9

INTERPLANETARY ORBITS

D. F. Lawden

Survey of the problems. The navigation of a spaceship presents three main problems. First, the trajectory of the ship through space must be computed. This is a complex task which will be performed with the assistance of an automatic computer some time before the commencement of the journey. The number of possible tracks connecting the two points of departure and arrival is clearly infinite, and it is therefore necessary to invoke some guiding principle to permit us to discover from amongst these the most convenient path. The most obvious criterion to select for this purpose is that the propellant expenditure shall be a minimum. However, this minimization procedure will be subject to certain limiting factors. An excessive time of transit between the end points of the track will result in a large stock of food, oxygen and water having to be carried to supply the needs of the crew, and this, in turn, will lead to an increase in the propellant requirement. The time of transit will therefore be limited to less than a certain extreme value. In the case of an unmanned ship, such as a probe projected from the Earth with the object of making

a preliminary investigation of conditions on one of the planets, this limitation will not be operative. Other conditions which will have to be satisfied by the optimal trajectory are (i) that it shall not approach too closely to the Sun or enter any regions of excessive cosmic ray or meteor intensity and (ii) that excessive heat shall not be generated by high rocket velocities along the arcs immersed in an atmosphere.

Having arrived at a decision regarding the trajectory to be followed by the ship, the journey may be commenced and the navigator will then be faced with the task of deciding whether his ship is following the correct course. He may fix his position in space by observation of the various bodies of the Solar System against the background of the fixed stars, and a series of such fixes will permit him to compute the actual trajectory of the ship. Any divergence of this track from that originally computed will have to be corrected by the application of appropriate thrusts from the motor. This is the third main problem of space navigation.

In the following sections we will examine these three problems in greater detail.

Optimal trajectories. The basic problem of spaceship trajectory computation may be expressed thus: a rocket is situated at a point A in a given gravitational field and has a specified velocity there. It is required to transfer the rocket to another point, B, of the same field in such a manner that it arrives with a certain velocity and the propellant expenditure is a minimum. A might be a launching point on the Earth's surface and B a point on the surface of Mars. In this case, the specified velocities at the terminals would be those of the two planets in their orbits. The instants of departure and arrival may, or may not, be given. Provided that the motion is entirely *in vacuo*, the optimal trajectory is found to comprise a number of arcs meeting at junction points at which the rocket velocity is changed, theoretically instantaneously. The rocket motor is energized at the junctions in order to effect these velocity changes. Along the arcs connecting the junctions, however, the motor is shut down and the ship orbits under its own momentum and the gravitational attraction alone. The conditions to be satisfied by the arcs of null-thrust and the junctions at which they intersect, if the resulting maneuver is to involve the expenditure of a minimum quantity of propellant, are known but, being somewhat technical, will not be given here. The reader interested in the general theory of optimal rocket trajectories may refer to three papers listed at the end of this chapter.[1, 2, 3] The instantaneous velocity changes at the junctions can only be brought about by the application of impulsive thrusts from the motor. Such thrusts cannot be produced in practice, but it is rarely difficult to achieve a close approximation, since a thrust applied over a period of a few minutes will generally effect the velocity change necessary, and this period is usually negligible by comparison with the times, often amounting to hundreds of days, spent negotiating the arcs of null-thrust.

The duration of an impulsive thrust is so short that the velocity changes for which finite forces, such as gravitational attraction and air resistance, are responsible during this period are negligible by comparison with the velocity increment caused by the motor thrust. In calculating the velocity change at a junction, it is accordingly permissible to disregard the gravitational field and then, provided the direction of thrust remains steady during its application, the velocity change produced will be in the direction of the thrust and of a magnitude which may be calculated from the equation

$$\Delta v = c \log_e R. \qquad ..(1)$$

In this equation, c is the exhaust velocity of the motor and R is the ratio of the rocket mass upon arrival at the junction to the rocket mass upon departure. R is termed the *mass ratio* of the maneuver. A proof of equation (1) will be found in Appendix XII.

Thus, to compute the thrust requirement at any junction, we proceed as follows: let the vectors v_0, v_1 (Figure 13) represent the velocities of arrival at and departure from a junction respectively. Then, by completing the triangle of veloci-

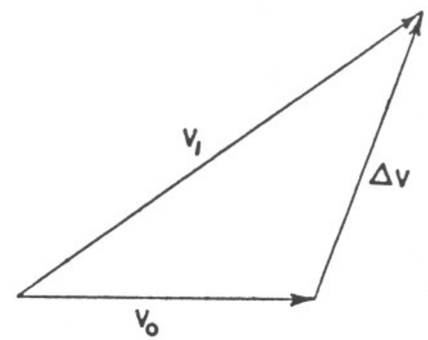

Figure 13. Computation of thrust required at junction

ties, we see that Δv is the velocity change which has to be made in the rocket. To effect this change, the thrust must be applied in the direction of Δv and its duration must be such that the mass ratio is given by

$$R = e^{\Delta v / c}, \qquad ..(2)$$

an equation which is another form of equation (1).

If the whole trajectory involves two junctions only, and if m_1, m_2 are the masses of the rocket upon arrival at and departure from the first junction respectively, and m_2, m_3 are the corresponding masses at the second junction, the product of the mass ratios R_1, R_2 at the two junctions is

$$R_1 R_2 = \frac{m_1 m_2}{m_2 m_3} = \frac{m_1}{m_3}, \qquad ..(3)$$

i.e., is equal to the ratio of the rocket mass at the commencement of the journey to the mass at its termination. This latter ratio is termed the *over-all mass ratio*. In general, if there are n junctions and $R_1, R_2 \ldots R_n$ are the corresponding mass ratios, that for the whole journey is R where

$$R = R_1 R_2 \ldots R_n. \qquad ..(4)$$

Suppose that $\Delta v_1, \Delta v_2 \ldots \Delta v_n$ represent the magnitudes of the velocity changes at the junctions. Then, by equation (1):

$$\begin{aligned}\Delta v_1 + \Delta v_2 \ldots \Delta v_n &= c(\log_e R_1 + \log_e R_2 + \ldots + \log_e R_n)\\ &= c \log_e R_1 R_2 \ldots R_n\\ &= c \log_e R. \qquad ..(5)\end{aligned}$$

The left-hand member of this equation is the numeral sum of all the velocity changes occurring during the complete maneuver and is called the *characteristic* (or *ideal*) *velocity* of the journey. If it is denoted by V, then we have

$$V = c \log_e R \text{ or } R = e^{V/c}, \qquad ..(6)$$

indicating that the over-all mass ratio requirement is easily determined when the characteristic velocity for a journey is known. A table of values of R for a range of values of V/c is given below:

TABLE 13

V/c	R	V/c	R	V/c	R
0·1	1·105	0·7	2·014	2·5	12·18
0·2	1·221	0·8	2·226	3·0	20·09
0·3	1·350	0·9	2·460	3·5	33·12
0·4	1·492	1·0	2·718	4·0	54·60
0·5	1·649	1·5	4·48	4·5	90·02
0·6	1·822	2·0	7·39	5·0	148·4

The computation of the null-thrust arcs may be performed using the standard techniques of celestial mechanics. Over the greater part of a rocket trajectory, the attraction of a single body will be predominant and all other gravitational forces may be neglected. This greatly simplifies the calculation, since motion under the attraction of a single body is of a particularly simple mathematical nature, and formulas are readily available by which numerical results can be quickly found. For example, if it is desired to transfer a rocket moving in a circular orbit about the Earth into a similar orbit about Mars, an impulsive thrust would first be applied to increase the rocket's momentum to a value sufficient to enable it to escape from the Earth's attraction. As the rocket moves along the arc of escape from the Earth, the influence of this body is paramount, the attractions of the Sun and of Mars being negligible by comparison. When the rocket has receded to a great distance from the Earth, only the Sun exerts an appreciable attraction upon the rocket, and its subsequent motion is dominated by this body until the vicinity of Mars is reached. The rocket then comes under the influence of this planet, and the effect of the Sun during the last few hours of the fall toward Mars may be neglected. Upon arrival at the level of the circular orbit, a second impulsive thrust is applied to convert the rocket's velocity into that appropriate to this orbit. It will be noted that the whole maneuver involves two junctions only.

The motion of a ship moving freely under the attraction of a single body being of prime importance, we will now describe this motion in some detail. If the attracting body is spherical, as it will be almost exactly in practice, its attraction decreases as the distance from the center increases, being inversely proportional to the square of this distance. Thus, the attraction is diminished by a factor of a quarter when the distance from the center is doubled, and at very great distances the attraction becomes so small that it can be neglected. It may be shown that, if a mass is projected in any direction from a point in this field of attraction, its subsequent path will be one of four types. In the exceptional case when the mass is projected directly toward or away from the center of attraction, its track will be a straight line. In all other cases the path will be curved. If the velocity of projection v_o is equal to or exceeds a certain minimum value, the mass will have sufficient momentum to recede to a great distance from the center in spite of the attraction and hence eventually to escape from the field altogether. The minimum velocity of projection for escape is termed the *escape velocity* at the point in

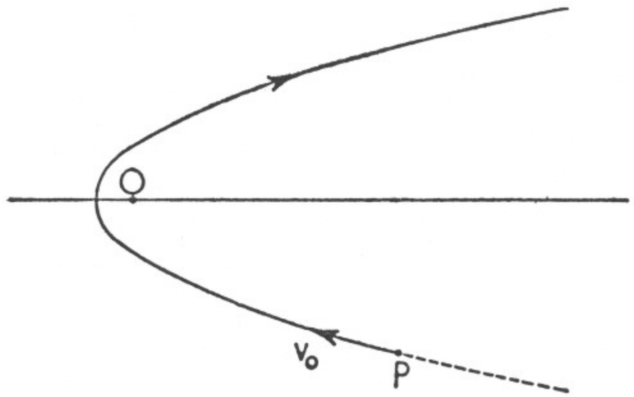

Figure 14. Parabolic trajectory

question, and will be denoted by v_e. If the mass be projected with escape velocity from a point *P*, its trajectory will be a *parabola*, as shown in Figure 14. The center of attraction O is called the *focus* of the parabola. As the mass recedes from the center of attraction along the parabola, its velocity steadily decreases and approaches zero. This is expressed by saying that the velocity at infinity is zero.

If the initial velocity of the mass is greater than escape velocity ($v_o > v_e$), its trajectory will be a *hyperbola*, as shown in Figure 15. O is the focus of the hyperbola. If v_1 is the

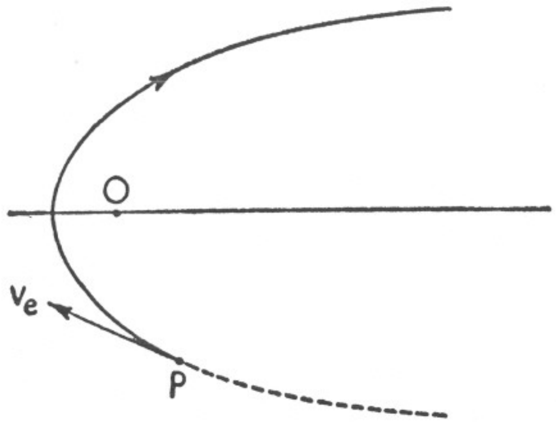

Figure 15. Hyperbolic trajectory

velocity of the mass at infinity, i.e., when it has receded to a great distance, it may be proved (see Appendix XIII) that

$$v_1^2 = v_o^2 - v_e^2. \qquad ..(7)$$

Finally, suppose that the velocity of projection is less than the escape velocity ($v_o < v_e$). The trajectory of the mass is then a closed orbit in the shape of an *ellipse* (Figure 16). The ellipse, being a symmetrical curve, has two foci O, O′ at

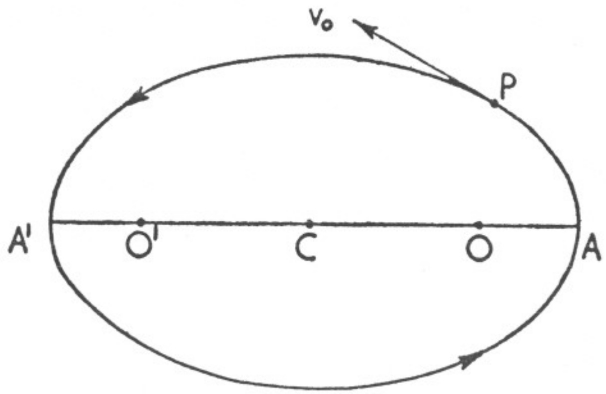

Figure 16. Elliptic trajectory

equal distances from its center, C. The focus O is the center of attraction. The line OO′, produced both ways to meet the ellipse at A and A′, is termed the *major axis* of the ellipse. If O is the center of the Sun, and the mass represents a planet moving under the Sun's attraction, the point A of closest approach to the Sun is referred to as *perihelion* and the point A′, where the planet is at its greatest distance from the Sun, is called *aphelion*. However, most of the planetary orbits approximate very closely to a circular shape, this being the particular case of an ellipse with coincident foci. To place a mass in a circular orbit about a center of attraction, it is necessary to project it in a direction at right angles to the line joining the point of projection to the center of attraction. The velocity of projection must be selected to be $v_o = v_e/\sqrt{2}$. This is termed the *circular velocity* at this point. The mass will then describe a circle at a steady speed equal to that of projection.

Velocities of escape from the surfaces of the various planets are given in Appendix XIV.

The form of the trajectory of a rocket undertaking an interplanetary journey can now be understood. Let us assume that the rocket is initially revolving in a circular orbit about the Earth under the attraction of this body. The escape velocity at points close to the surface of the Earth is about 7 miles per second and hence, provided the circular orbit is just outside the Earth's atmosphere, the circular velocity will be $7/\sqrt{2} = 5$ miles per second. Suppose that the rocket is subjected to an impulsive thrust from its motors, acting in the direction of its motion at any instant. If the velocity increment caused by the thrust is 3 miles per second, it leaves the circular orbit with a velocity of 8 miles per second. Thus $v_o = 8$, $v_e = 7$ and, by equation (7), $v_1^2 = 64 - 49 = 15$. Hence $v_1 = 3.9$ miles per second is the velocity of the rocket after it has receded along a hyperbolic trajectory to a great distance from the Earth. It should be noted that if the velocity increment had been only 2 miles per second, the rocket would have left the circular orbit with escape velocity exactly, and accordingly would have arrived at infinity with zero velocity. Thus, a velocity increment of 1 mile per second at the circular orbit is sufficient to yield a velocity increment of 3.9 miles per second at infinity. This is a particular example of a general principle which states that *it is always more economical to expend propellant at points near to a center of attraction than at points which are more distant.*

Having escaped from the Earth's field with a velocity of 3.9 miles per second relative to the Earth, by adding vectorially to this velocity the Earth's velocity in its orbit (see Appendix XIV for this and other planetary velocities), we find the rocket's velocity relative to the Sun. This is the velocity of entry of the rocket into the Sun's field, which now dominates its motion. This velocity of entry will generally be considerably less than the velocity of escape from the Sun's field, and hence the rocket will proceed to move in an elliptical orbit with the Sun at one focus. Eventually the rocket will arrive at the point where the elliptical orbit intersects the orbit of the destination planet, say Mars. Provided the instant of departure was chosen appropriately, Mars will also be in the vicinity of this point at this time and will commence to attract the spaceship. Knowing the velocities of the rocket and of Mars relative to the Sun, the rocket's velocity relative to Mars, whilst it is still at a great distance from this body, can be found. The order of events during the approach to Mars will be the reverse of the order experienced during the escape from the Earth. The rocket will fall toward Mars along a hyperbolic orbit as seen by an observer on that planet, and, when a certain height above the Martian surface has been reached, the motors will be energized to deliver an impulsive thrust opposing the motion and reducing the rocket's velocity to circular velocity at this height. The ship will now settle into a circular orbit and will remain at a constant height above the planet's surface.

If we approximate the planetary orbits by circles about the Sun as center, and suppose further that these circles are coplanar, the problem of the optimal transfer of a rocket between any two planets possesses a particularly simple solution. The orbit of transfer is then an ellipse tangential to both the

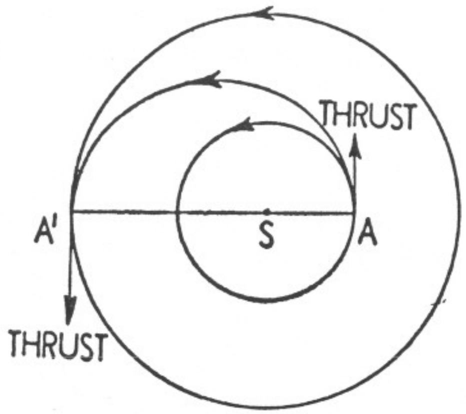

Figure 17. Orbit of transfer

circular orbits at the two ends of its major axis A and A′ (Figure 17) and with the Sun S at one of the foci. If the rocket is initially rotating in a circular orbit about the inner planet, an impulsive thrust is applied of such a magnitude that it recedes from the planet along a hyperbolic arc and arrives at a great distance with the velocity necessary to ensure its entry into the elliptical transfer orbit. The ship coasts along this orbit from A to A′ under the attraction of the Sun alone and, provided the instant of departure was correctly chosen, its arrival at A′ will coincide with the arrival of the destination planet at this point.

The sum of the two velocity changes caused by the impulsive thrusts applied at A and A′ is the characteristic velocity for the transfer. Values taken by this quantity for transfers between various pairs of planets are given in Table 14, the units

being miles per second. It is assumed that the circular orbits about the two planets are as close as possible to the planetary surfaces without being inside the planetary atmospheres (if any). The time of the transfer in days is also shown below each characteristic velocity.

TABLE 14

	Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus
Venus	6·1 76						
Earth	8·4 106	4·3 146					
Mars	10·9 171	4·4 218	3·6 259				
Jupiter	21 853	16 932	15 998	13 1,130			
Saturn	17 2,020	12 2,120	11 2,210	10 2,830	17 3,650		
Uranus	15 5,590	10 5,740	9 5,860	8 6,090	15 7,770	11 9,940	
Neptune	15 10,900	11 11,000	9 11,200	9 11,500	16 13,500	11 16,100	8 22,300

It will be observed that a transfer between circular orbits about the Earth and Mars corresponds to the smallest characteristic velocity, 3.6 miles per second. Assuming a motor jet velocity of 2 miles per second, $V/c = 1.8$ and entering Table 13 we find that this implies a mass ratio for the maneuver of about six. This value is at about the upper limit of practicability for a single-step rocket. If, however, the rocket is to carry a crew, it is necessary to allow for a return journey having the same characteristic velocity. A two-step rocket, designed to have a mass ratio of six for each stage, would be able to make the round trip. The first stage would be employed on the outward journey, and would be left in circular orbit about

Mars. The second stage would then return the crew to Earth. The expedition would be forced to spend a period of about 452 days in circular orbit about Mars, awaiting the time when the two planets are once again in suitable positions for the execution of this maneuver. The total period of time spent away from Earth would therefore be 970 days, or 2 years 8 months.

The periods of transit for transfers between the Earth and the outer planets are so great that the cotangential ellipse is unlikely ever to be employed for this purpose. Instead, nonoptimal paths involving larger characteristic velocities but shorter periods of transit will have to be followed and, until much higher exhaust velocities become available (e.g., by the harnessing of nuclear energy for rocket motor drives), such journeys will not be possible.

From what has been said, it will be appreciated that escape to infinity from a circular orbit about an attracting body is a maneuver of basic importance. It has been assumed that escape will be effected by the application of a single impulsive thrust which sets the rocket into a hyperbolic orbit. The magnitude of the thrust must be so chosen that the rocket arrives at a great distance from the planet with the velocity necessary for entry into the orbit of transfer. For most purposes this will constitute the optimal mode of escape, provided the direction of thrust is tangential to the circular orbit (Figure 18(a)). If, however, the velocity requirement at infinity is larger than a certain critical value (as, for example, it might be in the case of a nonoptimal transfer between the Earth and one of the outer planets), a more complex maneuver is found to save propellant (see Reference 4). An impulsive thrust, directly opposing the motion, is first applied. The consequent reduction in velocity causes the ship to fall toward the center of attraction along an elliptical arc with a focus at this center (Figure 18(b)). At the point of closest approach (known as *perigee* if the planet is Earth) to the center, the

motors are again energized and a thrust is applied in the direction of motion to increase the rocket velocity to a value in excess of the escape velocity at this point. The ship now recedes to infinity along a hyperbola, and the maneuver has been completed. It is found that the closer the approach that can be

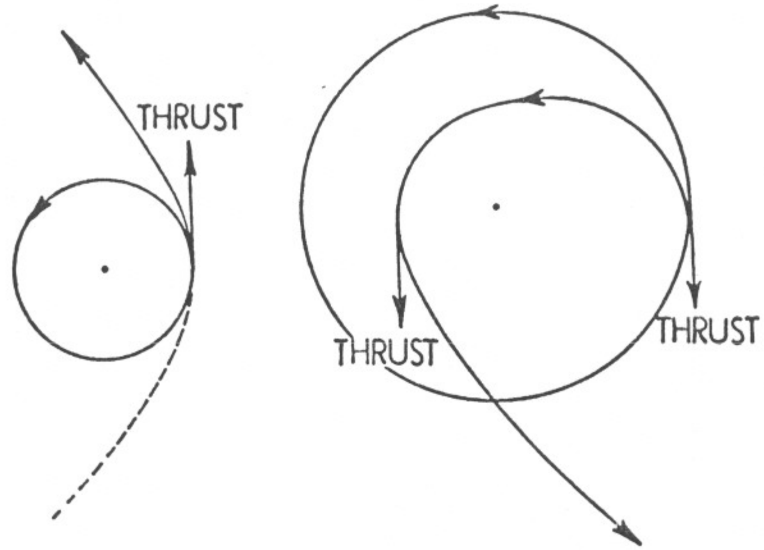

Figure 18. (a) Simple escape orbit, (b) Fuel-saving orbit

made to the center of attraction, the more economical this maneuver becomes. In any particular case, the point of closest approach must lie outside the atmosphere of the planet in question, and this limits the economy which can be achieved by the maneuver. We have here a further example of the superior effect of a thrust applied at a point close to a center of attraction as compared with that of a thrust applied at a more distant point.

Consider now the projection of a rocket from a launching point on the Earth's surface into a circular orbit. It appears that a compromise will have to be effected between too rapid an initial rate of fuel expenditure and the opposite extreme. If the thrust over the early part of the trajectory is large, the velocity of the rocket when it is passing through the denser layers of the atmosphere will also be large, and much energy will be wasted overcoming the resulting high air resistance. If, on the other hand, the thrust is small, the period of acceleration will be long and the velocity loss due to the operation of gravity over this period will be excessive. In the particular case of a rocket rising vertically to achieve maximum height, it is found[5] that the thrust must be adjusted so that during the operation of the motor the rocket velocity is always λc, where c is the exhaust velocity and λ satisfies

$$\lambda^3+\lambda=\frac{\text{rocket weight}}{\text{air resistance at jet speed}}. \qquad ..(8)$$

This equation is exact, provided that the air resistance is proportional to the air density and to the square of the velocity. The denominator of the right-hand member of equation (8) is the air resistance which would act upon the rocket if it were to move at velocity c in the atmosphere through which it is passing at the instant under consideration. The velocity appropriate to the launching point is not zero and must be attained, theoretically instantaneously, by application of an impulsive thrust. In practice, a booster would be employed, this falling away as the rocket leaves its launching tower. The numerator of the right-hand member of equation (8) is proportional to the cube of the rocket dimensions, whereas the denominator is only proportional to the square. It follows that this fraction increases in value with increase in the size of the rocket. For a large rocket of the size necessary to achieve circular velocity, this fraction, and therefore λ also, becomes so large that the optimal thrust program cannot be put into effect. Under these circumstances, maximum height is achieved by operating the motor at maximum thrust until all propellant has been consumed. Similarly, if the object is to attain

circular velocity, maximum thrust should be delivered by the motor throughout the period of its operation. There then remains the problem of programing the thrust direction.

A means of determining the solution to this problem has been given.[6] It is found that the thrust must be so directed that the rocket first rises vertically until the denser layers of the atmosphere have been cleared, when the trajectory bends

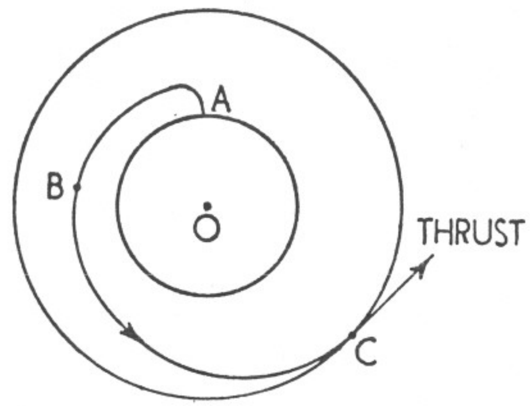

Figure 19. Trajectory to reach circular orbit

until it runs parallel to the Earth's surface. The rocket continues to accelerate along this arc until it has acquired sufficient momentum to be able to coast along an elliptical arc to the level of the circular orbit. In Figure 19, B is the point at which the motor cuts out and BC is the null-thrust arc with a focus at the Earth's center, O. The ellipse must be tangential to the circular orbit at C. Upon arrival at C, a short burst from the motor converts the rocket's velocity into circular velocity. The rocket has then been established in its orbit.

Finally in this section we will explain how propellant economies may sometimes be achieved by the execution of *perturbation maneuvers*. In this type of maneuver the transfer of a ship between two planets is facilitated by utilization of the attraction of a third body, such as a moon of one of the planets, for the rocket. This attraction may be arranged to cause a velocity change contributing to the characteristic velocity for the journey so that only a fraction of this velocity has to be provided by actual expenditure of propellant.

Thus, suppose a rocket, outward bound for Mars, escapes from a circular orbit about the Earth with velocity v_0 (Figure

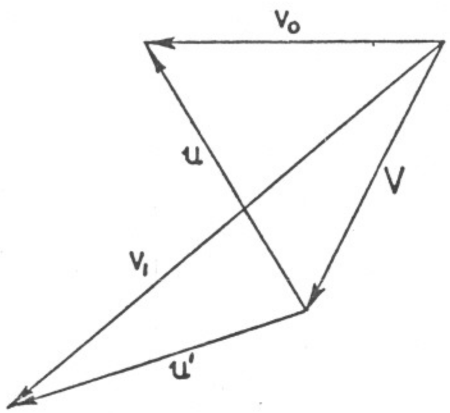

Figure 20. Velocities in perturbation maneuver

20) relative to this body. Let V be the velocity of the Moon. Then, if it is arranged for the rocket to pass close to this body, as seen by an observer on the Moon, it will approach from infinity with velocity $u = v_0 - V$. This observer will see the rocket first approach along one arm of a hyperbola and later recede along the other arm. The two arms of a hyperbola are mirror images of one another and, apart from direction, the motions along them are identical. The velocity of recession from the Moon is accordingly u', a vector having the same magnitude as u but a different direction. Relative to the Earth, the rocket's velocity is now $v_1 = u' + V$. It will be noted from the figure that v_1 can be of any magnitude greater

than v_0. Thus, the maneuver has resulted in a velocity increment without any expenditure of propellant.

Similarly, at Mars, Deimos or Phobos might be employed to yield a second velocity increment, thus reducing the propellant expenditure necessary at this terminal to place the rocket in its circular orbit.

The way in which maximum advantage may be taken of the attractions of perturbing bodies has not yet been investigated.

Determination of position and velocity. So long as a spaceship remains within the Solar System, the directions of the stars will not alter appreciably on account of their immense distances. These directions have been catalogued so that by observing any member of the Solar System against the stellar background and noting its angular distances from certain reference stars, the direction of this member as seen from the ship can be very accurately determined. This also fixes the direction of the ship from this body. Suppose by this means we determine the direction of the ship from two members, P_1, P_2, of the Solar System. Since the positions of P_1 and P_2 will be known, we can draw straight lines from P_1 and P_2 in the directions calculated, and these must intersect at the position of the ship. This is the principle of the calculation of the ship's position from observations made on any of the planets, the Sun or other members of the Solar System.

Alternatively, additional information could be obtained by the use of standard radar techniques. A pulse of radio waves transmitted from the Earth could be received at the ship and retransmitted, after a known short delay, back to Earth. The interval between transmission and reception of the pulse could be accurately measured on the Earth, and, the velocity of radio waves being known, quickly converted into the distance of the ship from the Earth. This information would then be radioed to the ship, where the direction of the ship from the Earth should already be available. The position of the Earth being known, that of the ship is then easily determined.

Since the frequency of the radio pulse received by the Earth from the ship would depend upon the ship's velocity along the line joining it to the Earth (Doppler principle), by careful measurement of this quantity, information concerning the rocket's velocity could also be made available. This could also be found by fixing the ship's position at the ends of measured intervals of time.

When space-travel is well established, radio responders might be placed on the Earth, Mars and Venus. These could be interrogated from any ship by transmission of a radio pulse of the correct frequency. The responders would receive and transmit this pulse back to the ship, where the distance of the responder could be found by the method already explained. The distances of the ship from the three planets being known, the ship's position is evidently calculable.

These methods of locating a ship in space become less accurate as the distances of the bodies observed become greater. Greatest accuracy will be achieved at the commencement and conclusion of an interplanetary journey, therefore, since there will then be planetary bodies relatively close and available for observation.

Correction of orbits. As a result of the observations described in the previous section, the orbit in which a spaceship is moving will be accurately known a few days after departure from a planet. In general, this orbit will differ appreciably from the track the ship should be following. The divergence between the actual and computed tracks will increase as time elapses, and a very small error in the direction or magnitude of the initial impulsive thrust may be magnified into an error of a million miles at the destination planet if left uncorrected. As soon as a divergence has been observed, therefore, steps

must be taken to make an early correction to the course of the ship.

Suppose that it is discovered that the orbit of the ship lies along the arc AP' (Figure 21) instead of the required arc AP. It is then necessary to compute the position, P', the ship

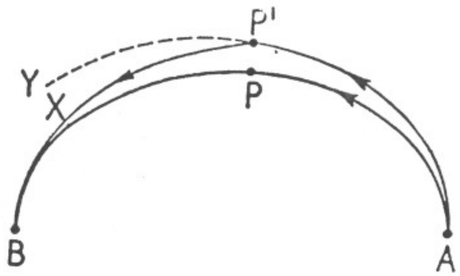

Figure 21. Correction of orbit

will occupy in a few hours' time, when the course correction is to be made. Let B be the next junction point at which an impulsive thrust is to be applied. Clearly, the ship must arrive at B at the previously calculated scheduled time, for otherwise it will fail to rendezvous with the destination planet. Knowing the time of arrival at P' and the scheduled time of arrival at B, the difference is the time of transit T from P' to B. It may be proved that there is but one orbit passing through points P' and B having the property that the time of motion along it is exactly T. This is shown in the figure as $P'XB$. Upon arrival at P', therefore, an impulsive thrust must be applied to the rocket to effect transfer into the orbit $P'XB$. This thrust will, in general, be very small, since the orbit $P'XB$ may be expected to differ but little from the continuation $P'Y$ of the orbit AP'. The velocity of arrival at B will, of course, be different from that originally computed, but this is easily allowed for when the impulsive thrust is applied at this junction.

The correction applied at P' will itself be subject to error, and hence other course corrections will have to be made later. To assist the navigator in the calculation of these corrections, tables will be prepared in advance, by a method which is fully explained in Reference 7.

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10

DIFFICULTIES OF SPACE NAVIGATION

J. G. Porter

Any forecast of the future trends of science is apt to fall short of the mark, and in a subject which is still in its infancy, the prophet is likely to be proved wrong more often than right. This is particularly true of space-travel, a subject which rouses some writers to a wild pitch of enthusiasm, while other authorities dismiss the whole idea as nonsense. One may, perhaps, be forgiven for taking a cautious attitude in attempting to envisage the triumphs and troubles of man's attempted conquest of space in the next few decades. In the immediate future we shall witness the launching of many artificial satellites of the Earth, and it is clear that the lessons which these will teach us will occupy our attention for many years to come. The next step may well be the launching of an unmanned rocket which will make a longer journey—still a satellite of the Earth, but this time able to reach the Moon. There can be no reasonable objection to the possibility of this form of space-travel, but the next step, in which the rockets are able to carry a crew, is an enormous advance. Each step, in fact,

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