Missions Normal to the Ecliptic

Certain heliocentric orbits offer interesting possibilities for observation, training, and interplanetary rendezvous

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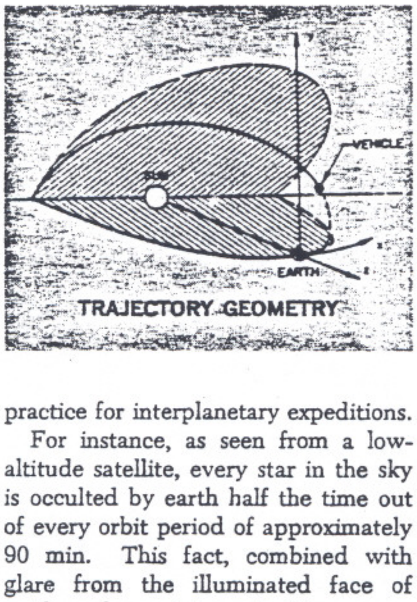
SPACECRAFT can be launched into heliocentric orbits similar to the earth's, but inclined to it, so that they return to our planet after several weeks or months. Here we will look at the value of such flight paths for space laboratories, for practice interplanetary expeditions, and for rendezvous with returning interplanetary rockets. We will also cover briefly the mathematics of these flight paths, or trajectories.

Disregarding perturbations, a spacecraft launched from earth with a hyperbolic excess speed in a direction normal to the plane of earth's orbit will enter an orbit similar to the earth's, but inclined to it. Such an orbit will intersect the earth's 180 deg from the launch point approximately six months after launch.

Excursions of several million kilometers from the plane of earth's orbit require only small hyperbolic excess speeds relative to earth, both at launch and at re-entry. Actually, for such small hyperbolic excess speeds, the combined effects of the masses of sun and earth act to bring the vehicle home after less than six months and even after as little as two weeks, if desired.

As can be seen from the graph on page 60, for total energies in the neighborhood of zero—in fact, for orbit-injection speeds differing from the theoretical escape speed by not more than 100 m/sec—the maximum distance from earth varies from less than $^1/_2$- to more than $7^1/_2$-million km, and the trip duration varies from less than 2 to more than $5^1/_2$ weeks.

Such trips offer several advantages over low-altitude satellite orbits around earth for various researches, including practice for interplanetary expeditions.



TRAJECTORY GEOMETRY

For instance, as seen from a low-altitude satellite, every star in the sky is occulted by earth half the time out of every orbit period of approximately 90 min. This fact, combined with glare from the illuminated face of earth, makes uninterrupted time exposures of more than 45 min in astronomical cameras impossible. Removal of the cameras to great distances from earth would permit longer exposures.

Moreover, earth satellites remain at all times within the zodiacal dust cloud. It has been estimated that photographic exposures outside earth's atmosphere, but within the zodiacal dust cloud, can be only one-third as long as would be possible outside the zodiacal dust cloud. If we assume the dust is mainly confined to a zone within a few million kilometers of the ecliptic plane, it will be easily possible to send vehicles outside it for considerable parts of any six-month period.

Such gains in seeing position may make the difference between being able or not being able to penetrate beyond the boundaries, if any, of the local universe of galaxies.

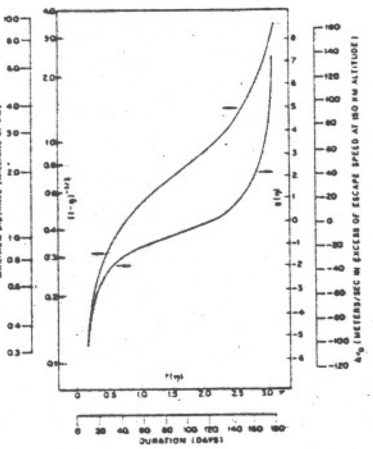
The inclined-orbit station, in addition, would be beyond the magnetic field of earth, outside the ion belt, and away from atmospheric molecules and possible dust belts. These factors are important in setting up studies of the solar-system environment. An uninterrupted watch of the sun could be maintained from a single station and from a single set of instruments.

With the earth a target for study, a time-lapse motion picture made from a station riding north of the ecliptic in summer would show the entire polar region during the long polar day. Every day a 2-min run of the past 48 hr of the north polar weather could be relayed to earth; any meteorologist could then make a direct reading of weather prospects. Practice with radar pictures of hurricanes has suggested that any interested person can quickly learn to make significant interpretations.

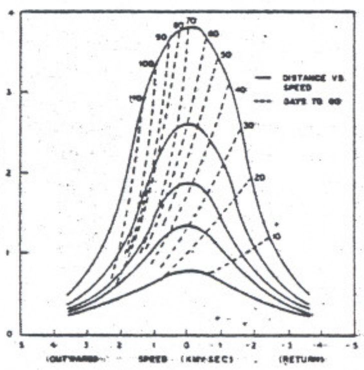
Since only the central regions of the hemisphere show clearly in a photograph, it might prove desirable to maintain more-complete coverage by means of two stations, one in the north and the other in the south. If, in addition, a third station could be maintained on the earth–sun line, at the metastable position about 1.5×10^6 km inside earth's orbit, to obtain an uninterrupted time-lapse picture of the sunlit hemisphere as it rotates beneath, a continuous coverage from pole to pole would result. Motion pictures thus obtained would not require an army of data reducers, and therefore could be used directly and immediately for interpreting current weather behavior.

Possibly now is the time to look to

INITIAL SPEED AND MAXIMUM DISTANCE VS. DURATION



MOTION HISTORIES



EQUATIONS

$$\frac{d^2y}{d\phi^2} + y + \frac{\rho}{y^2} = 0 \qquad (1)$$

$$\frac{1}{2}\left(\frac{dy}{d\phi}\right)^2 + \frac{1}{2}y^2 - \frac{\rho}{y} = E \text{ (a constant)} \qquad (2)$$

$$\phi - \phi_0 = \int_0^y \frac{y^{1/2}dy}{[2\rho + 2Ey - y^3]^{1/2}} \qquad (3)$$

$$\eta = \frac{2E}{y_1^2} = 1 - \frac{2\rho}{y_1^3} \qquad (4)$$

$$y_1 = \left(\frac{2\rho}{1-\eta}\right)^{1/3} \qquad (5)$$

$$\phi - \phi_0 = \int_0^{(y/y_1)} \frac{u^{1/2}du}{[(1-u)(1-\eta+u+u^2)]^{1/2}} \qquad (6)$$

$$\phi_2 - \phi_0 = 2(\phi_1 - \phi_0) = 2\int_0^1 \frac{u^{1/2}du}{[(1-u)(1-\eta+u+u^2)]^{1/2}} = f(\eta) \qquad (7)$$

$$\frac{1}{2}v_0^2 - \frac{GM_E}{r_0} = GM_S\left[\frac{1}{2}\left(\frac{dy}{d\phi}\right)^2 - \frac{\rho}{y}\right]_{(\text{at } y << \rho^{1/3})} \cong GM_SE \qquad (8a)$$

$$\text{Eq. 8a} = GM_S\frac{\eta y_1^2}{2} = \frac{GM_E}{r_0}\left[\frac{r_0\eta}{2\rho}\left(\frac{2\rho}{1-\eta}\right)^{2/3}\right] \qquad (8b)$$

$$v_0 = \left(\frac{2GM_E}{r_0}\right)^{1/2}\left\{1 + \frac{\eta}{[2(1-\eta)^2]^{1/3}}\cdot\frac{r_0}{\rho^{1/3}}\right\} \qquad (8c)$$

the future. Imagine a large rocket on an inclined heliocentric orbit. Its empty propellant tanks house upwards of 100 spacious laboratories and dormitories. Every six months shuttle rockets bring supplies and new men. Periodically some men return to earth. The complement of men gradually builds to 100 or so. Most of these astronauts might stay with the station a year.

A MAJOR, more-immediate use for the heliocentric orbit would be for practice with interplanetary vehicles. Equipment and operations could be tested under relatively easy and safe conditions, with test durations of any length from a few weeks to many months. Launches could be made any day of the year. Actual interplanetary journeys would then be routine extensions of the practice flights on heliocentric orbits.

The use of rockets on inclined heliocentric orbits to rendezvous with returning interplanetary spacecraft depends on the fact that only a small mass need be transferred by rocket propulsion, and that the change in speed is less than it would be at perigee. In a sample study based on a short stopover trip to Mars at the time of the opposition in 1971, it was found that this scheme allows a 30% reduction in size of the launch mass at earth.

Now let us look at an analysis of the vehicle's trajectory; this will be based on the supposition that not only does the distance from earth, measured in astronomical units, remain small, but that also the vehicle's direction from earth remains approximately perpendicular to the ecliptic plane.

More specifically, if a rotating coordinate system is introduced, centered at the earth as indicated in the sketch on page 59, the coordinates x, y, z are assumed to be such that $x, z \ll y \ll 1$. To the extent that $(x/y)^2$ and $(z/y)^2$ may be neglected, the y-motion turns out to be uncoupled from the small x and z motions, and may thus be studied separately.

As may be anticipated, this motion, which includes both the sun's attraction and the earth's attraction as "restoring" forces tending to reduce y to zero, depends on a single parameter—the one-dimensional total energy.

IDEALIZING the earth's orbit as circular and adopting as independent variable, in place of time, the angular position, ϕ, of the earth in its orbit, the equation for the y-motion is given by Eq. 1 in the table here at top where ρ = earth's mass, M_e/sun's mass, M_s. We immediately obtain an "energy" integral, given by Eq. 2. A further quadrature yields the time (through ϕ), as given by Eq. 3, where the lower limit is taken as zero rather than some small initial y_0 (at which distance the perturbing force due to the sum is much smaller than the earth's attraction), so that the contribution to ϕ between zero and y_0 is unimportant.

A convenient description of the motion, involving essentially one parameter, is obtained if we introduce y_1, the maximum y reached, given by $2\rho + 2Ey_1 - y_1{}^3 = 0$, and a non-dimensional energy parameter η given by Eq. 4 in the table on page 60, so that we arrive at the expression for y given by Eq. 5.

As the parameter η varies from $-\infty$ to $+1$, the maximum distance y_1 (in astronomical units) varies from 0 to ∞. Our solution, of course, is valid only over the range of values of η for which $y_1 \ll 1$ (e.g., $y_1 < 0.1$).

Equation 6 gives the earth's position during the vehicle's outward motion; and Eq. 7 gives the total angular travel of the earth, and hence the total time. In particular,

$$f\left(\frac{3}{4}\right) = \frac{2\pi}{3^{1/2}}\ (3^{1/2} - 1)$$

Other values of $f\ (\eta)$ may be obtained from tables of elliptic functions. Thus, if $\eta < 3/4$, $f\ (\eta)$ may be evaluated according to $3(d)$ on p. 48 of Gröbner and Hofreiter's *Integraltafeln*, Vol. 2, while if $\eta > 3/4$, $f(\eta)$ may be evaluated by $3(a)$ on p. 67 of this reference.

The parameter η, in turn, may be related to the speed, v_0, at a specified small radial distance r_0 from earth, at which distance the potential energy term $(^1/_2)y^2$ due to the sun's perturbative force may be assumed to be negligible by comparison with the potential energy term $-\rho/y$ due to the earth's attraction. We then arrive at the expressions for velocity given by Eq. 8a, 8b, and 8c.

Equations 5, 7, and 8 may be used to plot the relations between distance, "initial" speed, and flight duration. Thus, in the graph on page 60, the abscissa is $f(\eta)$, the earth's angular travel. The left-hand ordinate is $(1-\eta)^{-1/3}$ (or, rather, its logarithm), and the right ordinate is $\eta/[2(1-\eta)^2]^{1/3} = g\ (\eta)$, say, so that by suitable rescaling we can read the maximum distance, y, in millions of kilometers, $\Delta\ v_0 = v_0 - (2GM_E)^{1/2}/r_0$ in meters/sec, with r_0 chosen as 150 km in excess of the earth's equatorial radius, versus the flight duration in days.

By further rescaling, the graph on page 60 may be used for trips launched from other planets perpendicular to their heliocentric orbital planes, or from the moon perpendicular to its orbital plane, etc. The abscissa rescaling obviously corresponds to the various planetary half-years, or the lunar half-month, etc. The scales for the ordinates are determined from the values corresponding to $\eta = -1$ and $g(\eta) = -1/2$, some of which are given in the table below.

The sensitivity of the flight duration and the maximum distance reached to the initial speed is apparent from the graph on page 60. The sensitivity of the return time to midcourse changes, small or otherwise, in the y-velocity is indicated in the graph on page 60, which is based on Eq. 2 and 6, the latter being evaluated by numerical integration.

Corrections to this theory, taking account of the eccentricity of the earth's orbit, are being treated separately and will be published at a later date, together with a linearized treatment of the x and z deviations from the one-dimensional y-motion, due to initial conditions and sun as well as moon perturbations, the moon's orbit being treated as circular in the ecliptic plane.

Recent comparisons with accurate variation-of-parameters trajectory calculations indicate good agreement for three-month trips at various times of year and month and qualitative agreement for four-month trips, where the deviations grow considerably larger. The linearized treatment, of course, has the advantage not only in that the effects of initial conditions and sun and moon perturbations can be treated additively, but also in that the effects of the moon's initial phase can be accounted for by a suitable linear combination of the full-moon and last-quarter-moon effects, for example. The linearized treatment provides, in addition, a good indication of the effect of midcourse velocity-vector changes.

We conclude that trips lasting substantially less than six months cannot only be controlled but that they adhere closely to the one-dimensional approximate description presented here. ♦♦

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SCALE CHANGES FOR MISSIONS

					For $\eta = -1$, $g(\eta) = -\frac{1}{2}$		
Large center of attraction	Small center of departure and return	Mean distance of small center from large center, km	Departure radius at 150-km altitude (r_0), km	Escape speed (v_e), km/sec	Correction to escape speed (Δv_0), m/sec	Maximum distance from small center, km	Flight duration, days
Sun	Mercury	5.7×10^7	2.6×10^3	4.4	−9	3.4×10^5	23
Sun	Venus	1.08×10^8	6.2×10^3	10.8	−12	1.5×10^6	58
Sun	Earth	1.49×10^8	6.5×10^3	11.7	−9	2.1×10^6	94
Sun	Mars	2.3×10^8	3.5×10^3	5.2	−3	1.6×10^6	177
Sun	Jupiter	7.7×10^8	7.2×10^4 [a]	62.5	−16	7.7×10^7	1120
Earth	Moon	3.8×10^5	1.89×10^3	2.4	−12	8.8×10^4	7.6
Mars	Phobos[b]	9.3×10^3	8[b]	0.012	−1[c]	21	(1.96 hr)
Jupiter	Ganymede	1.06×10^6	2.7×10^3	2.9	−400[c]	4.7×10^3	1.84

(a) This is 1100 km above estimated surface.

(b) Phobos is assumed to be a sphere of 16-km diameter and mean density equal to the moon's; the departure radius is taken equal to the assumed radius of Phobos.

(c) In these cases Δv_0 is comparable with the escape speed, v_e, so that further increases of Δv_0 with $g(\eta)$ are appreciably non-linear, whereas $\Delta(v_0{}^2)$ remains linear in $g(\eta)$. (See derivation of Eq. (8) in the table on page 60.)