Table 10.1, values of the radii r_A of the planetary spheres of influence are given in millions of kilometers, in astronomical units, and in fractions of the planets' mean distances from the Sun, the figures being computed by using formula (5.70), namely,

$$r_A = \left(\frac{m}{M}\right)^{2/5} r_P,$$

where m and M are the masses of planet and Sun respectively, and r_P is the planet's semimajor axis. The consequence of the fall-off in intensity of the Sun's gravitational field with distance from the Sun is evident on comparing the sizes of the spheres of influence of Earth and Pluto (of comparable mass). The latter sphere is over thirty times as large as the former and, in fact, is two thirds as extensive as Jupiter's though the mass of Jupiter is about three hundred times that of Pluto.

The more flexible sphere-of-influence argument of Section 5.12 giving an outer and inner boundary led to the graph in Figure 8-13, where a shell about a planet could be defined for any accepted degree of perturbation, showing the range (namely, the thickness of the shell) over which special or general perturbation methods had to be used. Table 10.2 gives, for two values of $|\varepsilon|$, the boundaries of the shells about the planets in which such methods would be called for if perturbation ratios greater than $|\varepsilon|$ were not acceptable.

TABLE 10.1

PLANET	RADIUS (r_A) OF SPHERE OF INFLUENCE		
	MILLIONS OF KILOMETERS	FRACTION OF PLANETARY ORBIT'S SEMIMAJOR AXIS	A.U.
Mercury	0.112	0.00193	0.000747
Venus	0.615	0.00569	0.00411
Earth	0.925	0.00619	0.00619
Mars	0.579	0.00254	0.00387
Jupiter	48.1	0.0619	0.322
Saturn	54.6	0.0382	0.365
Uranus	52.0	0.0181	0.348
Neptune	86.9	0.0193	0.581
Pluto	34.0	0.00574	0.227

The figures in Tables 10.1 and 10.2 should be taken as merely giving the orders of magnitude of the spheres of influence sizes. It should be remembered too that the "spheres" are only approximately spherical. Nevertheless, the information embodied in the two tables and in Figure 8-13 does show how the planets in the Solar System can be divided into two classes where feasibility studies are concerned. In the first class are Mercury, Venus, Earth, Mars, and Pluto (also the asteroids); in this class the use of the formulas of a central force field—according to the methods of Chapter 8—in feasibility studies should be expected to yield fairly accurate data for interplanetary missions even when perturbation shells are neglected. For precision studies, of course, special perturbation methods within the shells must be used.

In the second class are the giant planets Jupiter, Saturn, Uranus, and Neptune. Feasibility studies of missions involving these planets, especially the first two, that neglect the perturbation shells about these bodies, will provide, at best, orders of magnitude data about transfer times and energy budgets and cannot give real information about the actual orbits of vehicles once they have approached to within the outer shell boundary. Precision studies, of course, can always be carried out for these bodies.

3. MINIMUM ENERGY INTERPLANETARY TRANSFER ORBITS

By assuming the planetary orbits to be coplanar and circular, the formulas of Chapter 8 may be used to give information about energy requirements and transfer and waiting times that are of the right order of magnitude;

TABLE 10.2

PLANET	$\lvert\varepsilon\rvert = 0.1$: RADIUS OF INNER BOUNDARY OF SHELL (millions of kilometers)	$\lvert\varepsilon\rvert = 0.1$: RADIUS OF OUTER BOUNDARY OF SHELL (millions of kilometers)	$\lvert\varepsilon\rvert = 0.01$: RADIUS OF INNER BOUNDARY OF SHELL (millions of kilometers)	$\lvert\varepsilon\rvert = 0.01$: RADIUS OF OUTER BOUNDARY OF SHELL (millions of kilometers)
Mercury	no shell exists with both $\lvert\varepsilon_P\rvert$ and $\lvert\varepsilon_S\rvert$ as large as 0.1		0.053	0.243
Venus			0.24	2.04
Earth			0.40	2.66
Mars			0.27	1.27
Jupiter	29.0	70.8	13.23	217.9
Saturn	35.4	80.5	15.7	243
Uranus	38.6	64.3	17.2	195
Neptune	64.4	107.9	30.6	342
Pluto	as for first four planets		13.5	112

more precise studies, acknowledging that in reality the orbits of the planets are ellipses of low eccentricity and low inclination to each other, do not change the picture by an order of magnitude.

A mission from the surface of a planet to the surface of another planet can be broken up into three phases: (1) ascent from the surface of the departure planet to the boundary of its sphere of influence; (2) transfer in heliocentric space to the boundary of the destination planet's sphere of influence; (3) descent to the surface of the destination planet.

Phase (1) may involve entry into a parking orbit about the departure planet as an intermediate step for checkout purposes before an impulse puts the vehicle into the prescribed planetocentric hyperbolic escape orbit giving the required hyperbolic excess velocity at the point where it leaves the sphere of influence of the departure planet. For high-thrust vehicles in terrestrial planet missions (Mercury, Venus, Earth and Mars), phase (1) will last a week at most.

Phase (2), apart from possible midcourse corrections, will consist of powerless flight under the dominant action of the Sun's gravitational field and will be described very closely by parts of ellipses (allowing for at least one midcourse correction). This phase accounts for most of the time spent in transit from one planet to another.

Phase (3) is the reverse operation of phase (1), involving a capture operation transforming the planetocentric hyperbolic encounter orbit into a parking orbit about the planet before the final descent to surface takes place. Phase (3) will last no longer than phase one in terrestrial planet missions in general.

A return mission requires the same three phases and is separated in all foreseeable practical cases from the outward mission by a waiting time whose length is specified by the orbital elements of both planets and the performance of the available vehicle. It will be remembered that this waiting time is the period that has to be spent at the destination planet before the planets and the Sun are suitably placed for the return trip to begin.

Total mission time for a return trip (there and back) will therefore be made up chiefly of two phase (2) transfer times (not necessarily equal) and a waiting time.

It was seen in Chapter 8 that the most economical transfer orbits between two particles in circular orbits in a single central force field consisted of cotangential ellipses (omitting the time consuming bi-elliptic transfer). A transfer from one planet to another and back again under the consideration that a minimum of fuel is to be expended will lead to a total mission time easily obtained by the formulas of Chapter 8. The first person to draw attention to such minimum energy orbits and compute mission times for them was Walter Hohmann (Reference 10.1). Taking the planetary orbits to be circular and coplanar, the Earth to be the departure body in all cases, and neglecting times spent in phase (1) and phase (3) maneuvers, the use of formulas (8.16) and (8.24) gives the transfer time t_T to be

$$t_T = \pi\left[\frac{(a_E + a_P)^3}{8\mu}\right]^{1/2},$$

where a_E, a_P are the semimajor axes of the orbits of Earth and planet respectively, while μ is the product of the Sun's mass and the gravitational constant.

Now the Earth's period of revolution T_E is given by

$$T_E = 2\pi\sqrt{\frac{a_E^3}{\mu'}},$$

where $\mu' = G(M + m_E) \sim GM$, since $m_E/M \sim 1/330{,}000$.

Hence

$$t_T = \sqrt{(1 + a)^3}/5.656 \text{ yr}, \tag{10.1}$$

the planetary semimajor axis a being now expressed in astronomical units.

The minimum waiting time t_W is found by using formulas (8.73) through (8.77) while the total mission time T equals $(2t_T + t_W)$. The eccentricity of the cotangential transfer orbit comes from (8.23), namely,

$$e = \frac{a_2 - a_1}{a_2 + a_1}.$$

For a superior planet,

$$e = \frac{a - 1}{a + 1}, \tag{10.2}$$

while for an inferior planet,

$$e = \frac{1 - a}{1 + a}, \tag{10.3}$$

where, as in Eq. (10.1), the planetary semimajor axis a is in astronomical units.

In Table 10.3, the transfer times, waiting times, and total mission times for round trips to all planets are given, using minimum energy cotangential ellipses. In addition the eccentricities of these transfer orbits are given.

On examining the table, several statements may be made immediately.

Manned voyages to the planets beyond Mars are rendered out of the question by the long mission times if orbits close to minimum energy have to be used. Even if unmanned probes were used, reliability of the components over such long intervals of time could not be guaranteed even if

TABLE 10.3

PLANET	TRANSFER TIME t_T (yr)	MINIMUM WAITING TIME t_W (yr)	TOTAL MISSION TIME $T = 2t_T + t_W$ (yr)	ECCENTRICITY OF TRANSFER ORBIT
Mercury	0.289	0.183	0.76	0.44
Venus	0.400	1.278	2.08	0.16
Mars	0.709	1.242	2.66	0.21
Jupiter	2.731	0.588	6.05	0.68
Saturn	6.048	0.936	13.03	0.81
Uranus	16.04	0.932	33.01	0.91
Neptune	30.62	0.766	62.01	0.94
Pluto	45.47	0.061	91.00	0.95

information collected by the probes' instruments could be transmitted over distances of many millions of kilometers.

The mission times for Venusian, Martian, and Mercurian round trips are not impossible to contemplate for manned voyages, the interesting fact emerging that the Mercurian mission lasts only about a third and a quarter as long respectively as the Venusian and Martian missions. The important factor in these cases is the long waiting time at Mars and Venus before the return journey can be begun. It suggests that the decrease of such long waiting times by the use of different transfer orbits compatible with available energies should have a high priority in the list of factors involved in planning such voyages.

It is also illuminating to consider the actual velocity requirements for such transfer orbits. Let us calculate the velocity increments necessary to place the vehicle into particular heliocentric orbits. The first increment places the vehicle in a parking orbit about the Earth. This orbit, taken to be circular, is assumed to be at a height of 460 km so that a circular velocity of 7.635 km/sec is required. To achieve parabolic or escape velocity from the Earth's field a further increment in velocity of $(\sqrt{2} - 1) \times 7.635$ km/sec must be added. We suppose that this is added tangentially. In theory, this would enable the vehicle to enter the heliocentric gravitational field just beyond the Earth's sphere of influence with almost zero geocentric velocity (zero hyperbolic excess) and a heliocentric velocity equal to the Earth's heliocentric velocity. In order to carry out any interplanetary mission, the actual escape should be made hyperbolically.

Expression (8.79) gives the hyperbolic excess V with which the vehicle leaves the Earth's sphere of influence, radius ρ, when it receives, at a geocentric distance ρ_0, an incremental velocity v_e in addition to escape velocity V_e, where

$$V_e = \sqrt{2}V_c = \sqrt{2Gm/\rho_0}. \tag{10.4}$$

Re-writing (8.79) we have

$$V = \left[\frac{2Gm}{\rho} + v_e(2V_e + v_e)\right]^{1/2}. \tag{10.5}$$

In Figure 10-2, for the parking orbit about the Earth of height 460 km and a radius of the outer sphere of influence ρ taken to be 2.66×10^6 km

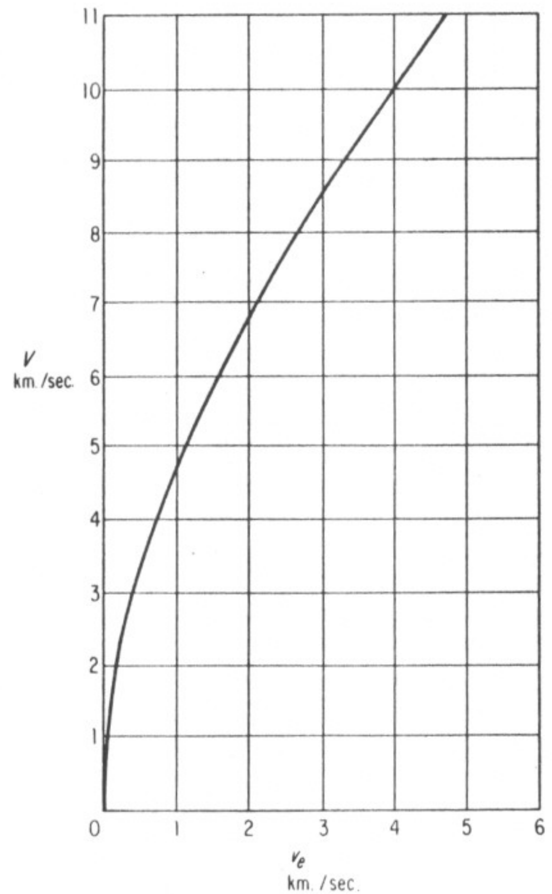

Figure 10-2

(such that $|\varepsilon_P| \leq 0.01$), the hyperbolic excess V is graphed against the excess v_e to escape velocity with which the vehicle leaves the parking orbit.

For a cotangential heliocentric transfer orbit the vehicle will leave the Earth's sphere of influence either in the direction in which the Earth is travelling or in the opposite direction. If the Earth's orbital velocity is $V_\oplus$, the first case gives the vehicle a heliocentric orbital velocity of

$$V_V = V_\oplus + V; \tag{10.6}$$

in the second case, the vehicle's heliocentric orbital velocity is

$$V_V = V_\oplus - V. \tag{10.7}$$

The first case places the vehicle in a transfer orbit whose perihelion distance is 1 A.U.; the second case gives a transfer orbit of aphelion 1 A.U.

Equations (8.21) and (8.22) may be used to calculate the required velocity increment V, inserting the Earth's orbital velocity of 29.8 km/sec in place of $\sqrt{\mu/a_1}$ when the transfer is to a superior planet and $\sqrt{\mu/a_2}$ when an inferior planet is the planet of destination. The second column in Table 10.4 gives the velocity increments required for cotangential transfer to the various planetary orbits.

The use of Figure 10-2 then allows the velocity v_e in excess of escape velocity at the parking orbit, corresponding to the required hyperbolic excess V to be found. Values of v_e appear in column three of Table 10.4. Also in the table are given the hyperbolic excess V and the velocity excess v_e to achieve *heliocentric* parabolic velocity at the Earth's distance from the Sun, that is to achieve escape from the Solar System.

TABLE 10.4

PLANET	VELOCITY V(KM/SEC) REQUIRED IN ADDITION TO EARTH'S CIRCULAR VELOCITY $V_\oplus$	VELOCITY v_e (KM/SEC) REQUIRED BEYOND ESCAPE VELOCITY
Mercury	7.537	2.362
Venus	2.497	0.290
Mars	2.947	0.396
Jupiter	8.797	3.139
Saturn	10.30	4.115
Uranus	11.29	4.831
Neptune	11.66	5.075
Pluto	11.82	5.197
Interstellar space	12.34	5.608

Thus to reach any of the planets, the vehicle must be capable of achieving a velocity increment of v_e km/sec in excess of the escape velocity (10.80 km/sec) from the parking orbit 460 km above the Earth's surface. Venus and Mars are well within the range of modern rockets; the asteroid belt and Mercury should be capable of being reached by the vehicles of the next decade.

It should be pointed out that no allowance has been made in the above calculations for transformation of the resulting hyperbolic encounter with the planet of destination to an elliptic or circular capture orbit about it. Such a maneuver will require a considerable velocity increment in itself, since the vehicle will have to reduce its planetocentric velocity below escape velocity. The size of increment in this maneuver will be of the same order of magnitude as that involved in leaving the parking orbit about the planet and entering the heliocentric transfer orbit for the return journey. It should be noted, however, that the amount of fuel used in the escape maneuver from the destination planet will be less than that burned in the preceding capture operation since the mass of the vehicle is diminished by the mass of fuel burned in the capture maneuver. This statement should be revalued in the light of the conclusions of Section 10.4.

Summing up, it may be stated that manned or unmanned missions into the domain of the giant planets of the Solar System are not practical by chemical rockets though fast unmanned reconnaisances as far as Jupiter by instrumented probes are probable marginal achievements within the next ten years. The unmanned exploration of the inner Solar System from the asteroid belt to Mercury is practical by present-day and near-future technological standards, but the figures arrived at in this section emphasize the need to wait for sources of power an order of magnitude better than chemical fuels can supply before men can be landed upon Mars and Venus and returned to Earth.

With such power sources, medium-fast transfer orbits can be chosen so that the long waiting times on these planets can be slashed especially since an added flexibility is achieved by virtue of the fact that outward and inward transfer paths need not be of the same eccentricity or have the same transfer time.

4. THE USE OF PARKING ORBITS IN INTERPLANETARY MISSIONS

Considerable saving in fuel can be achieved by the use of parking orbits as storage dumps about the planets of departure and destination. The well-known analogy to this procedure is the establishment of a number of base-camps on the route to the South Pole or up the slopes of Mt. Everest, in which supplies of food and fuel are left for the return journey; obviously

this results in a saving of energy. In the literature of astronautics there are many studies of this use of parking orbits with application to lunar and interplanetary voyages; Project Apollo (the proposed landing of men on the Moon) embodies this idea in the lunar landing phase of the mission.

We will consider the method in the following simple example of a journey conducted from the surface of planet P_1 to the surface of planet P_2 and back to the surface of planet P_1. In one case the mission is accomplished by one vehicle that uses a circum-P_1 and a circum-P_2 parking orbit only for checkout purposes; in the other case, the two parking orbits are used for storing fuel tanks. The mission phases are shown schematically in Figure 10-3 where S is the Sun. The return journey is indicated by the dotted line and it should be remembered that although it is shown in the diagram as a mirror image of the outward transfer orbit, in fact a finite waiting time on P_2 is necessary before take-off can occur. The orbits of P_1 and P_2 are assumed to be circular and coplanar. The sizes of the circular parking orbits are grossly exaggerated for the sake of clarity.

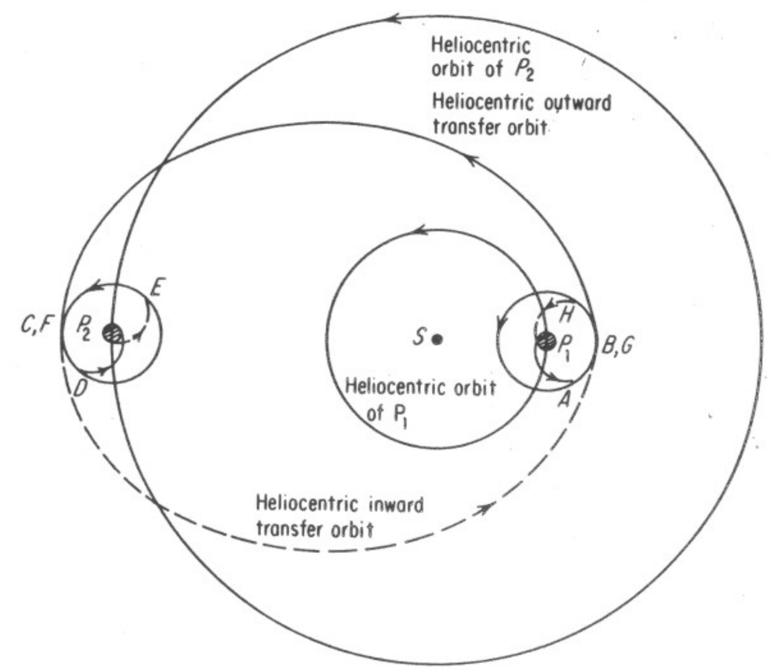

Figure 10-3

Then in "procedure one," the phases of the operation are as listed in Table 10.5.

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