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Preface

During the spring vacation of 1963, the Graduate Academy of the University of California had its inaugural meeting on the Los Angeles campus. Sponsored by the UCLA Graduate Student Association, in concert with the several graduate organizations and divisions of the University, the Academy will annually present research of uncommon merit by graduate students from the various departments and campuses of the University.

It is the aim of the Academy to bring together young University of California graduate scholars from diverse disciplines, working on the same topic from differing points of view, and thus promote intercampus and interdisciplinary exchange.

The *Proceedings* will in most instances be the first published efforts of these young scholars. But immodestly we would like to note that of the twenty-six papers originally prepared for the Academy and included in this first volume of the *Proceedings*, thirteen have subsequently been published in scholarly journals, and two have won prestigious awards.

MICHAEL A. MINOVITCH
Los Angeles Campus

The Determination and Potentialities of Advanced Free-Fall Interplanetary Trajectories

When an interplanetary space vehicle approaches a planet on a free-fall trajectory the gravitational influence of the planet can radically change the vehicle's trajectory about the sun. It is possible for such vehicles to take advantage of this influence by passing the planet on a precisely calculated trajectory, which will place the vehicle on an intercept trajectory with another planet. Of course, these advanced trajectories taking a vehicle from one planet to another will in general require very long flight times. There are however some advanced trajectories involving Mecury, Venus, Earth, and Mars which have been found to have remarkably short flight times and low launch energies. Since the favorable launch periods for missions to a given planet do not occur often, advanced trajectories taking one free-fall vehicle to many planets are particularly attractive.

The determination of free-fall trajectories to several planets is essentially the famous unsolved n-body problem. Thus in order to calculate these trajectories certain simplifying assumptions must be made. This paper is based upon one fundamental assumption: At any instant one and only one gravitating body influences the vehicle's motion. The primary goal of the theoretical part of this paper is to determine a trajectory in the vicinity of a passing planet which will enable the vehicle to pass out of its gravitational sphere of influence on a conic trajectory about the sun which will intercept another predetermined planet. Thus we shall assume that the missions begin and end at the centers of massless planets. The initial conditions are given by specifying the order in which the vehicle is to rendezvous with the given planets $P_1 - P_2 - \cdots - P_n$ along with the launch date T_1 and first planetary closest approach date T_2.

This paper also includes many important trajectories discovered at the Computer Facility of the University of California, Los Angeles, and at the computing complex at the Jet Propulsion Laboratory.

Conic Trajectories

The quantitative study of any branch of science dealing with forces, velocities, and positions (i.e., vectors) in a 3-dimensional space is always most conveniently done by making use of as much vector analysis as possible. With this point of

* This paper won first prize in the Ph.D. class in a recent contest sponsored by the Western Division of the American Institute of Aeronautics and Astronautics. An enlarged version is being published by the California Institute of Technology's Jet Propulsion Laboratory (Technical Report No. 32–464).

view, no assumptions regarding the geometry of the solar system will be necessary; indeed it will not matter how eccentric the planets' orbits are or how much their planes of motion differ from each other. Thus before attacking the above problem we must first develop a convenient mathematical technique for handling conic trajectories in 3-dimensional space.

We begin by equating the dynamic force on a space vehicle of mass m with the force of gravitational attraction set up by the presence of a body of mass M. If Σ is any inertial frame this equation becomes

$$m\,\frac{d\vec{V}}{dt} = -G\,\frac{Mm}{R^2}\,\hat{R}$$

where $\vec{V}$ is the velocity of the vehicle and $\hat{R}$ is a unit vector directed from the center of the body to the vehicle separated by a distance R. We shall adhere to the convention of denoting unit vectors by placing $\wedge$ over the letter.

Since the ratio m/M is very small we may assume that the body is at rest in Σ. We shall take the center of this body as the origin of Σ. By setting $GM = \mu$ and cancelling out m from both sides of the above equation we obtain

$$\frac{d\vec{V}}{dt} = -\frac{\mu}{R^2}\,\hat{R} \tag{1}$$

The $\vec{e}$ and $\vec{h}$ vectors of conic trajectories

By the differentiation formula for the cross product of two vectors it follows from (1) that

$$\frac{d}{dt}(\vec{R} \times \vec{V}) = 0$$

which implies

$$\vec{R} \times \vec{V} = \vec{h} \tag{2}$$

where $\vec{h}$ is some constant vector of integration. From this simple but important relation we notice that $\vec{R}$ always remains perpendicular to $\vec{h}$ and consequently the motion remains confined to a fixed plane in Σ. Now

$$\vec{h} = \vec{R} \times \frac{d\vec{R}}{dt} = \vec{R} \times \frac{d}{dt}(R\hat{R}) = \vec{R} \times \left(\frac{dR}{dt}\,\hat{R} + R\,\frac{d\hat{R}}{dt}\right)$$

Thus

$$\vec{h} = R^2\hat{R} \times \frac{d\hat{R}}{dt}$$

and employing this result together with (1) we obtain

$$\frac{d\vec{V}}{dt} \times \vec{h} = \mu\,\frac{d\hat{R}}{dt}$$

Since $\vec{h}$ and μ are constants this equation can be written in the form

$$\frac{d}{dt}(\vec{V} \times \vec{h}) = \frac{d}{dt}(\mu\hat{R})$$

which implies

$$\vec{V} \times \vec{h} = \mu(\hat{R} + \vec{e}) \tag{3}$$

where $\vec{e}$ is another constant vector of integration. Since $\vec{V} \times \vec{h}$ lies in the plane of motion, $\vec{e}$ also lies in the plane of motion.

Let Θ be the angle measured from $\vec{e}$ in the positive direction (i.e., counterclockwise) to $\vec{R}$. In view of (2) and (3) we have

$$h^2 = \vec{h}\cdot\vec{R} \times \vec{V} = \vec{R}\cdot\vec{V} \times \vec{h} = \vec{R}\cdot\mu(\hat{R} + \vec{e})$$

Consequently

$$R = \frac{h^2/\mu}{1 + e\cos\Theta} \tag{4}$$

This is the general equation of a conic with eccentricity e and semilatus rectum ℓ

$$\ell = \frac{h^2}{\mu} \qquad R = \frac{\ell}{1 + e\cos\Theta} \tag{5}$$

Thus we obtain the well-known fact that the trajectory of the vehicle moving under the influence of a single gravitating body is a conic section.

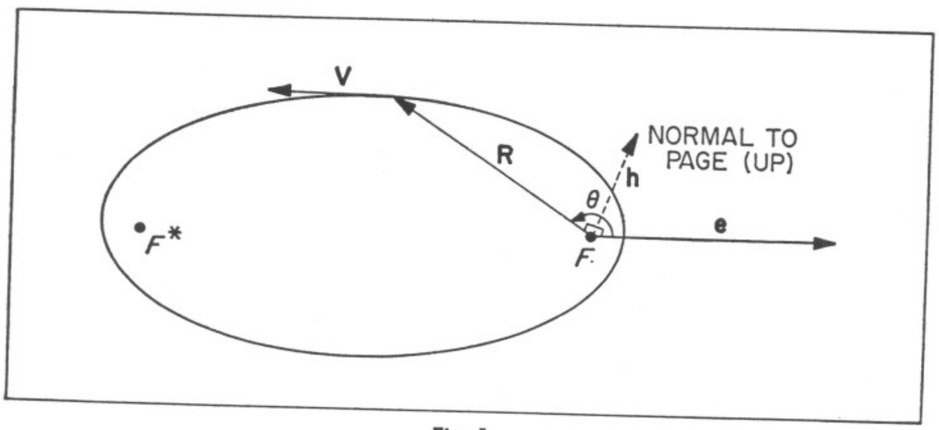

Fig. 1.

Since R is minimum when $\Theta = 0$, the direction of $\vec{e}$ is along the direction of perihelion (see fig. 1, where F and F^* denotes the occupied and vacant foci respectively).

Velocity vector as a fraction of $\vec{e}$, $\vec{h}$, and $\vec{R}$

We shall now derive a very useful formula that expresses the vehicle's velocity vector in terms of the $\vec{e}$ and $\vec{h}$ vectors and its unit position vector $\hat{R}$. By making use of the vector triple product formula it follows that

$$\vec{h} \times (\vec{V} \times \vec{h}) = (\vec{h}\cdot\vec{h})\vec{V} - (\vec{h}\cdot\vec{V})\vec{h}$$

But since $\vec{V}$ is perpendicular to $\vec{h}$ we obtain

$$h^2\vec{V} = \vec{h} \times (\vec{V} \times \vec{h})$$

and by employing (3) we have

$$\vec{V} = \frac{\mu}{h^2}\vec{h} \times (\hat{R} + \vec{e}) \tag{6}$$

As an immediate application of (6) we derive the well-known energy equation.

$$V^2 = \frac{1}{\ell}(\vec{V}\cdot\vec{h} \times \hat{R} + \vec{V}\cdot\vec{h} \times \vec{e})$$

$$= \frac{1}{R\ell}[\vec{h}\cdot\vec{R} \times \vec{V} + R(\vec{e}\cdot\vec{V} \times \vec{h})]$$

By employing (2), (3), and (4) this expression becomes

$$V^2 = \mu\left(\frac{2}{R} \pm \frac{1}{a}\right) \tag{7}$$

where the negative or positive sign is chosen according to whether the trajectory is elliptic (in which case $\ell = a(1 - e^2)$) or hyperbolic (in which case $\ell = a(e^2 - 1)$).

Lambert's Theorem

We now come to a fundamental theorem of Celestial Mechanics known as Lambert's Theorem. This profound result will play an important role in the determination of interplanetary trajectories. The theorem states that the time T required for a body to move from a point P with position vector $\vec{R}_1$ to a point Q with position vector $\vec{R}_2$ depends only on $R_1 + R_2$ and the distance c between P and Q. The proof of this theorem can be readily found in any advanced text and hence we shall merely state the functional relationships

$$T = f(\vec{R}_1, \vec{R}_2; a) \tag{8}$$

where a denotes the semimajor axis of the conic. The eccentricity is expressible as another function of $\vec{R}_1$, $\vec{R}_2$ and a.

$$e = g(\vec{R}_1, \vec{R}_2; a) \tag{9}$$

The calculation of the $\vec{e}$ and $\vec{h}$ vectors from two position vectors

Suppose two position vectors $\vec{R}_1$ and $\vec{R}_2$ are known together with the time required for the vehicle to pass from $\vec{R}_1$ to $\vec{R}_2$. Then by Lambert's Theorem ((8) and (9)) the trajectory's semimajor axis a and eccentricity e can be calculated. The trajectory's $\vec{h}$ vector can easily be calculated from

$$\vec{h} = \pm \frac{\vec{R}_1 \times \vec{R}_2}{|\vec{R}_1 \times \vec{R}_2|}\sqrt{a\mu|1 - e^2|} \tag{10}$$

which follows immediately from (5) and the fact that $\vec{h}$ is normal to the plane of motion. The plus sign is chosen if $0 < \measuredangle\, \vec{R}_1\vec{R}_2 < 180°$ and the negative sign if $180 < \measuredangle\, \vec{R}_1, \vec{R}_2 < 360°$. Since $\vec{e}$ lies in the plane of motion there exists two scalors α and β such that

$$\vec{e} = \alpha\vec{R}_1 + \beta\vec{R}_2 \tag{11}$$

If we dot multiply each side of (11) by $1/\mu\ \vec{V}_1 \times \vec{h}$ and $1/\mu\ \vec{V}_2 \times \vec{h}$ where $\vec{V}_1$ and $\vec{V}_2$ denotes the vehicle's velocity vector at $\vec{R}_1$ and $\vec{R}_2$, respectively, and make use of (3) and (5) we find

$$\alpha = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{D} \qquad \beta = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{D}$$

where

$$b_i = \frac{\ell}{R_i} + e^2 - 1$$

$$a_{ii} = \ell$$

$$a_{ij} = \hat{R}_i \cdot \vec{R}_j + \ell - R_j$$

and

$$D = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

Using the Gravitational Influence of a Passing Planet

We now consider the problem of determining a trajectory in the vicinity of a passing planet such that its influence will enable the vehicle to rendezvous with another planet. Let Σ denote any cartesian inertial frame with the sun's center as its origin. Let Σ' be a parallel translation of Σ with new origin located at the center of a planet influencing the motion of the vehicle. Let τ denote the region of gravitational influence about the planet. It can be shown that τ can be taken as a spherical region with center at the planet's center and radius ρ^* given by

$$\rho^* = \left(\frac{m}{M}\right)^{2/5} R$$

where R is the distance between the sun of mass M and the planet of mass m.

We recall that the problem is defined by specifying $P_1 - P_2 - \cdots - P_n$ where P_1 is the launch planet and P_n the last planet to be encountered along with the launch date T_1 and time of closest approach T_2 to P_2.

The following notation shall be employed through this section:

(a) $\widehat{P_iP_{i+1}}$ = the elliptical transfer trajectory from P_i to P_{i+1}

(b) $\vec{R}_i(t)$ = position vector of P_i with respect to Σ at time t ($i = 1, 2, \cdots n$)

(c) $\vec{R}(t)$ = position vector of vehicle with respect to Σ at time t.

(d) $\vec{\rho}(t)$ = position vector of vehicle with respect to Σ' at time t

(e) $\vec{V}_i(t)$ = velocity vector of P_i with respect to Σ at time t

(f) $\vec{V}(t)$ = velocity vector of vehicle with respect to Σ at time t

(g) $\vec{V}'(t)$ = velocity vector of vehicle with respect to Σ' at time t

(h) T_1^*, T_2^* = time at which vehicle enters and leaves τ of P_2, respectively

(i) $a_1, \ell_1; a_3, \ell_3$ = semimajor axis and semilatus rectum of $\widehat{P_1P_2}$ and $\widehat{P_2P_3}$, respectively

(j) $\vec{e}_1, \vec{h}_1; \vec{e}_3, \vec{h}_3$ = $\vec{e}$ and $\vec{h}$ vectors of $\widehat{P_1P_2}$ and $\widehat{P_2P_3}$ respectively

(k) $a_2, \vec{e}_2, \vec{h}_2$ = semimajor axis and $\vec{e}$ and $\vec{h}$ vectors of hyperbolic passing trajectory in τ of P_2 with respect to Σ' (with respect to Σ_1 this trajectory in τ is not a conic and hence these quantities have no meaning)

(l) d_2 = distance of closest approach to the surface of P_2

(m) μ_2 = m_2G where m_2 is the mass of P_2

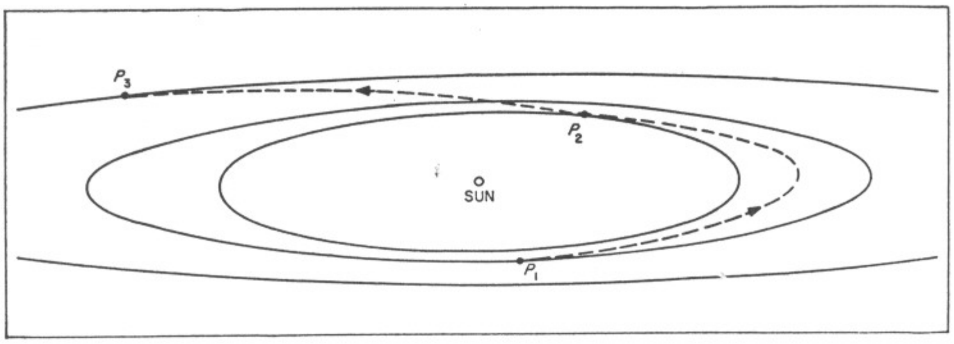

Fig. 2.

We shall determine the complete trajectory by first determining the trajectory in the vicinity of P_2 which will cause the vehicle to intercept P_3. We shall also determine $\widehat{P_1P_2}$ and $\widehat{P_2P_3}$ (see fig. 2).

The fundamental equation

If $T_1^* < t < T_2^*$ it follows from the above notations that

$$\vec{R}(t) = \vec{R}_2(t) + \vec{\rho}(t)$$

Hence

$$\vec{V}(t) = \vec{V}_2(t) + \vec{V}'(t) \, .$$

Since half of the total time that the vehicle spends in τ is very small compared to the period of P_2 about the sun we may write

$$\vec{V}(t) = \vec{V}_2 + \vec{V}'(t)$$

Consequently

$$\vec{V}(T_i^*) = \vec{V}_2 + \vec{V}'(T_i^*) \qquad (i = 1, 2) \tag{12}$$

Thus

$$V^2(T_i^*) = V_2^2 + 2\vec{V}_2 \cdot \vec{V}'(T_i^*) + V'^2(T_i^*) \tag{13}$$

By invoking the energy equation (7) for hyperbolic trajectories we write

$$V'^2(T_i^*) = \mu_2\left(\frac{2}{\rho(T_i^*)} + \frac{1}{a_2}\right)$$

The radius of τ at T_1^* which is $\rho(T_1^*)$ is almost identical with the radius of τ at T_2^* which is $\rho(T_2^*)$. Thus the above equation implies that the vehicle's energy with respect to Σ' as it enters τ is the same as its energy as it leaves τ.

$$V'^2\,(t_1^*) = V'^2(T_2^*) \tag{14}$$

Upon substituting this result into the difference of the equations given by (13) we find

$$V^2(T_2^*) - V^2(T_1^*) = 2\vec{V}_2 \cdot [\vec{V}'(T_2^*) - \vec{V}(T_1^*)] \tag{15}$$

With the aid of (12) this equation can be expressed as

$$V^2(T_2^*) - V^2(T_1^*) = 2\vec{V}_2 \cdot [\vec{V}(T_2^*) - \vec{V}(T_1^*)] \tag{16}$$

It should be borne in mind that this equation in essence says nothing more than (14). Its value lies in its form where the quantities are given with respect to Σ and not Σ'.

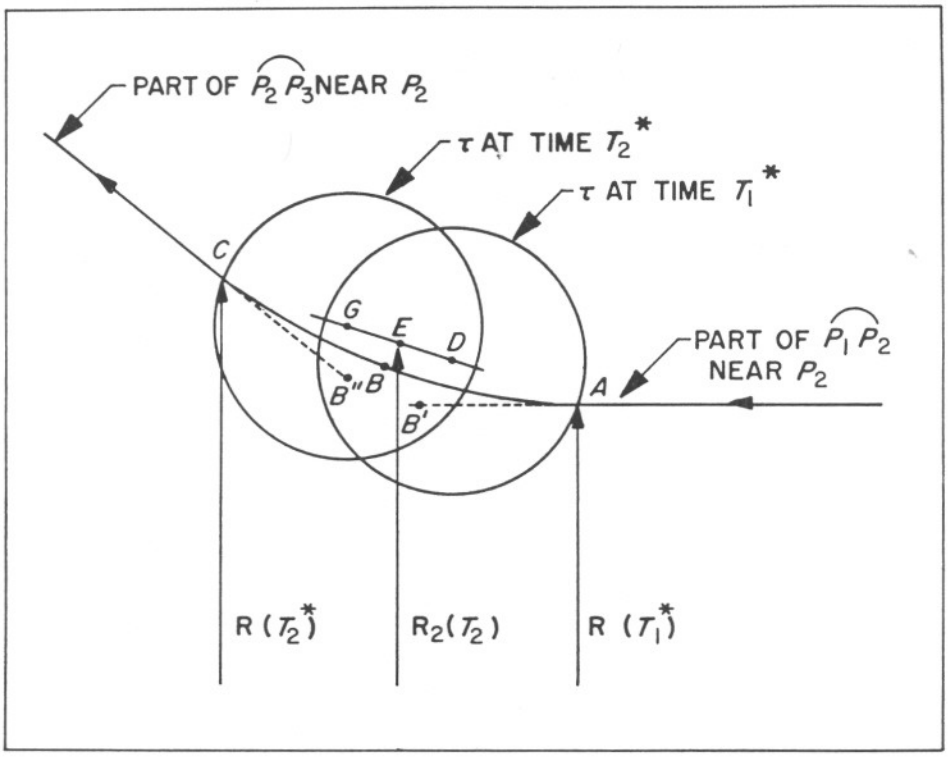

Fig. 3.

The determination of the elliptical orbits associated with the transfer trajectories

By the orbits associated with the transfer trajectories we mean the two closed elliptical orbits about the sun where $\widehat{P_1P_2}$ and $\widehat{P_2P_3}$ are sections. The elliptical trajectory $\widehat{P_1P_2}$ begins at the center of P_1 with position vector $\vec{R}_1(T_1)$ and ends at a point on the surface of τ at $T_1{}^*$ with position vector $\vec{R}(T_1{}^*)$. The elliptical trajectory $\widehat{P_2P_3}$ begins at a point on the surface of τ at $T_2{}^*$ with position vector $\vec{R}(T_2{}^*)$ and ends at the center of P_3 with position vector $\vec{R}_3(T_3)$.

In figure 3 the short solid line represents a small portion of P_2's orbit about the sun when the vehicle is nearby. The points D, E, and G are the planets' positions at $T_1{}^*$, T_2 and $T_2{}^*$, respectively. The longer solid line represents a small part of the vehicle's trajectory near P_2. The point A is the position of the vehicle at the time $T_1{}^*$ as it enters τ, B is its position at T_2 when it is closest to P_2 and the point C is the position of the vehicle at time $T_2{}^*$ as it leaves the moving region τ. The trajectory of the vehicle bounded by A and C is not conic since the figure is drawn with respect to Σ. When viewed from Σ' this part of the trajectory is hyperbolic. The vehicle's elliptical trajectories outside τ appear as straight line segments because of the scale of the figure. The sun is very far away and therefore the vectors $\vec{R}(T_1{}^*)$, $\vec{R}_2(T_2)$ and $\vec{R}_2(T_2{}^*)$ appear as parallel vectors. The dotted lines are continuations of $\widehat{P_1P_2}$ and $\widehat{P_2P_3}$. The points B' and B'' correspond to the positions of the vehicle moving on the orbits of $\widehat{P_1P_2}$ and $\widehat{P_2P_3}$ at the time T_2 as though P_2 did not exist. The figure clearly displays some very important facts.

It is easy to see that the position vectors of B' and B'' are almost identical with $\vec{R}_2(T_2)$. Thus by employing Lambert's Theorem with $T = T_2 - T_1$, $\vec{R}_1 = \vec{R}_1(T_1)$, and $\vec{R}_2(T_2)$, the semimajor axis a_1 of $\widehat{P_1P_2}$ can be calculated. Then by using (9) the eccentricity e_1 can be found. Consequently since $\ell_1 = a_1(1 - e_1^2)$ the vectors $\vec{e}_1$ and $\vec{h}_1$ corresponding to $\widehat{P_1P_2}$ can be calculated by (10) and (11).

Similarly by setting $T = T_3 - T_2$, $\vec{R}_1 = \vec{R}_2(T_2)$, and $\vec{R}_2 = \vec{R}_3(T_3)$ an application of Lambert's Theorem yields $a_3 = a_3(T_3)$. Since T_3 is unknown a_3 is written as $a_3(T_3)$ meaning that a_3 is a function of T_3. Following the above procedure the functions $e_3(T_3)$, $\ell_3(T_3)$, $\vec{e}_3(T_3)$, and $\vec{h}_3(T_3)$ can in theory be obtained. In practice these functions are not actually determined since high-speed digital computers make it possible to give T_3 an actual trial numerical value. Thus $a_3(T_3), e_3(T_3), \ell_3(T_3)$, $\vec{e}_3(T_3)$, and $\vec{h}_3(T_3)$ all take on actual numerical values corresponding to the trial value given to T_3. The actual value of T_3 can be obtained by noticing a second important fact suggested from figure 3. It is clearly evident that the vehicle's velocity vector at A and C are almost identical with the hypothetical velocity vectors at B' and B''. Consequently in view of our first observation these velocities can easily be obtained by (6).

$$\vec{V}(T_1^*) = \frac{1}{\ell_1}\,\vec{h}_1 \times [\hat{R}_2(T_2) + \vec{e}_1]$$

$$\vec{V}(T_2^*) = \frac{1}{\ell_3(T_3)}\,\vec{h}_3(T_3) \times [\hat{R}_2(T_2) + \vec{e}_3(T_3)]$$

Thus the actual value of T_3 is that value yielding a solution to (16). In general there are an infinite set of values of T_3 generating vectors $\vec{V}(T_2^*)$ which satisfies (16) but we shall choose the solution that gives $T_3 - T_2$ the smallest value. Thus a systematic search for T_3 can be initiated which, when found, determines the values of a_3, e_3, ℓ_3, $\vec{e}_3$, and $\vec{h}_3$ along with $\vec{V}(T_2^*)$. Hence the elliptical orbits associated with $\widehat{P_1P_2}$ and $\widehat{P_2P_3}$ can be completely determined. We emphasize at this point that even though $\vec{V}(T_1^*)$ and $\vec{V}(T_2^*)$ are known, T_1^*, T_2^*, $\vec{R}(T_1^*)$ and $\vec{R}(T_2^*)$ remain to be calculated.

The determination of the hyperbolic trajectory

We now consider that part of the vehicle's trajectory in τ. The seemingly difficult task of finding this trajectory turns out to be surprisingly easy.

Figure 4 is drawn with respect to Σ'. Hence the vehicle's trajectory in τ is hyperbolic. The points A, B, and C correspond to the points A, B, and C of figure 3. From (12) we calculate the hyperbolic excess velocity vectors at A and C

$$\vec{V}'(T_i^*) = \vec{V}(T_i^*) - \vec{V}_2 \qquad (i = 1, 2)$$

The quantities $V'^2(T_1^*)$ and $V'^2(T_2^*)$ are now calculated and in view of (14) we calculate their average V'^2.

$$V'^2 = \frac{1}{2}\,[V'^2(T_1^*) + V'^2(T_2^*)]$$

Thus by applying the energy equation (7) we calculate the semimajor axes a_2 of the hyperbolic trajectory.

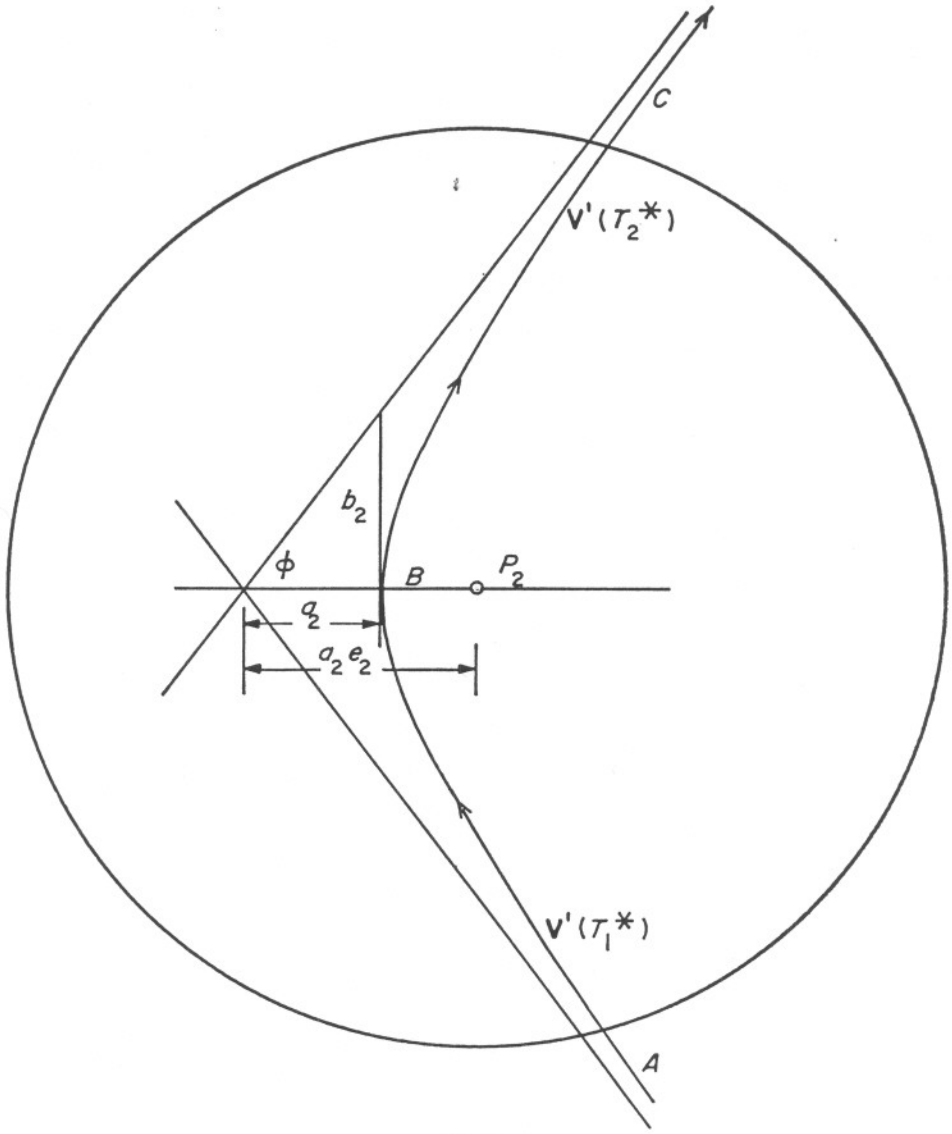

Fig. 4.

$$a_2 = \frac{\mu_2 \rho^*}{\bar{V}'^2 \rho^* - 2\mu_2} \tag{17}$$

where

$$\rho^* = \left(\frac{m}{M}\right)^{2/5} \qquad R_2(T_2)\ .$$

The term $2\mu_2$ is negligible compared to $\bar{V}'^2\rho^*$ since with respect to the hyperbolic trajectory the sphere of influence lies at infinity. Hence this term may be omitted from (17) with little or no effect.

If we denote the length of the conjugate axis of the hyperbolic path by b_2 as shown in figure 4, one observes that

$$\tan \phi = \frac{b_2}{a_2}$$

where ϕ is one half of the angle between the asymptotes. Thus since the eccentricity e_2 is related to a_2 and b_2 by

$$e_2 = \sqrt{1 + \left(\frac{b_2}{a_2}\right)^2}$$

we obtain

$$\cos \phi = \frac{1}{e_2} \tag{18}$$

Hence by studying figure 4 we find

$$\vec{V}'(T_1^*) \cdot \vec{V}'(T_2^*) = V'(T_1^*)V'(T_2^*) \cos 2\left(\frac{\pi}{2} - \phi\right)$$

which is expressible as

$$\vec{V}'(T_1^*) \cdot \vec{V}'(T_2^*) = V'(T_1^*)V'(T_2^*)(1 - 2\cos^2 \phi)$$

Thus by making use of (18) the eccentricity of the hyperbolic path can be calculated by

$$e_2 = \left\{\frac{2V'(T_1^*)V'(T_2^*)}{V'(T_1^*)V'(T_2^*) - \vec{V}'(T_1^*) \cdot \vec{V}'(T_2^*)}\right\}\frac{1}{2} \tag{19}$$

The distance of closest approach to the surface of P_2 can now be easily calculated by

$$d_2 = a_2(e_2 - 1) - \text{Radius of } P_2 \tag{20}$$

If this quantity turns out to be negative the trajectory is obviously physically unrealizable. The value of T_3 is then progressively increased by appropriate amounts until the next smallest value of T_3 is found.

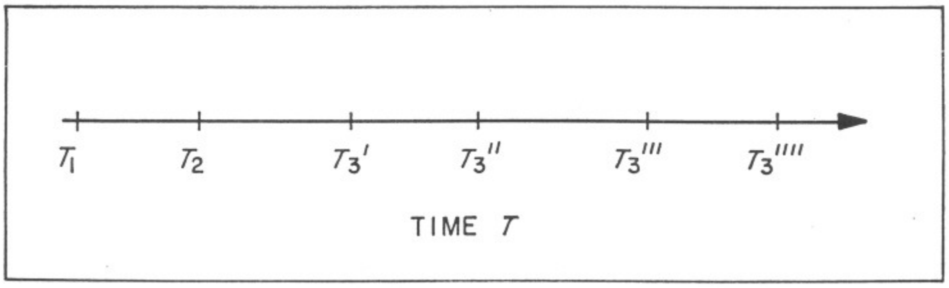

Fig. 5.

Figure 5 represents the general characteristic properties of all T_3 values in some interval of time after T_2. Suppose the values of T_3' and T_3'' satisfy (16) but give a negative value to the distance of closest approach. Let T_3''' be the next T_3 value but yielding a positive distance of closest approach. This is the value we choose to take. T_3'''' is the next possible T_3 value yielding a positive value to d.

After T_3 has been determined, the velocity at closest approach V'_{CA} with respect to P_2 can be obtained by again making use of the energy equation. One easily finds

$$V'_{CA} = \sqrt{\frac{\mu_2}{a_2}\left(\frac{e_2+1}{e_2-1}\right)} \tag{21}$$

The magnitude of the $\vec{h}$-vector can be calculated by

$$h_2 = \sqrt{\mu_2 a_2 (e_2^2 - 1)}\ . \tag{22}$$

By observing figure 4 of the trajectory of the vehicle in τ with respect to Σ', the $\vec{e}$ and $\vec{h}$ vectors can readily be obtained by

$$\vec{e}_2 = \frac{\vec{V}'(T_1^*) - \vec{V}'(T_2^*)}{|\vec{V}'(T_1^*) - \vec{V}'(T_2^*)|}\quad e_2 \tag{23}$$

$$\vec{h}_2 = \frac{\vec{V}'(T_1^*) \times \vec{V}'(T_2^*)}{|\vec{V}'(T_1^*) \times \vec{V}'(T_2^*)|}\quad h_2 \tag{24}$$

The position vectors of the points A and C with respect to Σ' can now be calculated by employing (3)

$$\vec{\rho}(T_i^*) = \left(\frac{1}{\mu_2}\,\vec{V}'(T_i^*) \times \vec{h}_2 - \vec{e}_2\right)\rho^* \qquad (i = 1, 2) \tag{25}$$

By employing Lambert's Theorem (8) with $\vec{R}_1$ and $\vec{R}_2$ replaced by $\vec{\rho}(T_1{}^*)$ and $\vec{\rho}(T_2{}^*)$ the total amount of time ΔT which the vehicle spends in τ can be calculated. Thus

$$T_1^* = T_2 - \tfrac{1}{2}\Delta T$$

$$T_2^* = T_2 + \tfrac{1}{2}\Delta T\ .$$

Consequently the position vectors of A and C can be calculated with respect to Σ by

$$\vec{R}(T_i^*) = \vec{R}_2(T_i^*) + \vec{\rho}(T_i^*) \qquad (i = 1, 2) \tag{26}$$

We have now completely determined both arriving and departing elliptical transfer trajectories and the trajectory of the vehicle near the planet P_2 which will take the vehicle to P_3. Of course, in view of the above approximations the trajectory is not exact; however, one may now, by an obvious iteration process, proceed to obtain a trajectory that is arbitrarily close to the desired elliptical and hyperbolic parts of the total trajectory. In practice it turns out that these approximations introduce very little error so that it is impractical to obtain greater accuracy.

We sum up this section with a very important observation. Recalling the method of solution we first determined $\widehat{P_1P_2}$ from the given initial conditions. Then we proceeded to find $\widehat{P_2P_3}$ by solving (16) such that $d > 0$. Finally the trajectory in τ was calculated. Now instead of terminating the mission at P_3 we wish to use P_3 to go on to some other planet P_4. Since the initial conditions specifying T_1, T_2, P_1, and P_2 are equivalent (on an interplanetary scale) to specifying $\widehat{P_1P_2}$ we simply take $\widehat{P_2P_3}$ as an initial condition to proceed to P_4. Thus $\widehat{P_3P_4}$ and the hyperbolic trajectory in the vicinity of P_3 can be determined by a completely analogous manner until P_n is reached. The numerical results which we take up in the next section clearly display the feasibility of such advanced missions.

Numerical Results

In the preceding section we were able to determine very general and complicated interplanetary free-fall trajectories. This was accomplished by employing the vector techniques developed in the section on conic trajectories and assuming that only one body of the solar system at any given time influences the vehicle's motion. The following examples depict typical advanced free-fall interplanetary trajectories which we are concerned with.

i) Trajectories of vehicles launched from Earth at a given time T_1 which makes a closest approach to Venus at a given time T_2 (T_2 shall continue to be called the Venus intercept date) such that the gravitational influence of Venus sends the vehicle back to Earth (Earth-Venus-Earth).

ii) Trajectories of vehicles launched from Earth at a given time T_1 which intercepts Mars at a given time T_2 such that the gravitational influence of Mars sends the vehicle back to Earth (Earth-Mars-Earth).

iii) Trajectories of vehicles launched from Earth at T_1 making a closest approach to Venus at T_2 whose gravitational influence sends the vehicle on an intercept course with Mercury (Earth-Venus-Mercury).

iv) Trajectories of vehicles launched from Earth at T_1 making a closest approach to Venus at T_2 where the gravitational influence of Venus causes the vehicle to intercept Mars such that the gravitational influence of Mars sends the vehicle back to Earth (Earth-Venus-Mars-Earth).

v) Trajectories of vehicles launched from Earth at T_1 making a closest approach to Venus at T_2 which causes the vehicle to intercept Mars which in turn causes the vehicle to return to Earth where the Earth's gravitational influence causes the vehicle to repeat the same flight; sending it on to Venus such that Venus's influence sends it to Mars whereupon the Martian gravitational influence causes the vehicle to return to Earth (Earth-Venus-Mars-Earth-Venus-Mars-Earth).

These examples represent only a very small fraction of the total number of possible different types of advanced interplanetary free-fall trajectories having $n - 1$ planetary encounters. For example, the total number of different types of advanced trajectories having only two planetary encounters of the form P_1–P_2–P_3 where P_1 = Earth is 9^{3-1} or 81! In general the total number of different trajectories of the form P_1–P_2–$\cdots$–P_n having $n - 1$ planetary encounters is 9_n. Thus the trajectory given by iv is only one of the 729 different types possible of the form P_1–P_2–P_3–P_4 where P_1 = Earth.

Precise numerical calculations of even the most simple types of these advanced trajectories having only two planetary encounters have (as far as I know) never been carried out. A few round-trip trajectories to Mars of the type given in example ii have been calculated at the Massachusetts Institute of Technology by R. Battin, but these did not include the important Mars hyperbolic approach trajectories.

Battin reported on six different trajectories of which the shortest flight time was about 1,050 days (see "The Determination of Round-Trip Planetary Reconnaissance Trajectories," *Journal of the Aero/Space Sciences* [Sept., 1959]). This minimum total flight time is obviously too long for serious consideration. Consequently, as the program for the digital computer was being written corresponding to the preceding section, many important questions remained unanswered. For example, since Battin calculated flight times in excess of 1,000 days for relatively simple trajectories like ii, how much time would trajectories like iv require τ. Moreover, it was not known whether trajectories like iii were even possible, to say nothing of trajectories like v.

When the early numerical calculations were confirmed by elaborate integrating programs at the Jet Propulsion Laboratory, the Computing Facility at the University of California, Los Angeles, where the program was written, began the first extensive analytical analysis of these advanced trajectories. These early calculations at UCLA not only proved the feasibility of such missions, they showed that in some instances they could become an economic necessity.

As the numerical calculations were stepped up by also utilizing the computing complex at the Jet Propulsion Laboratory, three distinct types of advanced mission began to crystallize. These missions follow in a natural chronological order.

1) Unmanned exploration of the inner planets by instrumented space vehicles.

2) Initial interplanetary missions by manned vehicles.

3) Interplanetary transportation networks to support manned bases on Venus and Mars.

Since short flight times and low launch energies are always desirable, the first category of missions must be of the form

$$P_1\text{–}P_2\text{–}P_3$$

where $P_1 =$ Earth, $P_2 \neq P_3$ and are either Mercury, Venus, or Mars.

The six possibilities are: Earth–Mercury–Venus; Earth–Mercury–Mars; Earth–Venus–Mercury; Earth–Venus–Mars; Earth–Mars–Mercury; Earth–Mars–Venus. The first two possibilities were immediately eliminated because launch energies required for Earth–Mercury transfer are very high. The last two possibilities were found to require either high launch energies or long flight times. It was discovered that the Earth–Venus–Mercury trajectories required considerably less launch energies than the direct flight Earth–Mercury trajectories. These advanced trajectoreies would permit the payload weight reaching Mercury to be increased by over 100 per cent during the decade 1965 through 1974. Of course, these trajectories do require greater flight times than the direct flight Earth–Mercury trajectories, but in some cases these flight times were found to be less than those required for direct flight Earth–Mars trajectories.

Now missions to Mars on conventional Earth–Mars trajectories can take place only during launch periods lasting only a few weeks. These favorable launch periods are separated by about 780 days (the "synodic period of Mars"). This represents a definite time barrier for trips to Mars. If a mission is not successful then we are forced to wait about 780 days until the next launch opportunity occurs. Now favorable Earth–Mars launch periods occur in 1969, 1971, and 1973. It was discovered that the Earth–Venus–Mars trajectories for 1970 and 1972 have low launch energies and relatively short flight times. Thus by utilizing these trajectories the three launch opportunities for missions to Mars can be increased to five: 1969, 1970, 1971, 1972, and 1973.

Of course, advanced trajectories of the form $P_1\text{–}P_2\text{–} \cdot \cdot \cdot \text{–}P_n$ will require highly accurate planetary approach guidance. These guidance systems on the other hand will not require any significant scientific breakthroughs. The guidance systems now being developed for the Apollo moon mission could perhaps meet these high guidance requirements.

TABLE 1

Launch date	HEV_1	T_{12}	Θ_{12}	HEV_2	$TISI$	$DOCA$	$VACA$	DA	T_{23}	Θ_{23}	HEV_3	TFT	Trajectory profile
1/20/65.........	7.20	98.00	137.57	16.39	....	.....	.....	.....	......	......	.....	98.00	Earth-Mercury
12/18/65.........	3.97	170.17	249.76	6.86	2.01	1560.	11.48	56.57	105.62	227.95	9.83	275.43	Earth-Venus-Mercury
1/ 3/66.........	6.86	98.00	144.55	16.44	....	.....	.....	.....	......	......	.....	98.00	Earth-Mercury
12/11/66.........	6.58	104.00	157.36	16.21	....	.....	.....	.....	......	......	.....	104.00	Earth-Mercury
6/19/67.........	3.70	96.28	107.84	6.56	2.09	311.	12.01	65.49	71.49	190.36	9.25	167.77	Earth-Venus-Mercury
11/21/67.........	6.46	106.00	169.50	15.15	....	.....	.....	.....	......	......	.....	106.00	Earth-Mercury
11/12/68.........	6.85	102.00	175.42	12.84	....	.....	.....	.....	......	......	.....	102.00	Earth-Mercury
1/23/69.........	3.76	189.38	256.88	6.39	2.14	645.	11.71	65.57	107.67	214.64	9.79	279.05	Earth-Venus-Mercury
10/ 3/70.........	6.85	116.00	216.75	13.02	....	.....	.....	.....	......	......	.....	116.00	Earth-Mercury
8/18/70.........	3.50	101.22	110.31	7.59	1.82	3768.	11.11	42.68	59.00	145.66	11.91	160.22	Earth-Venus-Mercury
1/31/71.........	7.50	96.00	129.16	17.04	....	.....	.....	.....	......	......	.....	96.00	Earth-Mercury
1/12/72.........	7.05	98.00	138.06	17.13	....	.....	.....	.....	......	......	.....	98.00	Earth-Mercury
4/ 1/72.........	4.03	196.58	265.01	7.30	1.88	521.	12.30	57.27	85.00	151.67	14.40	281.58	Earth-Venus-Mercury
12/19/72.........	6.74	104.00	150.99	16.94	....	.....	.....	.....	......	......	.....	104.00	Earth-Mecrury
11/ 4/73.........	4.25	93.79	103.34	8.10	1.70	2983.	11.71	41.31	58.00	137.83	11.14	151.79	Earth-Venus-Mercury
12/ 2/73.........	6.49	106.00	163.25	15.73	....	.....	.....	.....	......	......	.....	106.00	Earth-Mercury

TABLE 2

Launch date	HEV_1	T_{12}	Θ_{12}	HEV_2	$TISI$	$DOCA$	$VACA$	DA	T_{23}	Θ_{23}	HEV_3	TFT	Trajectory profile
2/28/69.........	2.97	180.00	141.11	.....	....	.....	.....	.....	......	......	5.05	180.00	Earth-Mars
8/12/70.........	3.26	129.28	151.68	5.47	2.45	3850.	9.76	62.87	180.00	173.01	6.75	309.28	Earth-Venus-Mars
5/19/71.........	2.84	205.00	155.03	.....	....	.....	.....	.....	......	......	2.82	205.00	Earth-Mars
5/21/72.........	4.03	172.00	257.62	8.29	1.66	966.	12.67	47.26	117.70	112.77	12.61	289.70	Earth-Venus-Mars
7/27/73.........	3.80	195.00	144.04	.....	....	.....	.....	.....	......	......	2.99	195.00	Earth-Mars

Table 1 contains some important characteristics of near minimum launch energy direct flight Earth–Mercury and Earth–Venus–Mercury trajectories. Table 2 is a similar table of Earth–Mars and Earth–Venus–Mars trajectories. All of the tables in this paper will adhere to the following notation:

HEV_k = hyperbolic excess velocity (km/sec) at P_k

$T_{k,\,k+1}$ = time taken by vehicle to pass from P_k to P_{k+1}

$\Theta_{k,\,k+1}$ = heliocentric angle swept out by the vehicle passing from P_k to P_{k+1}

$TISI_k$ = amount of time (days) vehicle spends in P_k's sphere of gravitational influence

$DOCA_k$ = distance of closest approach (km) to P_k's surface

$VACA_k$ = velocity at closest approach (km/sec) to P_k

DA_k = angular difference between the vehicle's velocity vectors as it enters and leaves P_k's sphere of influence

TFT = total flight time (days)

If some symbols do not have any subscripts it will be understood to be 2. For example $DOCA$ means $DOCA_2$, etc. The time corresponding to all calendar dates that appear without any reference to a particular time shall always be taken as 1200 GMT. In discussing the launch energy of a particular trajectory we shall refer to the trajectories vis-viva energy which is simply the square of the hyperbolic excess velocity. These quantities however shall be omitted from the tables.

An examination of table 1 shows that the launch energies required for direct flight Earth–Mercury missions require approximately 3 to 4 times as much launch energy as those required for Earth–Venus–Mercury missions. We also notice that the Mercury approach energies for the direct Earth–Mercury trajectories are 4 to 6 times higher than those associated with the Earth–Venus–Mercury trajectories. This will be an important consideration for missions requiring orbiting payloads about Mercury.

Table 2 shows that the Earth–Venus–Mars trajectories very neatly fills the gap between the 1969 and 1971 Earth–Mars launch opportunities and the 1971 and 1973 launch opportunities.

Let us now consider possible applications of these multiple planetary encounter free-fall trajectories to early manned interplanetary flight. The simplest of all such trajectories applicable for manned vehicles are Earth–Venus–Earth and Earth–Mars–Earth. Table 3 contains some important properties of near minimum launch energy Earth–Venus–Earth and Earth–Mars–Earth trajectories. We observe that the Earth–Venus–Earth trajectories are much more reasonable than the Earth–Mars–Earth trajectories. The former could probably be achieved with a non-nuclear Nova-type launch vehicle. The Earth–Mars–Earth trajectories could be made shorter by increasing the launch energy but this increase is of the order of 200 to 400 per cent. Consequently, with respect to the manned planetary reconnaissance missions we find ourselves faced with another very sad time-energy barrier situation. A non-nuclear Nova-type launch vehicle could probably be used for the launch vehicle for Earth–Venus–Earth manned reconnaissance missions but not for the Earth–Mars–Earth missions.

During the early part of June, 1962, while I was checking some newly calculated multiple planetary trajectories of the form Earth–Venus–Mars–Earth, a very

TABLE 3

Launch date	HEV_1	T_{12}	Θ_{12}	HEV_2	$TISI$	$DOCA$	$VACA$	DA	T_{23}	Θ_{23}	HEV_3	TFT	Trajectory profile
8/20/70.........	2.92	114.00	132.23	5.44	2.46	728.	11.29	76.16	250.96	227.48	7.13	364.96	Earth-Venus-Earth
6/ 8/71.........	3.97	316.00	204.84	4.21	3.23	2251.	5.74	34.88	795.83	530.43	5.85	1111.83	Earth-Mars-Earth
4/ 3/72.........	3.69	114.00	134.07	5.02	2.69	3983.	9.47	68.29	260.95	235.43	8.18	374.95	Earth-Venus-Earth
8/20/73.........	4.60	236.00	151.68	2.53	5.46	7024.	3.83	46.06	790.72	502.25	6.56	1027.70	Earth-Mars-Earth
11/ 4/73.........	3.76	116.00	139.29	4.47	2.94	5344.	8.76	71.80	269.46	241.00	7.90	385.46	Earth-Venus-Earth

TABLE 4

EARTH-VENUS-MARS-EARTH

Launch date	HEV_1	T_{12}	Θ_{12}	$DOCA_2$	DA_2	T_{23}	Θ_{23}	$DOCA_3$	DA_3	T_{34}	Θ_{34}	HEV_4	TFT
8/12/70........................	3.26	129.28	151.68	3848.	62.87	180.00	173.01	6590.	9.89	312.36	290.86	9.34	621.63
5/27/72........................	4.16	170.16	258.61	6552.	30.01	141.94	121.74	1249.	13.40	157.59	79.72	13.04	469.68

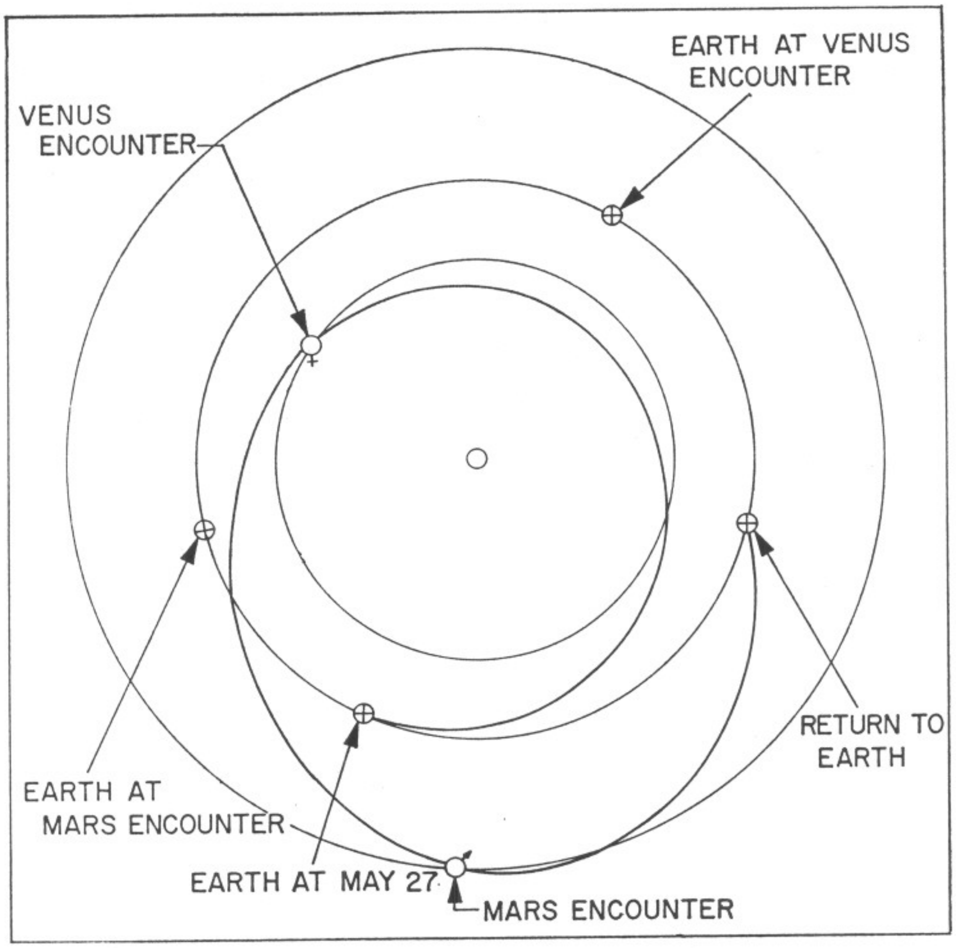

Fig. 6.

remarkable fact was discovered. Now it was already known at that time that the minimum energy Earth–Mars–Earth trajectories required very long flight times. Thus, it was believed that the most favorable Earth–Venus–Mars–Earth manned reconnaissance trajectories would have flight times much greater than 1,000 days. The fact is, however, that this is not always true. It was discovered that in some cases this assumption was false by a very wide margin. These cases very conveniently turned out to be the 1970 and 1972 Earth–Venus launch periods.

Table 4 contains a description of some important characteristics of these Venus–Mars reconnaissance trajectories. These trajectories show that by employing Earth–Venus–Mars–Earth trajectories instead of Earth–Mars–Earth, the flight times can be greatly reduced. These important trajectories remove the time-energy barriers inherent in simple Mars reconnaissance trajectories and open the door to the utilization of non-nuclear launch vehicles for early manned Mars fly-by missions. *Moreover, these trajectories also permit a Venus reconnaissance at the same time.* It has been determined that the 1973, 1975, and 1977 Earth–Venus launch periods

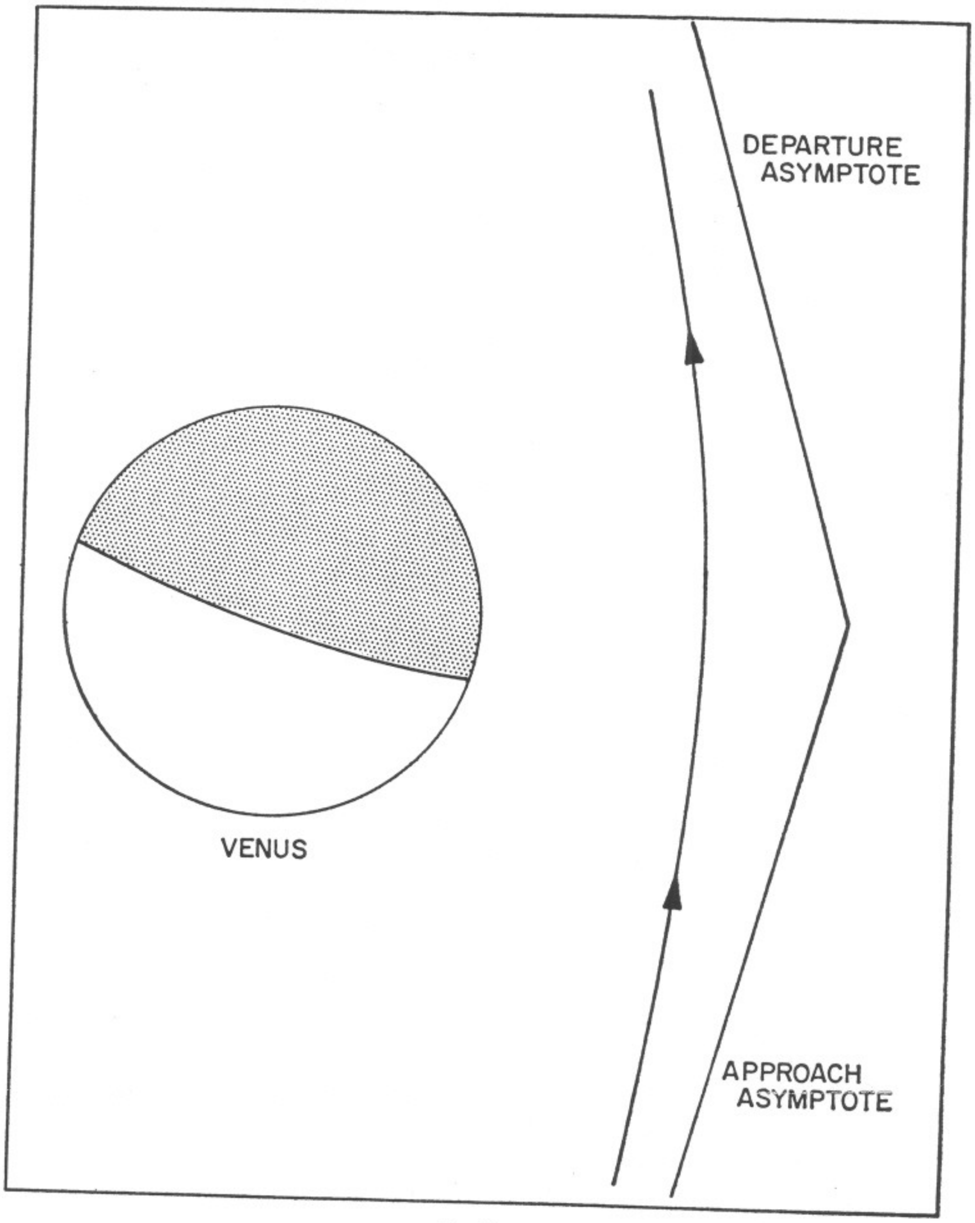

Fig. 7.

do not possess nearly as favorable Earth–Venus–Mars–Earth trajectories as those given in table 4. Since the 1970 and 1972 trajectories make it unnecessary to perform separate Earth–Venus–Earth and Earth–Mars–Earth missions they would make ideal early manned interplanetary missions. Figure 6 displays the planetary configuration for the 1972 Earth–Venus–Mars–Earth trajectory. Figures 7 and 8 show how this trajectory looks as it passes Venus and Mars, respectively.

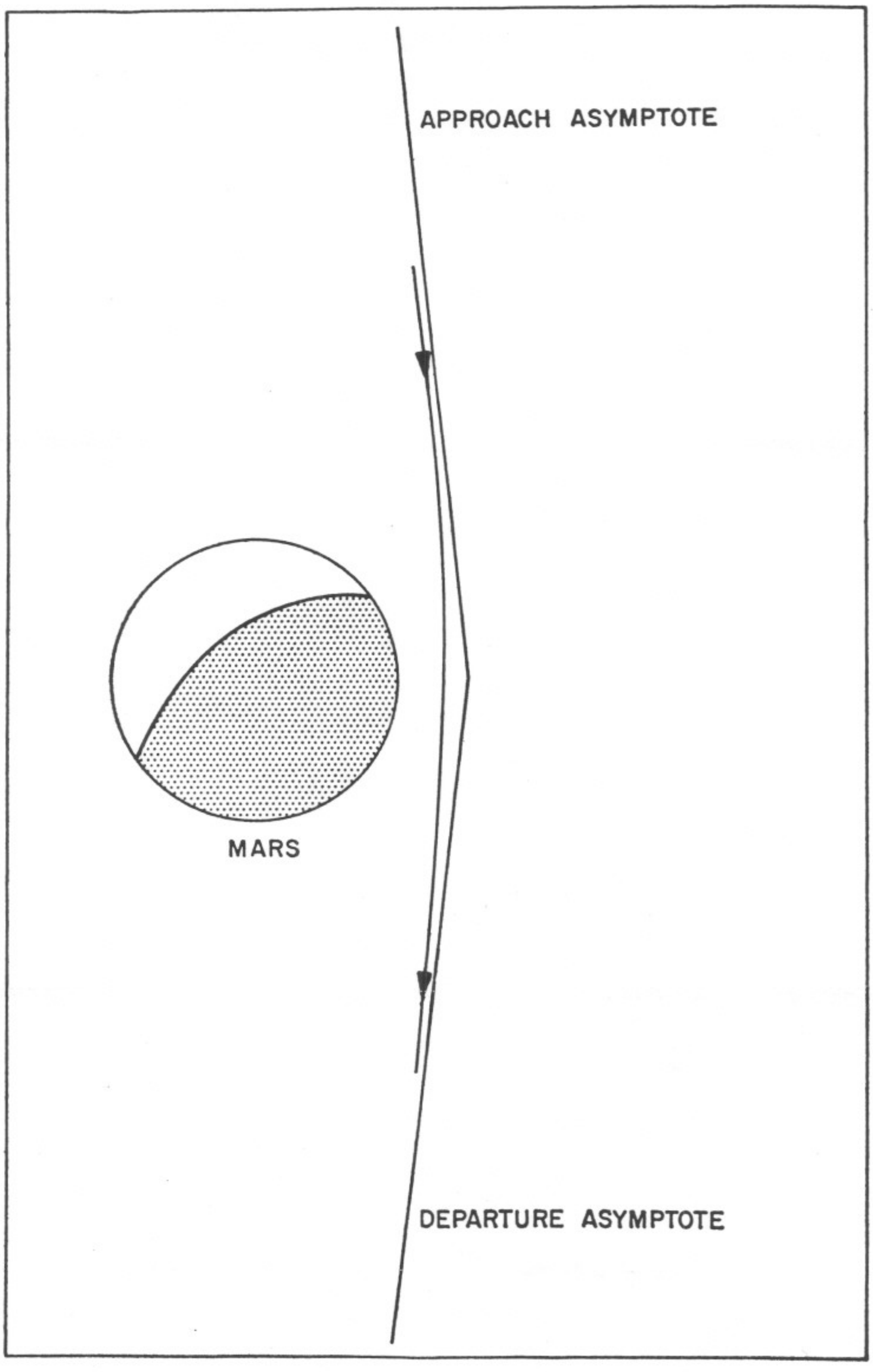

Fig. 8.

Recent studies regarding manned interplanetary missions to Mars show that a Mars landing can be carried out with almost total atmospheric braking. The studies show that the dry weight of the smallest possible vehicle of this type would be approximately 13,000 lb (including the heat shield). This vehicle called a Mars Excursion Module could carry 4 men. It has also been calculated that the weight of an interplanetary mission module carrying all the life-support equipment and related supplies to last a crew of 4 for 1 year would be almost 38,200 lb. An Apollo-type (6-man) aerodynamic-braking Earth reëntry vehicle (reëntering at 13.3 km/sec) would weigh about 13,180 lb. Using these design parameters one may wonder if it is possible to attempt a manned Mars landing mission with a Nova-type launch vehicle. Unfortunately, since the favorable Mars–Earth launch periods occur 40 to 60 days *before* the favorable Earth-Mars launch dates, the launch energies required for the conventional mission profiles are almost insurmountable. Very high energy nuclear stages would be an absolute necessity along with many complicated orbital refueling operations. *There is, however, a way which could permit a manned exploration of Mars using a conventional non-nuclear Saturn 5 Apollo launch vehicle.* In order that this method be clearly understood we shall consider an actual example.

Before attempting an actual manned landing mission, it would probably be desirable to first carry out a single manned Mars fly-by mission. This mission could provide an ideal means of testing man's endurance under prolonged interplanetary space flights. For this reason, the August 8, 1970, Earth–Venus–Mars–Earth trajectory appearing in table 4 may prove to be ideal. Now assuming a crew of 3 or 4, the weight of the primary interplanetary mission module could be taken to be approximately 43,000 lb. Let us take the weight of the Earth reëntry module to be 13,000 lb. We recall the well-known formula

$$\frac{M_1}{M_2} e \exp \left(\frac{\Delta V}{c}\right)$$

where M_1 and M_2 are the masses of a vehicle before and after obtaining a velocity change of ΔV using a rocket with an exhaust velocity of c km/sec. Consequently since our trajectory's Earth approach hyperbolic excess velocity is 9.34 km/sec the vehicle would reënter the Earth's atmosphere at about 14.5 km/sec. Thus we shall employ partial rocket braking to permit our 13,000-lb reëntry module to reënter the Earth's atmosphere at about 13.3 km/sec. Thus the mass before retro should be about 19,000 lb using a rocket motor weighing 500 lb, and an Isp of 367 sec. This means that the total weight at the start of the mission would be approximately 19,000 + 43,000 lb or 62,000 lb. The Saturn 5 launch vehicle will have the capacity to send 90,000 lb on an escape trajectory (parabolic), thus since the Earth's escape velocity is 11.19 km/sec, this rocket will be able to send approximately

$$90{,}000 \cdot e \exp \left(\frac{11.19 - \sqrt{11.19^2 + 3.26^2}}{3.6}\right) = 79{,}500 \text{ lb}$$

on the required trajectory! The Saturn 5 should be a highly reliable standard launch vehicle by 1970.

We shall now show how this launch vehicle could be used to carry out a manned Mars landing mission. To carry out this mission two vehicles will be used. The first, which we shall denote by A, shall be a simple Earth–Venus–Mars–Earth fly-by vehicle. The second vehicle, denoted by B, shall be launched on an Earth–Venus–Mars fly-by trajectory and in place of an Earth reëntry module it shall carry a small Mars excursion module. The mission profile consists of launching A and B on particular trajectories that will bring B to Mars several days before A makes its closest approach. During this time the crew of B can be exploring the surface of Mars. Then as A begins to make its closest approach, the crew of B launches their small Mars excursion vehicle and rendezvouses with A where they abandon the excusion module to complete the journey in A.

Let us now consider a definite trajectory profile for the manned Mars landing mission. Now just before the Mars excursion module is launched to rendezvous with A it is literally stripped of all equipment (heat shield, etc.) not absolutely necessary to effect a successful rendezvous with A. Thus at the moment it joins A we may assume it weighs 12,000 lb. The particular trajectories that we shall choose do not appear in the paper. They have, however, been carefully calculated. The first vehicle to be launched is B, which takes place on May 31, 1972. Its Earth–Venus–Mars trajectory will take it by Venus on November 17, 1972, with a distance of closest approach of 9,223 km. It will reach the vicinity of Mars on May 12, 1973, after a total flight time of 346 days. The vehicle's departing hyperbolic excess velocity will be 4.27 km/sec and will approach Mars with a hyperbolic excess velocity of 6.03 km/sec. Four days after launching B, A is launched. Its Earth–Venus–Mars–Earth trajectory will take it past Venus on November 19, 1972, with a distance of closest approach of 9,164 km. This vehicle will pass Mars on May 23, 1973, when it meets the Mars excursion module of B. The mission is completed on October 17, 1973, when the Earth reëntry module lands with its original crew plus the crew of B. The trajectory that A takes requires a departing hyperbolic excess velocity of 4.33 km/sec. Its return hyperbolic excess velocity is 9.51 km/sec. This mission profile will allow a crew of B to spend 11 days of exploration on Mars. The total flight time is 499 days. For this reëntry profile a retro thrust must be applied as in the previous case to dissipate about 3.1 km/sec to enable the intial atmospheric braking to take place at about 13.3 km/sec. Consequently, before retro the module will weigh approximately 31,000 lb. Although the flight of A in this case will be 122 days shorter than that of the 1970 mission, it will probably be required to carry more control equipment. Thus the injected weight of A will be about 72,000 lb. The Saturn 5 will be able to inject approximately

$$90{,}000 \cdot e \exp\left(\frac{11.19 + \sqrt{11.19^2 + 4.33^2}}{3.6}\right) = 72{,}100 \text{ lb}$$

The hyperbolic excess velocity of A as it approaches Mars will be 5.97 km/sec. Thus the weight of the Mars excursion module prior to launch should be about 103,000 lb. Assuming that 2 tons of supplies and equipment are left behind on Mars and assuming that B's interplanetary mission module weighs 40,000 lb, we find that the necessary injected weight at the beginning of its flight is 145,000 lb. The Saturn 5 will be able to inject about 72,500 lb on B's trajectory. This means that only two Saturn 5's would be necessary to send B on its required trajectory (via orbital assembly and refueling).

It should be stressed at this point that these calculations are optimistic (e.g., fuel requirements for guidance are omitted), however the trajectories chosen for A and B are not the best possible combinations. Others exist which would be more desirable. It should also be pointed out that we have taken only the *minimum* launch capabilities of the Saturn 5 as seen at the present time. This vehicle may develop a high growth potential, and by 1972 it may be capable of sending payloads weighing considerably more than 90,000 lb on an escape trajectory. Our main purpose for the above analysis is to show that it may be possible that the Saturn 5 could be used as the launch vehicle for the first manned landing mission to Mars. No new launch vehicles or advanced propulsion systems would be required.

After the first flights to the inner planets, man will naturally construct bases on these planets. These bases, no matter when they are constructed, will naturally require a means by which men and equipment can be taken to and from these bases. In the distant future when propulsion systems far more advanced than those currently being studied are developed, it will probably be possible to make interplanetary voyages such as Earth–Mars transfers with flight times as short as one or two weeks. But for the near future all interplanetary transfers will have to be made on near optimum transfer trajectories with low departure and arrival hyperbolic excess velocities. Consequently, a great deal of life-support equipment will be necessary to transport a few persons from one planet to another. In short, cargo vehicles shall probably be robot-type vehicles carrying no equipment necessary for manned flights, and the manned vehicles will probably carry very limited amounts of cargo. In addition to carrying all the extra equipment for manned flight, these vehicles will probably also be required to be able to induce some artificial gravity. Hence, the transportation of just 10 men, for example, from Mars to Earth should ordinarily require a large and very expensive rocket. Methods of recovery will become a necessity. This problem of economics can be conveniently solved by constructing a long-lasting interplanetary transportation network designed for the sole purpose of transporting personnel from one planet to another.

Preliminary calculations have shown that if the planets P_i are restricted to Mercury, Venus, Earth, and Mars where P_1 = Earth and $P_i \neq P_{i+1}$ for $i = 1, 2, \cdots, n$ it is possible to find sequences $\{P_1\text{–}P_2, - \cdots -P_n\}$ such that the flight times $T_{i+1}\text{–}T_i$ are comparable to those required for optimum $P_i\text{–}P_{i+1}$ transfers. Moreover some of these trajectories were found to have very low launch energies.

The network can be established by first constructing many large space vehicles that are to be used in the transportation system. This can be done by methods of prefabrication and orbital assembly. These vehicles can be designed to accommodate 20 to 60 persons, and since artificial gravity will be highly desirable the geometry of the vehicles could be a torus with an outside diameter of perhaps 200 to 300 feet. When each individual vehicle is completed one simply awaits its launch date T_1 when the vehicle (i.e., space bus) is injected into its prescribed interplanetary trajectory. This could be accomplished by convenient strap-on solid propellant rocket engines. The vehicle will carry extra provisions and life-support equipment to last until it makes its first Earth rendezvous, whereupon its supply of provisions and life-support equipment can be replenished to last until it makes its second Earth rendezvous, etc. As a vehicle approaches a planet P_i, a small excursion module orbiting P_i and containing a few men wishing to go to P_{i+1} is injected onto an intercept trajectory with the space bus. Upon making a rendezvous the excursion module containing very accurate planetary approach guidance systems could be left a few miles behind the space bus to be used to transport the men from the space bus onto an orbit about P_{i+1}. A tanker vehicle following the space bus could be used to refuel the planetary excursion modules. Other transportation systems could be established on each separate planet for the purpose of bringing the men from the circular orbit down to the planet's surface. Tanker vehicles orbiting each planet in the system could refuel the excursion modules that never actually land on a planet.

The transportation network outlined above seems attractive for several reasons. One notices that all the vehicles involved in the network (which could be made up of as many as 30 or 40 space buses all on their own interplanetary trajectories) can be used as often as desired. These interplanetary transfer vehicles could be very large and hence could be designed so that these necessarily long planetary transfers can become quite acceptable. The large number of transfer vehicles in the network would greatly preclude the possibility of a crew running out of supplies and becoming isolated on a planet. Table 5 contains an example of one such trajectory of the form Earth–Venus–Mars–Earth–Mars–Earth–Venus–Earth.

TABLE 5

	$i = 1$ Earth	$i = 2$ Venus	$i = 3$ Mars	$i = 4$ Earth	$i = 5$ Mars	$i = 6$ Earth	$i = 7$ Venus	$i = 8$ Earth
T_i	8/14/70	12/20/70	6/17/71	4/25/72	12/11/73	5/31/74	3/26/75	7/30/75
HEV_i	3.28	5.44	6.74	9.34	13.46	11.81	10.83	7.91
$T_{i,i+1}$	....	128.20	179.30	312.22	595.03	171.58	298.37	126.26
$\Theta_{i,i+1}$	....	151.25	171.65	290.81	203.10	191.74	196.13	220.57
$DOCA_i$	....	3817.	6838.	8089.	592.	6249.	9455.	
$VACA_i$	....	9.75	7.34	11.93	14.23	14.23	12.61	
$TISI_i$	....	2.47	1.85	2.26	.97	1.82	1.29	
DA_i	....	63.44	9.69	27.79	6.41	21.27	17.39	

Now there remains one very important aspect that must be considered. This involves the question of accuracy. Just how good is the assumption that one and only one body influences the vehicle's motion at any given time?

The Jet Propulsion Laboratory has a program such that if the geometry of the trajectory near P_i is sufficiently close to the exact trajectory required for the vehicle to intercept P_{i+1}, an integration, iteration process is begun by first determining the miss vector at P_{i+1} resulting from the initial approximation and then using it to modify the trajectory near P_i etc., until the miss vector becomes smaller than a specified amount. This program is highly unstable in the sense that if the initial trajectory in the vicinity of P_i is not extremely close to the real one required to permit the vehicle to rendezvous with P_{i+1}, the iteration process will not converge. It has been observed that each time this program was employed to check the trajectories resulting from the solution given in this paper, very rapid convergence resulted. This means that our fundamental assumption must not give trajectories very far from those which would occur in the real situation.

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