Technical Notes

Launch Parameters for Interplanetary Flights

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IT IS assumed that the heliocentric transfer orbit from Earth to the target planet, both considered as massless points, is known. This may be determined by using two-body equations to describe transfer ellipses and an iterative process to match flight time with planet positions. The initial heliocentric velocity vector, then, is known. This is transformed to a geocentric vector by subtracting the velocity vector of Earth. The direction of this vector is called the target direction. The magnitude is the hyperbolic excess of the departure orbit, and the square of the magnitude is taken to be the energy of this orbit. Intervals of time and distance which occur between launch and the practical attainment of the vector are small compared to the interplanetary distances and times involved.

The development is carried out with the idea that there will be no powered change of plane. Hence, the missile plane relative to a nonrotating Earth is defined as that plane which contains the Earth-centered target direction vector and the launch site position vector at the instant of launch.[2] For any time of day such a plane is possible. With each plane a launch azimuth is associated. The relationship between launch azimuth and launch hour is obtained.

If departure is made from a circular coasting orbit, the point of entering the hyperbolic orbit is easily found, since, for tangential impulse, it will be the perigee of the hyperbola. The coasting orbit should be circular for greater efficiency. If the coasting orbit is elliptic, iteration is needed, since the elements of the hyperbola depend upon the point of departure which, in turn, depends upon the required energy and asymptote direction of the hyperbola. For the direct shot, iteration may again be needed, since the launch time, and therefore launch azimuth, will depend upon the burnout conditions of downrange, altitude and flight path, and these in turn depend to some extent upon the launch azimuth.

Method

The initial velocity vector of the heliocentric transfer orbit is referred to the center of Earth by subtracting Earth's velocity vector. Its direction now is the target direction from Earth. The expression of the vector may take various forms. A typical and convenient sequence of steps is the following

$$l_v = \frac{1}{v}\begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} \text{ ecliptic} \qquad [1a]$$

$$\begin{aligned} v_x &= (V \sin \Gamma - V_e \sin \Gamma_e) \cos w - \\ &\qquad (V \cos \Gamma \cos I - V_e \cos \Gamma_e) \sin w \\ v_y &= (V \sin \Gamma - V_e \sin \Gamma_e) \sin w + \\ &\qquad (V \cos \Gamma \cos I - V_e \cos \Gamma_e) \cos w \\ v_z &= V \cos \Gamma \sin I \\ v &= [V^2 + V_e^2 - 2VV_e (\sin \Gamma \sin \Gamma_e + \\ &\qquad \cos \Gamma \cos \Gamma_e \cos I)]^{1/2} \end{aligned}$$

.... [1b]

$$l_v = \frac{1}{v}\begin{pmatrix} v_x \\ v_y \cos \epsilon - v_z \sin \epsilon \\ v_y \sin \epsilon + v_z \cos \epsilon \end{pmatrix} \text{ equatorial} \qquad [2]$$

$$l_v = \begin{pmatrix} \cos A \cos \delta \\ \sin A \cos \delta \\ \sin \delta \end{pmatrix} \qquad [3]$$

Equating Equations [2 and 3] gives the right ascension and declination of the target direction.

A launch into a plane containing the target vector may be made at any time of day, the launch azimuth varying with launch hour. The relation between azimuth and launch hour may be obtained by equating the expression of the normal to a plane passing through the launch site with a given azimuth to that for a plane passing through the launch site and the target vector. It is

$$\cos \alpha = \frac{\sin (A - H) \cos \delta}{\sin \phi} = \frac{\sin (A - H) \cos \delta}{\{1 - [\cos (A - H) \cos l \cos \delta + \sin l \sin \delta]^2\}^{1/2}} \qquad [4]$$

Letting $\alpha = 0$ gives

$$\cos (A - H) = \tan \delta / \tan l \qquad [5]$$

Thus when $|l| \geqslant |\delta|$ a due east launch may be made.

In case $|l| < |\delta|$, the least $|\alpha|$ is found, from spherical trigonometry, to be obtained when

$$\cos (A - H) = \tan l / \tan \delta \qquad [6]$$

and then

$$\cos \alpha = \cos \delta / \cos l \qquad [7]$$

In both cases the plane is that having the smallest inclination to the Equator. In the first case the inclination is equal to the latitude of the launch site, and in the second case to the declination of the target vector. The launches would be into coasting orbits, since it is highly unlikely that other conditions would be right for a direct entry into the hyperbola.

The point of initiating the hyperbola may be found. The true anomaly of the asymptote is given by

$$\cos \eta_a = -1/e \qquad \tan \eta_a = -(e^2 - 1)^{1/2}$$

For the circular coasting orbit the injection point is the

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[2] This neglects the generally unavoidable deviations from this plane that occur during the powered launch phase and which are a result of Earth's rotation, guidance commands, aerodynamic effects, etc.

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perigee and thus is η_a back from the target vector. In this case

$$\tan \eta_a = -\frac{v}{v_c}\left(\frac{v^2}{v_c^2} + 2\right)^{1/2} \qquad [8]$$

In case an elliptic coasting orbit occurs, a tentative initial point is chosen. The flight path angle and radius of this point are determined from the orbital elements as found from the burnout conditions of the surface launch phase. Now

$$\tan \eta_a = -\frac{v}{v_c} \cos \gamma_i \left(\frac{v^2}{v_c^2} + 2\right)^{1/2} \qquad [9]$$

The true anomaly in the hyperbola of the initial point is found from

$$\tan \eta_i = \frac{(v^2/v_c^2 + 2) \sin 2\gamma_i}{2[(v^2/v_c^2 + 2) \cos^2 \gamma_i - 1]} \qquad [10]$$

If the selected point is at an angle u in the forward direction from the target vector, then it required that

$$u + (\eta_a - \eta_i) = 360 \text{ deg}$$

The process is repeated until a point is found for which this occurs.

In the case of the direct shot, kinematics and geometry are interrelated. The angle $(\eta_a - \eta_i)$ is found from Equations [9 and 10]. Adding to this the downrange angle of the power phase gives ϕ. H and α are then found from

$$\cos (A - H) = (\cos \phi - \sin l \sin \delta)/\cos l \cos \delta \qquad [11]$$

$$\cos \alpha = \sin (A - H) \cos \delta/\sin \phi$$

Since launch is from a rotating Earth, the power phase is affected by α, so that a cyclic iteration may be needed. Also, Equation [11] may give an imaginery $(A-H)$, so that the power phase may have to be revised to obtain a compatible ϕ. A low γ_i will usually require a large $|\alpha|$.

The hyperbolic injection velocity for all cases is given by

$$v_i = (v^2 + 2v_c^2)^{1/2}$$

$$\frac{v_i}{v_c} = \left(\frac{v^2}{v_c^2} + 2\right)^{1/2} \qquad [12]$$

The launch azimuth relative to a rotating Earth may be approximated by

$$\cot \alpha' = \frac{\cos \alpha - v_0/v_c}{\sin \alpha} \qquad [13]$$

The distance of the asymptote from the target vector is given by

$$\rho = r_i[1 + 2/(v/v_c)^2]^{1/2} \qquad [14]$$

As a simple example let $w = 0$, $\Gamma = \Gamma_e = 0$, $I = 1.5$ deg, $V/V_e = 1.137$, $\epsilon = 23.5$ deg, $l = 28.5$ deg, and let launch be into a 100-nautical mile altitude circular coasting orbit. From Equation [2]

$$l_v = \frac{V_c}{v}\begin{pmatrix} 0 \\ 0.1136 \\ 0.0819 \end{pmatrix}$$

Using Equation [3]

$$A = 90 \text{ deg} \qquad \delta = \tan^{-1}(0.0819/0.1136) = 35.8 \text{ deg}$$

From Equations [6 and 7] we find

$$(A-H) = \pm 41.1 \text{ deg}$$

$$H = \begin{Bmatrix} 90 \text{ deg} + 41.1 \text{ deg} = 131.1 \text{ deg} \\ 90 \text{ deg} - 41.1 \text{ deg} = 48.9 \text{ deg} \end{Bmatrix}$$

$$= \begin{Bmatrix} 8.73 \text{ hr sidereal} \\ 3.26 \text{ hr sidereal} \end{Bmatrix}$$

$$\alpha = \pm 22.8 \text{ deg}$$

Equation [13] gives $\alpha' = \pm 24$ deg. For minimum $|\alpha'|$ then, launch should be made at approximately 3.26 hr sidereal and 24 deg north of east, or at 8.73 hr sidereal and 24 deg south of east.

Using Equation [1b], v/v_c is equal to 0.535, and from Equation [8], η_a is equal to 141.1 deg. The perigee, therefore, lies 141.1 deg back from l_v in the missile plane. At this point, from Equation [12]

$$v_i/v_c = 1.514$$

From Equation [11] it is found that $\phi = 35.3$ deg. The angle from launch to hyperbolic initiation is

$$360 - 141.1 + 35.3 = 254.2 \text{ deg}$$

for the northeast launch, or

$$360 - 141.1 - 35.3 = 183.6 \text{ deg}$$

for the southeast launch.

From Equation [14], ρ is about 10,000 nautical miles or some 3 Earth-radii.

Nomenclature

l_v = target direction vector, Earth centered
v = velocity relative to Earth at initiation of planetary transfer, i.e., the hyperbolic excess velocity
V = velocity relative to the sun at initiation of planetary transfer
V_e = orbital velocity of Earth at time of initiation of planetary transfer
Γ = initial flight path angle of interplanetary orbit
Γ_e = path angle of Earth's orbit at initiation of interplanetary orbit
I = inclination to the ecliptic plane of the interplanetary orbit
w = longitude of Earth at the initiation of the interplanetary orbit
ϵ = inclination of the Equator
A = right ascension of the target vector
δ = declination of the target vector
α = launch azimuth relative to a nonrotating Earth, measured from due east, positive to north
α' = launch azimuth, but relative to a rotating Earth
H = sidereal hour angle of the launch site at instant of launch
l = latitude of launch site
η_a = true anomaly of the hyperbolic asymptote
e = eccentricity
v_c = local circular velocity
v_0 = Earth surface velocity
γ = flight path angle
ϕ = angle in the missile plane from launch site to target vector
ρ = distance from target vector to hyperbolic asymptote

Subscript i refers to the initiation of the hyperbola

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