THE DETERMINATION AND POTENTIALITIES OF ADVANCED FREE-FALL INTERPLANETARY TRAJECTORIES.

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INTRODUCTION

When an interplanetary space vehicle approaches a planet on a free-fall trajectory the gravitational influence of the planet can radically change the vehicle's trajectory about the sun. It is possible for such vehicles to take advantage of this influence by passing the planet on a precisely calculated trajectory which will place the vehicle on an intercept trajectory with another planet.

The determination of free-fall trajectories to several planets is essentially the famous unsolved n-body problem. Thus in order to calculate these trajectories certain simplifying assumptions must be made. The solutions obtained in this investigation were based upon the fundamental assumption: at any instant one and only one gravitating body influences the vehicle's motion. The primary goal of the theoretical portion of the study was to determine a trajectory in the vicinity of a passing planet which would enable the vehicle to pass out of its gravitational sphere of influence on a conic trajectory about the sun that will intercept another pre-determined planet. Thus it was assumed that the missions began and ended at the centers of massless planets. The initial conditions were given by specifying the order in which the vehicle was to rendezvous with the given planets $P_1 - P_2 - \cdots - P_n$ along with the launch date, T_1, and first planetary closest approach date, T_2. The solution was programmed for the IBM 7090, and calculations were carried out at the computer facilities of the University of California at Los Angeles and the Jet Propulsion Laboratory. The results of these calculations comprise the most significant portion of the study. The development of the mathematical techniques employed may be found in Ref. 1.

NUMERICAL RESULTS

The following examples depict typical advanced free-fall interplanetary trajectories of concern: (i) Earth-Venus-Earth; (ii) Earth-Mars-Earth; (iii) Earth-Venus-Mercury; (iv) Earth-Venus-Mars-Earth; (v) Earth-Venus-Mars-Earth-Mars-Venus-Earth.

These examples represent only a very small fraction of the total number of possible different types of advanced interplanetary free-fall trajectories having $n - 1$ planetary encounters. For example the total number of different types of advanced trajectories having only two planetary encounters of the form $P_1 - P_2 - P_3$ where $P_1 =$ Earth is 9^{3-1} or 81. In general, the total number of different trajectories of the form $P_1 - P_2 - \cdots - P_n$ having ${}_n - \text{v}$ planetary encounters is 9^n.

When the early numerical calculations were confirmed by elaborate integrating programs at the Jet Propulsion Laboratory, the Computing Facility at the University of California at Los Angeles, where the program was written, the first extensive numerical determination of these advanced trajectories began. These early calculations at UCLA not only proved the feasibility of such missions but also showed that in some cases they would become an economic necessity.

As the numerical calculations were stepped-up by also utilizing the computing complex at the Jet Propulsion Laboratory, three distinct types of advanced missions began to crystallize. These missions follow in a natural chronological order:

(1) Unmanned exploration of the inner planets by instrumented space vehicles
(2) Initial interplanetary missions by manned vehicles
(3) Interplanetary transportation networks to support manned bases on Venus and Mars.

Since short flight times and low launch energies are always desirable, the first category of missions must be of the form $P_1 - P_2 - P_3$ where $P_1 =$ Earth, $P_2 \neq P_3$ and are either Mercury, Venus, or Mars, e.g., Earth-Mercury-Venus, Earth-Mercury-Mars, Earth-Venus-Mercury, Earth-Venus-Mars, Earth-Mars-Mercury, Earth-Mars-Venus.

The first two possibilities were immediately eliminated because launch energies required for Earth-Mercury transfers are very high. The last two possibilities were found to either require high launch energies or long flight times. It was discovered that the Earth-Venus-Mercury trajectories required considerably less launch energies than the direct flight Earth-Mercury trajectories. These advanced trajectories permitted the payload

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weight reaching Mercury to be increased by over 100% during the decade 1965 through 1974. Of course these trajectories do require greater flight times than the direct flight Earth-Mercury trajectories, but in some cases these flight times were found to be less than those required for direct flight Earth-Mars trajectories.

Now missions to Mars on conventional Earth-Mars trajectories can take place only during launch periods of a few weeks duration. These favorable launch periods are separated by about 780 days (the "synodic period of Mars"). This represents a definite time barrier for trips to Mars. If a mission is not successful, it is necessary to wait about 780 days until the next launch opportunity occurs. Favorable Earth-Mars launch periods will occur in 1969, 1971, and 1973. It was discovered that the Earth-Venus-Mars trajectories for 1970 and 1972 have low launch energies and relatively short flight times. Thus by utilizing these trajectories the three launch opportunities for missions to Mars can be increased to five: 1969, 1970, 1971, 1972, and 1973.

Of course, advanced trajectories of the form $P_1 - P_2 - \cdots - P_n$ will require highly accurate planetary approach guidance. These guidance systems on the other hand will not require any significant scientific breakthroughs. The guidance systems now being developed for the Apollo moon mission could perhaps meet these high guidance requirements.

Table 1 contains some important characteristics of near minimum launch energy direct flight Earth-Mercury and Earth-Venus-Mercury trajectories. Table 2 is a similar table of Earth-Mars and Earth-Venus-Mars trajectories. All of the tables in this paper shall adhere to the following notation:

HEV_{k} = hyberbolic excess velocity (km/sec) at P_{k}

$T_{\text{k, k+1}}$ = time taken by vehicle to pass from P_{k} to $P_{\text{k+1}}$

$\theta_{\text{k, k+1}}$ = heliocentric angle swept out by the vehicle passing from P_{k} to $P_{\text{k+1}}$

$TISI_{\text{k}}$ = amount of time (days) vehicle spends in P_{k}'s sphere of gravitational influence

$DOCA_{\text{k}}$ = distance of closest approach (km) to P_{k}'s surface

$VACA_{\text{k}}$ = velocity at closest approach (km/sec) to P_{k}

DA_{k} = angular difference between the vehicle's velocity vectors as it enters and leaves P_{k}'s sphere influence

TFT = total flight time (days)

If some symbols do not have subscripts it will be understood that $_{\text{k}} = 2$. In discussing the launch energy of a particular trajectory reference is made to the trajectory's vis-viva energy which is simply the square of the hyperbolic excess velocity. These quantities however shall be omitted from the tables. The vis-viva energy is approximately proportional to the actual launch energy from a high Earth parking orbit.

An examination of Table 1 clearly shows that the launch energies required for direct flight Earth-Mercury missions requires approximately 3 to 4 times as much launch energy as those required for Earth-Venus-Mercury missions. Also notice that the Mercury approach energies for the direct Earth-Mercury trajectories are 4 to 6 times higher than those associated with the Earth-Venus-Mercury trajectories. This will be an important consideration for missions requiring orbiting payloads about Mercury.

Table 2 shows that the Earth-Venus-Mars trajectories very neatly fill the gap between the 1969 and 1971 Earth-Mars launch opportunties and the 1971 and 1973 launch opportunities.

Consider possible applications of these multiple planetary encounter free-fall trajectories to early manned interplanetary flight. The simplest of all such trajectories applicable for manned vehicles are Earth-Venus-Earth and Earth-Mars-Earth. Table 3 contains some important properties of near minimum launch energy Earth-Venus-Earth and Earth-Mars-Earth trajectories. Observe that the Earth-Venus-Earth trajectories are much more reasonable than the Earth-Mars-Earth trajectories. The Earth-Mars-Earth trajectories could be made shorter by increasing the launch energy, but this increase is of the order of 200 to 400%. Consequently, with respect to the manned planetary reconnaissance missions another time-energy barrier situation is encountered. A non-nuclear Saturn 5 launch vehicle could probably be used as the booster for Earth-Venus-Earth manned reconnaissance missions but not for the Earth-Mars-Earth missions.

During the early part of June 1962 while the author was checking some newly calculated multiple planetary trajectories of the form Earth-Venus-Mars-Earth, a very remarkable fact was discovered. It was already known at that time that the minimum energy Earth-Mars-Earth trajectories required very long flight times. Thus, it was believed that the most favorable Earth-Venus-Mars-Earth manned reconnaissance trajectories would have flight times much greater than 1000 days. The fact is, however, that this is not always true. It was discovered that in some cases this assumption was false by a very wide margin. These cases most conveniently turned out to be the 1970 and 1972 Earth-Venus launch periods.

Table 4 contains a description of some important characteristics of these Venus-Mars reconnaissance trajectories. These trajectories show that by employing Earth-Venus-Mars-Earth trajectories instead of the Earth-Mars-Earth reconnaissance trajectories, the flight times can be greatly reduced. These important trajectories with low departure and arrival hyperbolic excess velocities. Consequently, a great deal of life support equipment will be necessary to transport a few persons from one planet to another. In short, cargo vehicles shall probably be robot type vehicles carrying no equipment necessary for manned flight, while the manned vehicles will probably carry very limited amounts of cargo. In addition to carrying all the extra equipment for manned flight these vehicles will also probably be required to be able to induce some artificial gravity. Hence, the transportation of just ten men, for example, from Mars to Earth should ordinarily require a large and very expensive rocket. Methods of recovery will become a necessity. This problem of economics can be conveniently solved by constructing a long-lasting interplanetary transportation network designed for the sole purpose of transporting personnel from one planet to another.

Preliminary calculations have shown that if the planets P_i are restricted to Mercury, Venus, Earth, and Mars where P_i = Earth and $P_i \neq P_{i+1}$ for $i = 1, 2, \cdots, n$ it is possible to find sequencies $P_1 - P_2, \cdots, - P_n$ such that the flight times $T_{i+1} - T_i$ are comparable to those required for optimum $P_i - P_{i+1}$ transfers. Moreover some of these trajectories were found to have very low launch energies.

The network can be established by first constructing many large space vehicles which are to be used in the transportation system. This can be done by methods of prefabrication and orbital assembly. These vehicles can be designed to accommodate 20 to 60 persons and since artificial gravity will be highly desirable, the geometry of the vehicles could be a torus with an outside diameter of perhaps 200 to 300 feet. When each individual vehicle is completed one simply awaits its launch date, T_1, when the vehicle (i.e., space bus) is injected into its prescribed interplanetary trajectory. The vehicle will carry extra provisions and life support equipment to last until it makes its first Earth rendezvous, whereupon its supply of provisions and life support equipment can be replenished to last until it makes its second Earth rendezvous, etc. As a vehicle approaches a planet, P_i, a small excursion module orbiting P_i and containing a few men wishing to go to P_{i+1} is injected onto an intercept trajectory with the space bus. Upon making a rendezvous the excursion module containing very accurate planetary approach guidance systems could be left a few miles behind the space bus to be used to transport the men from the space bus onto an orbit about P_{i+1}. A tanker vehicle following the space bus could be used to refuel the planetary excursion modules. Other transportation systems could be established on each separate planet for the purpose of bringing the men from the circular orbit down to the planet's surface. Tanker vehicles orbiting each planet in the system could refuel the excursion modules which never actually land on a planet. Many of these vehicles could be on different trajectories at the same time. Table 5 is an example of a trajectory which might be used for this system.

There remains one very important aspect which must be considered. This involves the question of accuracy. Just how good is the assumption that one and only one body influences the vehicle's motion at any given time? The Jet Propulsion Laboratory has a program such that if the geometry of the trajectory near P_i is sufficiently close to the exact trajectory required for the vehicle to intercept P_{i+1}, an integration, interation process is begun by first determining the miss vector at P_{i+1} resulting from the initial approximation and then modifying it etc., until the miss vector becomes smaller than a specified amount. This program is highly unstable in the sense that if the initial trajectory in the vicinity of P_i is not extremely close to the real one required to permit the vehicle to rendezvous with P_{i+1}, the iteration process will not converge. It has been observed that each time this program was employed to check the trajectories resulting from the solution given in this paper, very rapid convergence resulted. This means that the fundamental assumption must not give trajectories very far from those which would occur in the real situation.

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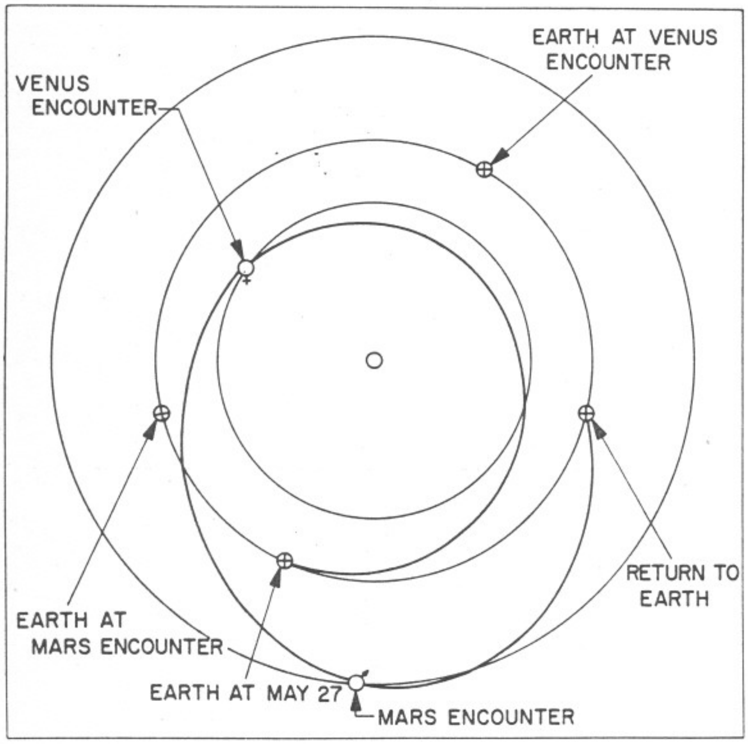
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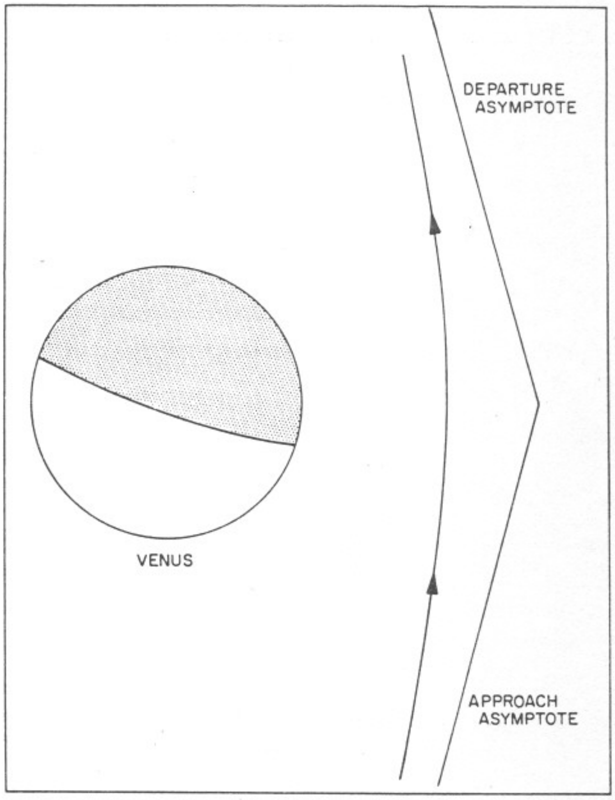
VENUS ENCOUNTER
RETURN TO EARTH
EARTH AT MARS ENCOUNTER
EARTH AT MAY 27
MARS ENCOUNTER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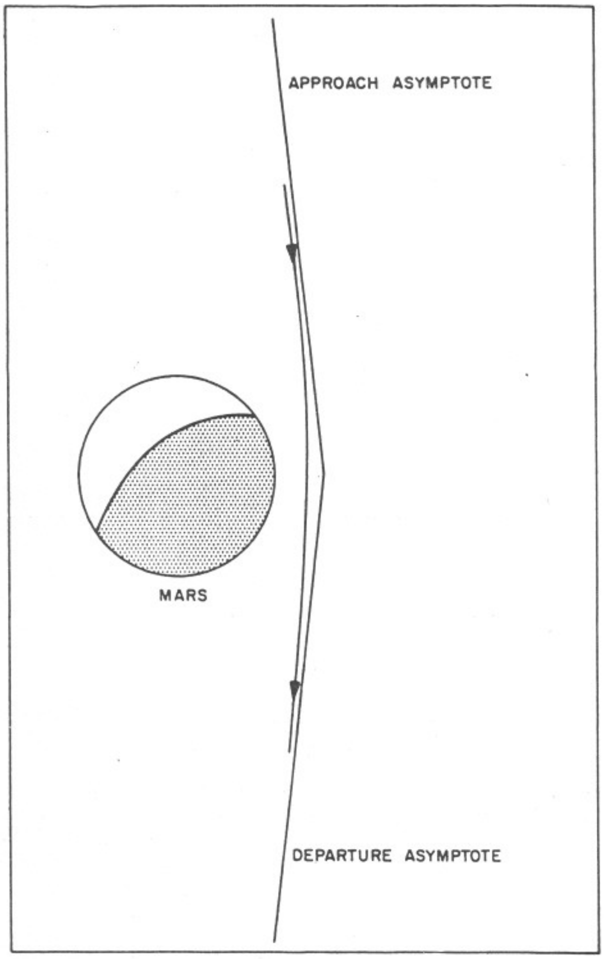
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TABLE 1

LAUNCH DATE	HEV_1	T_{12}	Θ_{12}	HEV_2	TISI	DOCA	VACA	DA	T_{23}	Θ_{23}	HEV_3	TFT	TRAJECTORY PROFILE
1/20/65	7.20	98.00	137.57	16.39	-	-	-	-	-	-	-	98.00	Earth-Mercury
12/18/65	3.97	170.17	249.76	6.86	2.01	1560.	11.48	56.57	105.62	227.95	9.83	275.43	Earth-Venus-Mercury
1/3/66	6.86	98.00	144.55	16.44	-	-	-	-	-	-	-	98.00	Earth-Mercury
12/11/66	6.58	104.00	157.36	16.21	-	-	-	-	-	-	-	104.00	Earth-Mercury
6/19/67	3.70	96.28	107.84	6.56	2.09	311.	12.01	65.49	71.49	190.36	9.25	167.77	Earth-Venus-Mercury
11/21/67	6.46	106.00	169.50	15.15	-	-	-	-	-	-	-	106.00	Earth-Mercury
11/12/68	6.85	102.00	175.42	12.84	-	-	-	-	-	-	-	102.00	Earth-Mercury
1/23/69	3.76	189.38	256.88	6.39	2.14	645.	11.71	65.57	107.67	214.64	9.79	279.05	Earth-Venus-Mercury
10/3/70	6.65	116.00	216.75	13.02	-	-	-	-	-	-	-	116.00	Earth-Mercury
8/18/70	3.50	101.22	110.31	7.59	1.82	3768.	11.11	42.68	59.00	145.66	11.91	160.22	Earth-Venus-Mercury
1/31/71	7.50	96.00	129.16	17.04	-	-	-	-	-	-	-	96.00	Earth-Mercury
1/12/72	7.05	98.00	138.06	17.13	-	-	-	-	-	-	-	98.00	Earth-Mercury
4/1/72	4.03	196.58	265.01	7.30	1.88	521.	12.30	57.27	85.00	151.67	14.40	281.58	Earth-Venus-Mercury
12/19/72	6.74	104.00	150.99	16.94	-	-	-	-	-	-	-	104.00	Earth-Mercury
11/4/73	4.25	93.79	103.34	8.10	1.70	2983.	11.71	41.31	58.00	137.83	11.14	151.79	Earth-Venus-Mercury
12/2/73	6.49	106.00	163.25	15.73	-	-	-	-	-	-	-	106.00	Earth-Mercury

TABLE 2

LAUNCH DATE	HEV_1	T_{12}	Θ_{12}	HEV_2	TISI	DOCA	VACA	DA	T_{23}	Θ_{23}	HEV_3	TFT	TRAJECTORY PROFILE
2/28/69	2.97	180.00	141.11	-	-	-	-	-	-	-	5.05	180.00	Earth-Mars
8/12/70	3.26	129.28	151.68	5.47	2.45	3850.	9.76	62.87	180.00	173.01	6.75	309.28	Earth-Venus-Mars
5/19/71	2.84	205.00	155.03	-	-	-	-	-	-	-	2.82	205.00	Earth-Mars
5/21/72	4.03	172.00	257.62	8.29	1.66	966.	12.67	47.26	117.70	112.77	12.61	289.70	Earth-Venus-Mars
7.27/73	3.80	195.00	144.04	-	-	-	-	-	-	-	2.99	195.00	Earth-Mars

TABLE 3

LAUNCH DATE	HEV_1	T_{12}	Θ_{12}	HEV_2	TISI	DOCA	VACA	DA	T_{23}	O_{23}	HEV_3	TFT	TRAJECTORY PROFILE
8/20/70	2.92	114.00	132.23	5.44	2.46	728.	11.29	76.16	250.96	227.48	7.13	364.96	Earth-Venus-Earth
6/8/71	3.97	316.00	204.84	4.21	3.23	2251.	5.74	34.88	795.83	530.43	5.85	1111.83	Earth-Mars-Earth
4/3/72	3.69	114.00	134.07	5.02	2.69	3983.	9.47	68.29	260.95	235.43	8.18	374.95	Earth-Venus-Earth
8/20/73	4.60	236.00	151.68	2.53	5.46	7024.	3.83	46.06	790.72	502.25	6.56	1027.70	Earth-Mars-Earth
11/4/73	3.76	116.00	139.29	4.47	2.94	5344.	8.76	71.80	269.46	241.00	7.90	385.46	Earth-Venus-Earth

TABLE 4

EARTH-VENUS-MARS-EARTH

LAUNCH DATE	HEV_1	T_{12}	Θ_{12}	$DOCA_2$	DA_2	T_{23}	Θ_{23}	$DOCA_3$	DA_3	T_{34}	Θ_{34}	HEV_4	TFT
8/12/70	3.26	129.28	151.68	3848.	62.87	180.00	173.01	6590.	9.89	312.36	290.86	9.34	621.63
5/27/72	4.16	170.16	258.61	6552.	30.01	141.94	121.74	1249.	13.40	157.59	79.72	13.04	469.68

TABLE 5

	i = 1 EARTH	i = 2 VENUS	i = 3 MARS	i = 4 EARTH	i = 5 MARS	i = 6 EARTH	i = 7 VENUS	i = 8 EARTH
T_i	8/14/70	12/20/70	6/17/71	4/25/72	12/11/73	5/31/74	3/26/75	7/30/75
HEV_i	3.28	5.44	6.74	9.34	13.46	11.81	10.83	7.91
$T_{i,i+1}$		128.20	179.30	312.22	595.03	171.58	298.37	126.26
$\Theta_{i,i+1}$		151.25	171.65	290.81	203.10	191.74	196.13	220.57
$DOCA_i$		3817.	6838.	8089.	592.	6249.	9455.	
$VACA_i$		9.75	7.34	11.93	14.23	14.23	12.61	
$TISI_i$		2.47	1.85	2.26	.97	1.82	1.29	
DA_i		63.44	9.69	27.79	6.41	21.27	17.39	

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