in the last section using simple single-stage construction. Multi-stage construction, in which a small rocket forms the payload of a larger rocket and the two together form the payload of a still larger rocket and so on, would be essential in order to obtain the lowest characteristic velocities required for astronautical purposes.

As a rough rule it may be said that if a characteristic velocity v_c can be obtained, for a given propellant mixture, in a single-stage rocket, then in a rocket having n stages, the same propellants and similar structural factors a characteristic velocity of nv_c would be obtainable. Secondly, if in the single-stage vehicle a fraction $1/p$ of the all-up weight is useful payload, then in the n-stage rocket the fractional payload is $1/p^n$.

These rules are approximate in so far as the structural problems of multi-stage vehicles are hardly likely to be the same as those for single-stage vehicles and the structural factors obtained in individual stages of the former may not be the same as those obtained in the latter. The first rule suggests that there is no serious limitation on the characteristic velocity even if the exhaust velocity is limited. The second, however, indicates where the limitation lies. Thus, it would be reasonable, within the limitations set by existing possible exhaust speeds, to construct a single-stage vehicle having $v_c = 4$ km./sec. and $p = 6$. A three-stage vehicle, on the same basis, would have $v_c = 12$ km./sec. and $p^3 = 216$, i.e. a take-off weight 216 times as great as the useful payload. This would be reasonable as an engineering possibility, though not very economical. However, if we wanted a characteristic velocity of 24 km./sec., then the ratio of take-off mass to payload would be p^6 or almost 50,000!

The inevitable conclusion is that with existing propellants the projection of a vehicle into a circular orbit around the Earth is a real possibility, but the lunar vehicle with direct take-off from Earth is not a feasible proposition.

2

The Dynamics of Space Flight*

ARTHUR C. CLARKE
B.Sc., F.R.A.S.

Introduction

The number of books on astronautics is now very considerable, but they fall rather sharply into two categories—the "popular" or semi-technical, and the highly mathematical. This paper is an attempt to fill the gap between these extremes, and to give a reasonably accurate quantitative treatment of many of the problems of astronautics without advanced mathematics. Indeed, it is hoped that many of the conclusions reached can be understood without much trouble by those with no mathematical training at all.

The subject divides itself naturally into three sections: first, the motion of a body in the Earth's gravitational field; then the problem of the lunar voyage; and finally, true interplanetary journeys. The cases are of increasing difficulty and complexity and will therefore be dealt with in this order.

It may be as well to point out that this discussion is perfectly general and does not presuppose any particular form of propulsion—chemical rocket, atomic rocket, or even rockets at all. To travel from one body in space to another involves the expenditure of a definite amount of energy: from this point of view it does not matter in the least by what technical means the journey is made. It is true that the rocket is the only conceivable form of interplanetary locomotion at the moment: but even if the anti-gravity screen beloved of early science-fiction writers appears, it will still have to obey the same fundamental laws.

These laws are extremely simple and have been well understood since their formulation by Newton nearly 300 years ago. It is, indeed, a curious thought that there is probably nothing in this paper which would be unfamiliar to Newton—whereas such relatively far less spectacular aspects of our subject as aerodynamics or thermodynamics would be almost incomprehensible to him.

* First published, March 1949.

Astronautics is a branch of celestial mechanics, the science dealing with the movement of heavenly bodies. Spaceships will obey the same laws, and will hence move along the same sort of paths, as planets and comets—except during those very short periods of time when they are under power. It is a fortunate fact that, with accelerations which the human body can withstand comfortably, a spaceship need operate its motors for only about 10 minutes to attain velocities which would take it anywhere in the Solar System—even on journeys that might last for decades. It follows, therefore, that for practically the whole duration of any voyage a spaceship would be "inert," like any heavenly body, entirely under the control of the gravitational forces in the surrounding space.

Gravitational Fields

The gravitational field with which we are most familiar, and from which all our journeys must commence, is the Earth's, which at sea level produces an acceleration on a falling body of 9·81 metres second/second (32·2 feet/second/second). Table I shows how the

TABLE I

Body	Gravity E = 1	Time to fall 4·9 metres (sec.)	Escape velocity (km./sec.)	Circular velocity (km./sec.)
Sun	28	0·2	618	437
Mercury	0·26	2·0	3·5	2·5
Venus	0·90	1·1	10·4	7·3
Earth	1	1	11·2	7·9
Moon ..	0·16	2·5	2·3	1·6
Mars	0·38	1·6	5·0	3·6
Phobos ..	0·001*	30*	0·01*	0·01*
Jupiter	2·65	0·6	60	42·5
Ganymede ..	0·2*	2*	3*	2*
Saturn	1·14	0·9	36	25
Titan	0·2*	2*	3*	2*
Uranus	0·96	1·0	22	15·5
Neptune	1·0	1·0	23	16

* Approximate figures.

value of g varies for the most important bodies in the Solar System, and it will be seen that there are only two planets on which human beings would feel heavier than they do on Earth. To bring the meaning of these figures home more vividly, the second column shows how long it would take to fall 4·9 metres from rest on each of these bodies—the distance one falls in the first second on Earth.

The gravitational field of a body falls off with distance according

to Newton's inverse square law—if one doubles the distance from the centre, "g" is reduced to a quarter, and so on. Thus the fields of the planets are effective only over very small distances, astronomically speaking. Fig. 1 shows the variation of the Earth's field with distance. Despite the fact that beyond 100,000 kilometres it is too small to be presented accurately on the graph, it never reaches zero however far away one goes. Even at 385,000 kilometres it is sufficient to keep the not inconsiderable mass of the Moon chained to its orbit!

When a body is lifted against the Earth's gravitational field, the work that has to be done equals the product of the vertical distance and the force. But since the force is steadily decreasing, it follows that the work done over equal distances is also decreasing, according to the same inverse square law. The *total* work that has to be done in lifting a body from the Earth's surface to a point where gravity is negligible (to infinity, as the mathematician would put it) is therefore proportional to the total area beneath the "g" curve in Fig. 1. This can be calculated very easily, and leads to the important and surprisingly simple result that for unit mass:

$$E = gR$$

where R is the radius of the Earth.

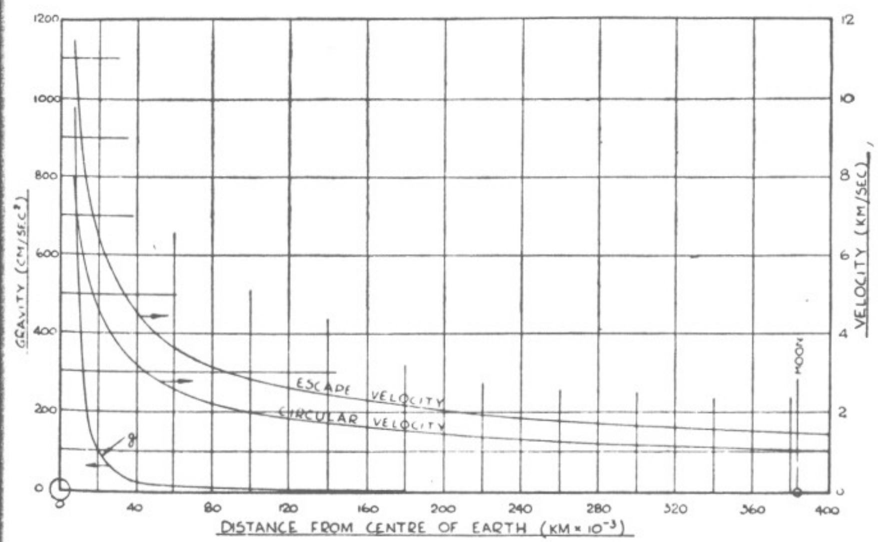


Fig. 1.

Variation of gravity, and escape and circular velocities, with distance from Earth's centre.

This means, for example, that to lift one ton completely away from the Earth ($R = 4{,}000$ miles) we must do 4,000 mile-tons of work. *The task is therefore exactly equivalent to that of climbing a mountain* 4,000 *miles high, assuming that gravity remained constant.* Perhaps this brings home more vividly than anything else the order of magnitude of the energies involved.

The formula $E = gR$ applies to all other bodies, if g and R stand for the appropriate surface gravities and radii. In this way we can calculate that to escape from the Moon is equivalent to climbing vertically 180 miles under one terrestrial gravity, a height attainable by today's single-stage rockets.

The Earth's gravitational field, therefore, may be regarded as producing a sort of valley or pit 4,000 miles (6,360 kilometres) deep, out of which we have to climb if we wish to travel to other worlds. The walls of the pit are very steep near the beginning, but rapidly flatten out with increasing distance (Fig. 2). At the Moon's distance the gravitational slope is very gentle; it is never perfectly flat, but beyond about a million kilometres it is so nearly level that scarcely any more work need be done in moving away from the Earth. As far as energy considerations are concerned, we have reached

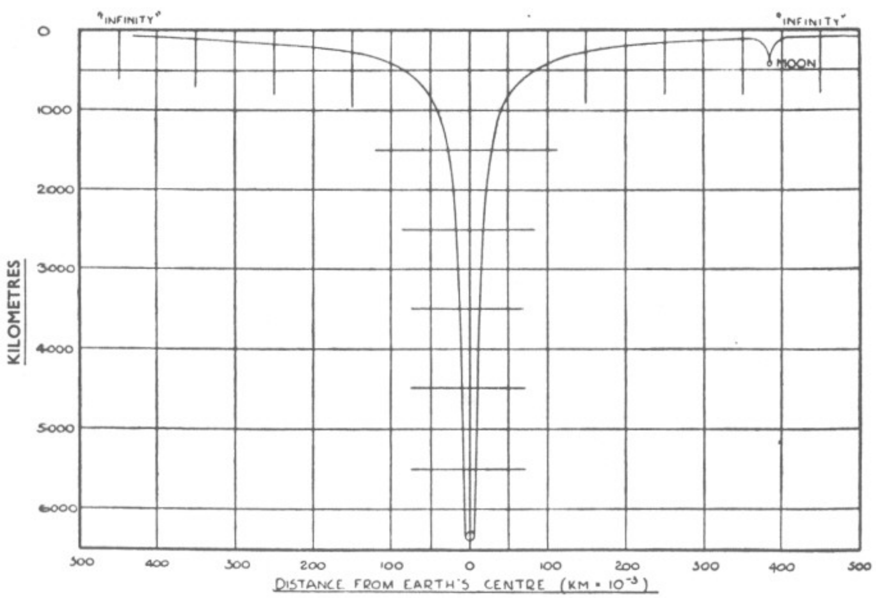


Fig. 2.
Potential Energy Diagram of Earth–Moon System.

"infinity," though by astronomical standards we are still very near the Earth.

If we happen to be going towards the Moon, we presently come across its private gravitational pit, only 278 kilometres (180 miles) deep, and with much more gently sloping sides. Fig. 2 shows very clearly how relatively easy it is to leave the Moon, and how important it may be as a base for expeditions to other planets. Though so close, astronomically speaking, it is really nine-tenths of the way to infinity.

There are two ways of climbing out of a valley. The usual method is to proceed up the slope at a low but more or less constant speed, exerting a fairly steady effort all the time. The second method, not unfamiliar to cyclists, is to build up such a speed at the bottom of the hill that one can then relax and rely on one's momentum to carry one to the top without any further effort—trading velocity for height.

Both methods are, in theory, possible for a spaceship leaving the Earth. If a rocket, for example, had unlimited power supplies, it could climb to the stars at a steady 100 miles an hour. But this slow, steady departure would require an enormous expenditure of energy, since most of the machine's effort would be wasted merely maintaining its position. (The extreme case of this is the rocket just balanced on its exhaust, which burns the whole of its fuel getting nowhere.) We are therefore forced to consider only the case in which a sufficient initial velocity is built up, in a relatively short distance, to enable the rocket to make the rest of the journey on momentum alone.

The velocity needed to enable us to "coast" up our 4,000-mile-high hill is almost exactly 25,000 miles an hour (11·2 km./sec.). This is the famous "velocity of escape," and its physical meaning is nothing subtler than this. A body projected vertically from the Earth at 11·2 kilometres (or 7 miles) a second would travel outwards, always slowing down, but would never fall back. At a lower speed, it would come to rest before reaching the top of the "hill," and would come falling down the slope again. It would return to Earth with exactly its initial speed, as would any body projected up a frictionless slope with a velocity insufficient for it to reach the top.

Escape and Circular Velocities

Every planet has its characteristic escape velocity, usually quoted for the surface of the body. Escape velocity falls off with increasing height; as might be expected, the velocity needed to reach the top of the "gravitational slope" is reduced if one starts some way from the bottom. The rate of decrease is rather slow, depending inversely on the square *root* of the distance. (See Appendix.)

Related to escape velocity is the conception of circular velocity. This is the velocity a body needs, not to escape from a planet, but to circle it like a moon. At the Earth's surface this velocity is about 18,000 miles an hour (7·9 km./sec.). If a body reached this speed in horizontal flight just outside the atmosphere the outward centrifugal force would exactly balance gravity. It would circle the Earth once every 90 minutes or so. At greater heights the velocity required is naturally less, the variation with distance being the same as for escape velocity. In fact, at any point the escape velocity is equal to $\sqrt{2} \times$ circular velocity.

Fig. 1 shows the variation of the Earth's escape and circular velocities with distance: it will be seen that they fall off very much more slowly than does "g." Fig. 3 shows the value of circular velocity for points close to the Earth, and also the period of revolution of a body in a circular orbit. These orbits will be the first to be of practical importance in astronautics, as the development of satellite vehicles has shown.

Table I lists the escape and circular velocities for the chief bodies in the Solar System. They range from 60 km./sec. for Jupiter down

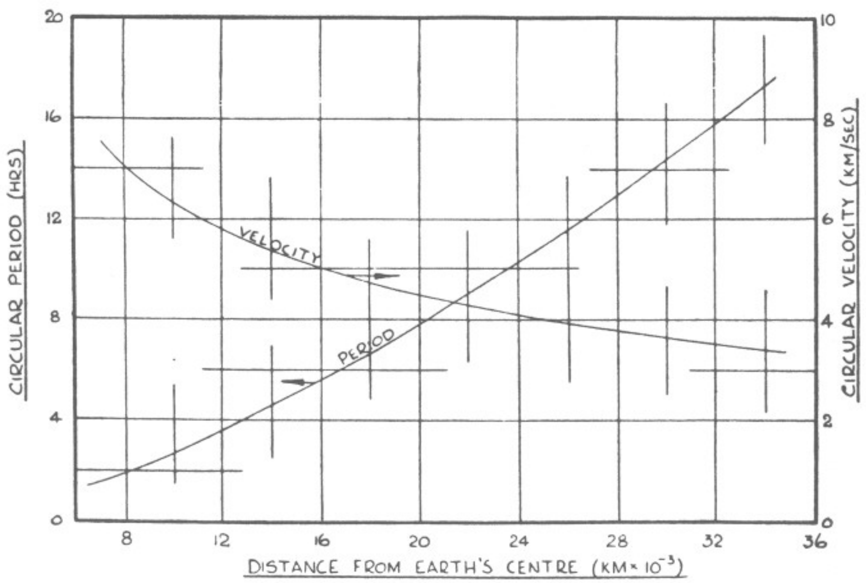


FIG. 3.
Period and Velocity in Circular Orbit round Earth.

to about 10 metres/sec. for Phobos, the inner moon of Mars. It would be possible to jump completely away from a body a little smaller than this; Phobos' "gravitational pit" is only about five *metres* deep, as against the Earth's 6,360 kilometres!

We are now in a position to discuss the requirements for a voyage from the Earth to the Moon. Looking at Fig. 2 again, we see that the velocity needed to reach the Moon from the Earth is a little less than that needed to reach "infinity." The difference, however, is only about a tenth of a kilometre a second, so we will assume that the full velocity of escape, 11·2 kilometres a second, is required.

A spaceship given this velocity in the direction of the Moon would slowly lose speed until it came almost to rest about 40,000 kilometres from the Moon. Then, as it began to enter our satellite's "gravitational pit" it would begin to accelerate again. If unchecked, it would reach the Moon at a little less than that body's velocity of escape, 2·3 kilometres a second. The ship's motors must therefore be used to neutralize this speed, and hence the *absolute minimum* velocity requirement for the journey is 11·2 + 2·3 or 13·5 kilometres a second (30,000 miles an hour). The ship would, of course, never actually reach this speed; but it must be potentially capable of doing so.

This figure is a theoretical minimum which must be exceeded in practice owing to (*a*) air resistance at take-off, (*b*) "gravitational losses" due to the retarding effect of the Earth's field, and (*c*) margins for navigational corrections, which might be considerable at the lunar end of the voyage. These additional figures are rather indefinite and depend a good deal on particular cases. In general, they should not exceed 20 per cent., making the total "velocity budget" for the one-way lunar voyage about 16 kilometres a second (36,000 miles an hour).

The air-resistance loss, though it is the one most often mentioned, is in fact the least important. A rocket leaving Earth is travelling relatively slowly in the atmosphere, and on a large ship the velocity loss due to frictional drag would be only a fraction of a kilometre a second—a negligible quantity compared with the requirements for the rest of the voyage.

The return journey is the same as the outward one, but with this difference, that the Earth's atmosphere may be used to produce a certain amount of frictional braking at the end of the voyage. Ignoring this, the requirement for the round trip would be 2×16 or 32 kilometres a second (72,000 miles an hour). Allowing for atmospheric braking, it would be about 30 kilometres a second (67,000 miles an hour), or perhaps a little less. Once again it should be emphasized that the maximum velocity the ship would ever actually reach would be less than a third of this.

We have been able to use this relatively simple treatment in the case of the Earth–Moon voyage because both bodies are at the same distance from the Sun, and so the Sun's gravitational field acts on them equally and may, therefore, be ignored. If we wish to consider true interplanetary journeys, however, we can no longer do this. Not only must our spaceship escape from one planet and lower itself safely on to another, but it must also do work either moving outwards against the Sun's gravitational field, or decelerating itself after falling down the solar field. It is therefore necessary to consider how this field varies across the orbits of the planets.

The Sun's Gravitational Field

The Sun's gravitational field obeys exactly the same laws as the Earth's; the only difference is that it is far more intense and is effective over a far greater volume of space. Escape velocity at the surface of the Earth is 11·2 kilometres a second; at the surface of the Sun it is no less than 618 kilometres a second. Thus the energies needed to leave the two bodies are of an altogether different order of magnitude. The Earth's 6,360-kilometre-deep "gravitational pit" seemed quite impressive; but to escape from the Sun is equivalent to climbing, under one gravity, out of a pit 20,000,000 kilometres (12,000,000 miles) deep!

Luckily for astronautics, this "pit" is so very narrow that even Mercury, the innermost planet, is far up on the shallow, higher slopes. All the planets, in fact, are within a quarter of a million kilometres of the top, and as a result the velocities needed to go from one orbit to another are in most cases less than those needed to escape from the planets themselves. This is certainly very fortunate. One could easily imagine Solar Systems with very massive suns in which it might be quite simple to escape from a planet and go to its satellite, whereas the journey from one planet to another might be almost impossible.

We will now construct an "energy diagram" of the Solar System on much the same lines as we have already done for the Earth and Moon. This time, however, we will measure the depth of the "gravitational pit" in terms of the velocity needed to escape from it at any point, since it is this in which we are primarily interested. To escape from the Sun requires a velocity of 618 kilometres a second, which gives the maximum depth of the pit. To escape from the orbit of the innermost planet, Mercury, requires about a tenth

of this—68 kilometres a second. But Mercury's orbital speed is already 48 kilometres a second: hence additional velocity of nearly 20 kilometres a second is needed. This additional or excess velocity, which we will call "transfer velocity," is much less for the outer planets. In the Earth's case it requires only an extra 12·3 kilometres a second above orbital speed to reach infinity.

The left-hand side of Fig. 4 shows these transfer velocities for points out to the orbit of Jupiter. If the planets *had no gravitational fields of their own*, these would be the velocities needed by a spaceship to leave them and to escape completely from the Solar System.

But, of course, the planets have their own gravitational fields, each with its characteristic escape velocity. Thus we can, to a good degree of approximation, represent the true state of affairs by attaching to the main curve subsidiary "icicles" whose depths, in kilometres a second, are the escape velocities of the planets concerned.

This type of diagram was first constructed, as far as we know, by Dr. Robert S. Richardson, though we believe he employed energy units instead of velocities as ordinates. Physicists have, of course,

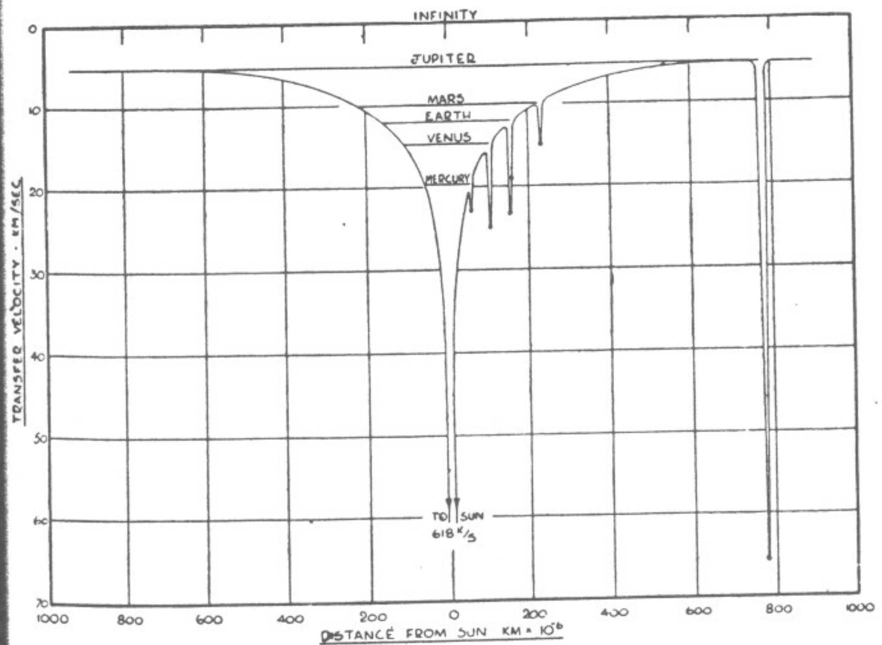


Fig. 4.

Energy Diagram of Solar System, in terms of Required Transfer Velocities.

used such figures for a long time to represent the potential barriers around atomic nuclei.

The resulting picture is not much like one's ordinary idea of the Solar System; but it is much more useful for astronauts. It shows, for example, the surprising fact that a journey to the outer planets could start more easily from Mercury than from Venus! It also demonstrates very vividly the difficulty of getting away from Jupiter, should one be unfortunate enough to be born there. Mercury is much farther away from the Sun, dynamically speaking, than is the surface of Jupiter!

To sum up, Fig. 4 indicates that the velocities needed to travel from one orbit to the next are, in general, less than those needed to escape from the planet of origin. The diagram even gives a rough idea of the *total* velocity needed for an interplanetary journey, from surface to surface, but more accurate results require a fuller treatment, which we will briefly discuss in the next section.

Orbits and Trajectories

So far, our treatment has been somewhat abstract; we have not considered the actual shapes of the paths that spaceships must follow, but only their initial velocities of projection. To fix ideas, we will first discuss possible orbits in the Earth's field.

We have already seen that a body travelling horizontally just outside the atmosphere at 7·9 kilometres a second would maintain itself in a circular orbit, never falling to Earth. At any lesser speed, it would return to the planet, though it might travel half-way round the world before doing so. But what of speeds *greater* than 7·9 kilometres a second?

Fig. 5 shows what happens in these cases. As the speed of projection increases, the orbit elongates into an ellipse of greater and greater eccentricity. As the velocity of escape—11·2 kilometres a second—is approached, the ellipse becomes larger very rapidly indeed. At exactly 11·2 kilometres a second it opens out, as it were, into a parabola, and the body never returns.

It should therefore be noted that a spaceship will escape from Earth at 11·2 kilometres a second whether its direction of motion is horizontal or vertical or at any intermediate angle.

When the velocity exceeds 11·2 kilometres a second, the orbit is a hyperbola which becomes more and more nearly a straight line as the velocity approaches infinity.

These orbits are of fundamental importance because they arise whenever a body moves in a gravitational field, whether it be that of the Earth or the Sun. Planets, comets and asteroids give all

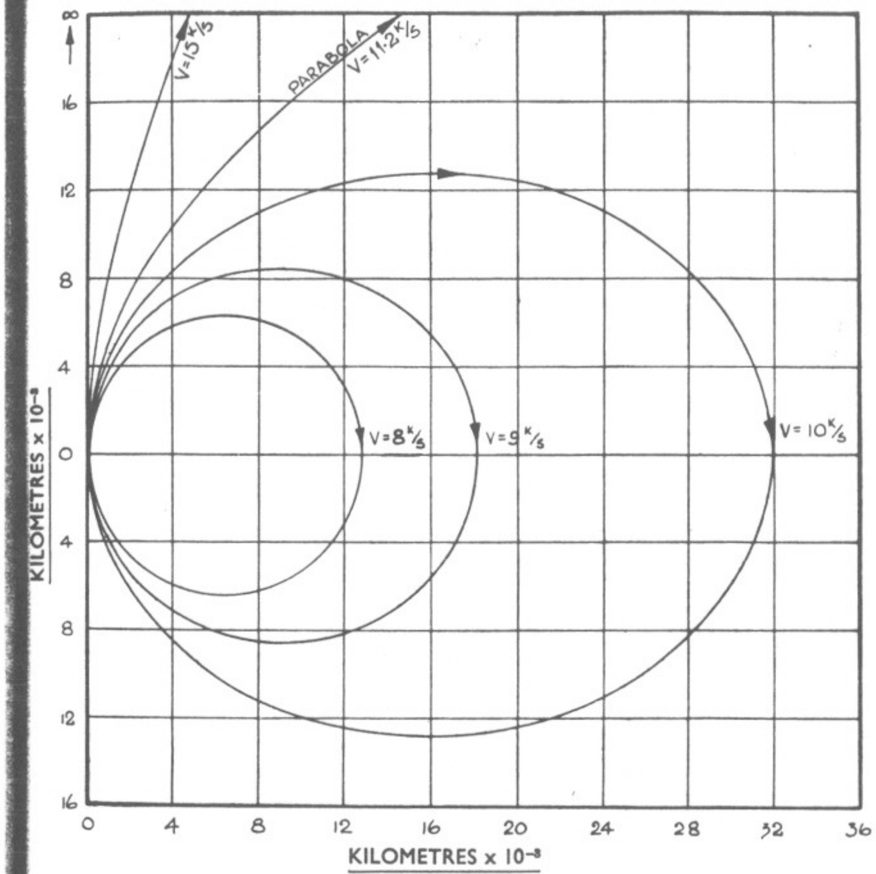


FIG. 5.
Orbits in Earth's Field.

possible types of closed orbit from almost perfect circles to very elongated ellipses. It has often been suggested that some meteors and comets may travel on hyperbolic paths, so that they pass through our system only once; but it is difficult to prove this, since (as will be seen from Fig. 5) all the paths are very similar at the nearest point to the central force.

From the astronautical point of view the elliptical paths are the most important, since they can be used to link one circular orbit to another. Fig. 6 (*a*) shows such an orbit touching the paths of, say, two "space-stations" S_1 and S_2 circling the Earth.

The body in the elliptical orbit will touch the space-station orbits at A and B; but at A it will be travelling *faster* than S_1 while at B

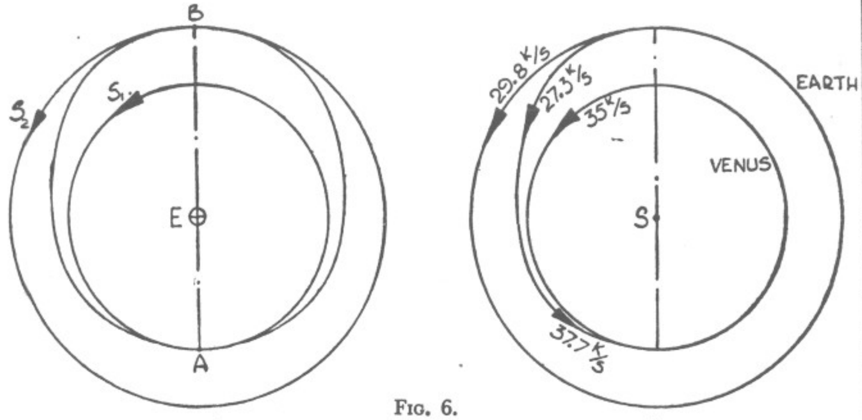


FIG. 6.
Tangential Ellipses.

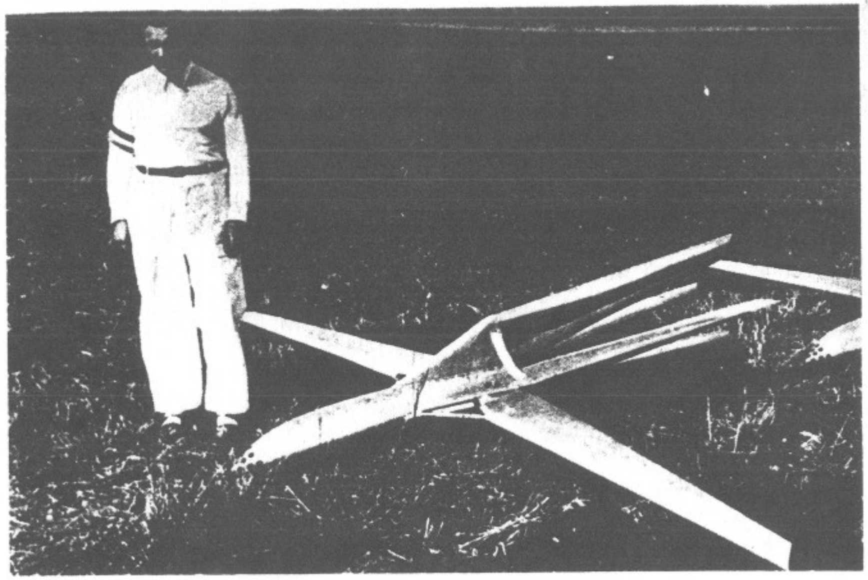
it will be travelling *more slowly* than S_2. Small velocity increments would therefore be needed at A and B to change from one type of orbit to another.

If one is in a circular orbit, and *increases* the speed slightly, the orbit becomes elliptical, lying outside the original circle—as happens at A. If one *decreases* the speed, the orbit becomes an interior ellipse, as at B.

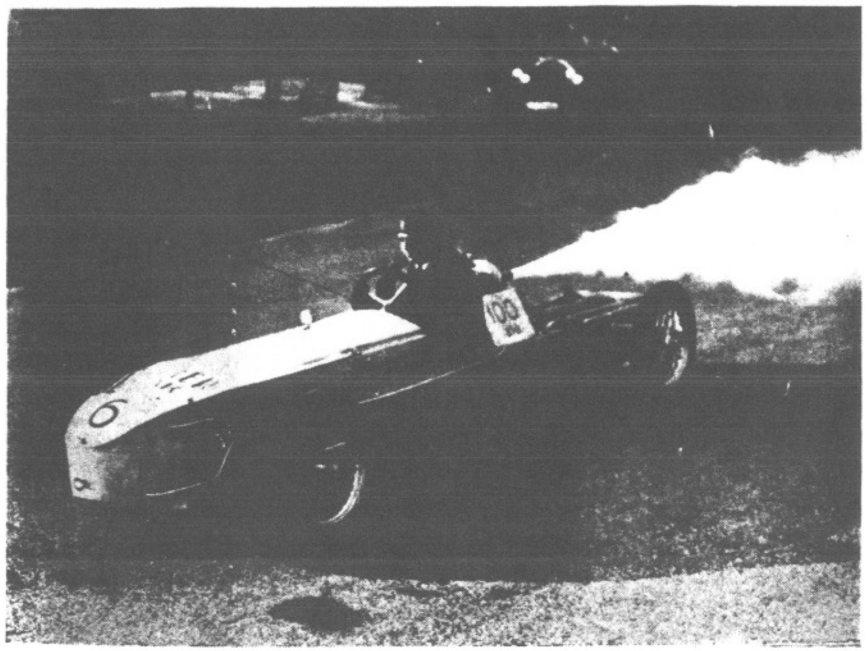
Exactly the same argument applies for journeys between planets. Fig. 6 (*b*) shows the actual velocities involved in a journey from Earth to Venus (or vice versa). It will be seen that the transfer velocities where the orbits touch are quite small. They are within the range of present-day rockets; a V2 starting from space in the Earth's orbit could reach Venus quite easily with full payload!

(At this point we recall an instructive mistake made by the author of a space-travel story many years ago. In this tale an interplanetary journey was made by sending a rocket to an asteroid which happened to be passing close to the Earth and which, owing to its elliptical orbit, gave its passengers a "free lift" to their destination. Unfortunately, of course, the presence of the asteroid would have made no difference at all. Once the spaceship had matched the asteroid's velocity—which it would have to do to make a landing—it would have travelled on exactly the same elliptic orbit whether the asteroid was there or not. So the expedition would have gained nothing but a few acres of probably rather uninspiring scenery.)

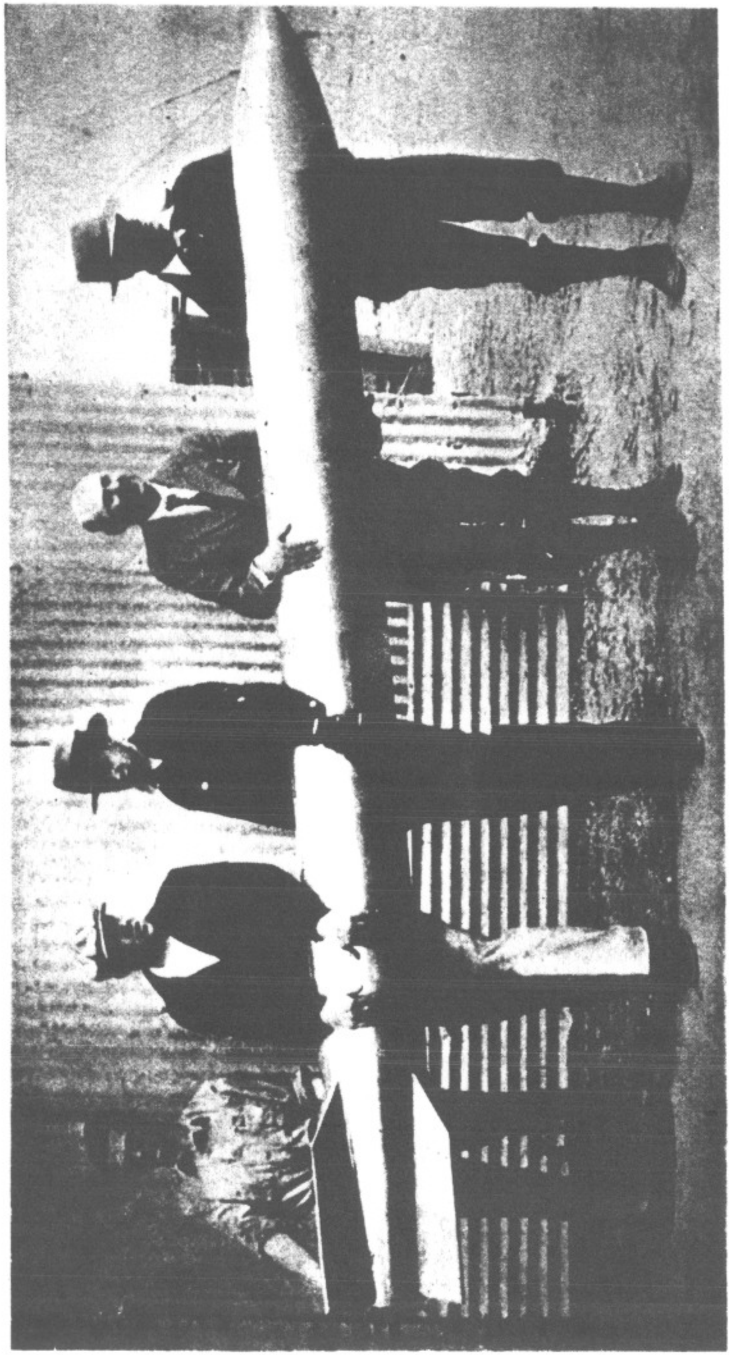
This type of orbit, which just grazes two planetary orbits, is by no means the only possible one. It will, however, be almost intuitively obvious that it requires the minimum energy. In the case



1. Tiling Rocket.



2. Valier Rocket Car, Berlin, December 22, 1929.



3. Goddard Rocket, during experiments at Roswell, U.S.A.

of the Venus journey, simple addition gives a total "velocity budget" as below:

	km./sec.
Escape from Earth	11·2
Earth orbit to voyage orbit	2·5
Voyage orbit to Venus orbit	2·7
Landing on Venus	10·4
	26·8

A rather subtle point, however, arises here. This straightforward addition does *not* give the minimum velocity budget for the voyage; the requirement is actually a good deal smaller. To understand why, let us consider the first part of the mission—the escape from Earth and the entry into the voyage orbit.

Carried out as *two separate operations*, this would require a total velocity of 11·2 + 2·5 km./sec. If the manœuvre were done in a single burst of power, however, while the ship was still close to the Earth, the velocity needed would be only 11·5 km./sec. (Obtained by squaring and adding 11·2 and 2·5 and taking the square root of the total.) This follows from the fact that the problem is then one of kinetic energies. We wish the ship to have a residual speed of 2·5 km./sec. when it has escaped from the Earth's gravitational field. Giving it an initial speed of 11·5 km./sec. will ensure this.

In the same way, the velocity needed for the manœuvre at the end of the voyage is not 2·7 + 10·4 km./sec., but only 10·7 km./sec., calculated as above. For the whole mission, therefore, the total velocity budget is:

	km./sec.
Escape from Earth and transfer to voyage orbit	11·5
Voyage orbit to Venus orbit and landing on Venus	10·7
Total	22·2

Navigational corrections might bring this up to about 26 km./sec., and the journey would last approximately 146 days.

For Mars the requirements are as follows, assuming mean distance between the planets. (As the orbit of Mars is somewhat eccentric, the figures vary slightly.)

	km./sec.
Escape from Earth and transfer to voyage orbit	11·6
Voyage orbit to Mars orbit and landing on Mars	5·7
	17·3

This would be at least 20 kilometres a second in practice—considerably less than for the voyage to Venus. But the journey would last longer—237 days.

These times of transit, though long, are not impossibly so. They could be reduced indefinitely by travelling in more eccentric orbits, or even along hyperbolic paths. Such orbits, however, would cut across the paths of the planets at steep angles and so would require very high transfer velocities. The fuel requirements for such journeys would thus be multiplied by very large factors indeed, making them out of the question for the first and perhaps even the second generation of atomic spaceships.

Orbits of this type would be necessary for journeys to the outer planets, which would last many decades if the tangential paths were employed.

Summary

We may now summarize our main results in Table II, which show the theoretical velocities needed for various typical journeys, as well as estimates of what these values may be in practice. In many cases the latter figures may turn out to be very pessimistic, since they make no allowance for atmospheric braking. Most authorities consider that landings on Earth (and presumably on Venus and Mars) could be made almost entirely by air braking, with very little use of the rockets. We hope that this is the case, but have felt it safer to ignore the possibility.

TABLE II

	Mission	Theoretical velocity*	Approximate actual value†	
		km./sec.	km./sec.	m.p.h.
One-way journeys	Orbit round Earth	8	10	22,000
	Escape from Earth	11·2	13	29,000
	Earth to Moon	13·5	16	36,000
	Earth to Mars	17·3	20	45,000
	Earth to Venus	22·2	26	58,000
Return journeys	Earth–Moon–Earth (no landing)	22·4	25	56,000
	Lunar return trip (with landing)	27	32	72,000
	Earth–Mars–Earth (no landing)	23·2	26	58,000
	Mars return trip (with landing)	34·6	40	90,000
	Earth–Venus–Earth (no landing)	23	26	58,000
	Venus return trip (with landing)	44·4	52	115,000

* Ignoring air resistance and gravitational losses.
† Including allowance for losses.

Conclusion

Our survey has now gone as far as is possible without employing mathematics of great complexity and extreme ugliness. Most of the problems in astronautics can only be discussed by numerical, step-by-step calculations, and no exact general solutions are possible. The conclusions we have reached must therefore be regarded as first approximations to the truth, but in most cases they are very good approximations indeed—correct, that is, to a few per cent. As such, they are quite sufficient to give a good idea of the general nature of the problem, which we now hand on to those engineers who are brave—or rash—enough to tackle it.

APPENDIX

For convenience in reference, some of the more important results in interplanetary dynamics are summarized below.

Earth's Gravitational Field

If g is surface gravity, r is distance from Earth's centre, R is radius of Earth, then gravity g_1 at any point r is, by inverse square law,

$$g_1 = gR^2/r^2 \qquad (1)$$

Hence work done in moving unit mass to infinity from the Earth's surface is

$$\int_R^\infty \frac{gR^2}{r^2}\,dr = gR \qquad (2)$$

This is the result used in Section 2 and is true for all bodies if g and R are given their appropriate values.

At the velocity of escape, the kinetic energy of the projected body must equal gR, i.e. $\frac{1}{2}v_R^2 = gR$.

Hence
$$v_R = \sqrt{2gR} \qquad (3)$$

Similarly the velocity of escape at any point r is given by

$$v_R = \sqrt{\frac{2gR^2}{r}} \qquad (4)$$

Circular velocity is given by the condition that at r, the outward centrifugal force due to the body's motion must equal the inward gravitational force.

Hence
$$\frac{v^2}{r} = \frac{gR^2}{r^2},$$
(mistake)

or
$$v = \sqrt{\frac{gR^2}{r^2}} \quad \sqrt{\frac{gR^2}{r}} \qquad (5)$$
$$= \sqrt{gR} \text{ at Earth's surface.}$$

Hence escape velocity at any point equals $\sqrt{2}$ times circular velocity at that point.

The height obtained by a body projected vertically from the Earth's surface (ignoring air resistance) is given by

$$H = \frac{2gR^2}{2gR - v^2} - R \qquad (6)$$

This becomes infinite as $v \rightarrow \sqrt{2gR} = 11.2$ km./sec.

ORBITS (Fig. 7.)

The velocity at any point in an orbit under a central force is given by:

$$v^2 = \mu\left(\frac{2}{r} \pm \frac{1}{a}\right) \qquad (7)$$

the minus sign being taken in the elliptic case, the plus in the hyperbolic case. r is the distance from the centre of force, a the semi-major axis, and μ is a gravitational constant $= gR^2$ ($= 4 \times 10^5$ km.^{3}/sec.^{2}, very nearly, for the Earth).

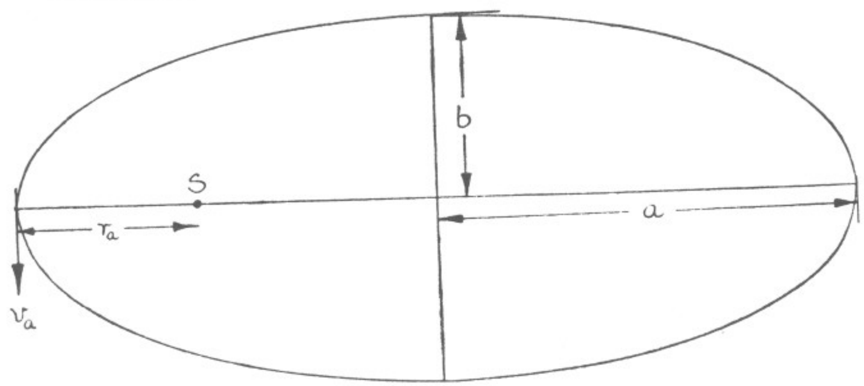


FIG. 7.
Ellipse: leading dimensions.

In the parabolic case this reduces to

$$v^2 = \frac{2\mu}{r} \qquad (8)$$

If the perigee or perihelion velocities and distances v_a, r_a are known, we may then calculate a from equation (7), which becomes

$$a = \frac{\mu}{(2\mu/r_a) - v_a^2} \qquad (9)$$

The semi-minor axis b is given by:

$$b = \frac{v_a\, r_a}{\sqrt{(2\mu/r_a) - v_a^2}} \qquad (10)$$

Equations (9) and (10) enable us to calculate the orbits in Fig. 5, while from equation (7) we can calculate the v_a needed to travel from one circular orbit to another. In this case the major axis $2a$ will clearly be the sum of the radii of the two circular orbits.

REALITIES OF SPACE TRAVEL

Selected Papers of the British Interplanetary Society

EDITED BY
L. J. CARTER
A. C. I. S.

1957

McGRAW-HILL BOOK COMPANY, INC.
NEW YORK TORONTO