HIGH-SPEED COMPUTING DEVICES

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outputs Z_1 and Z_2 of the two integrators are then added to give the product XY.

$$XY = Z_1 + Z_2 = \int X\,dY + \int Y\,dX \qquad (11\text{-}4)$$

The factor k_2 may be eliminated by adjusting the outputs of each integrator by this constant.

Where the two variables appear within the integral,

$$\int XY\,d\phi$$

two integrators will provide the required product. Since

$$\int XY\,d\phi \equiv \int X\,d(\int Y\,d\phi) \equiv \int X\,dS \qquad (11\text{-}5)$$

the integrator, with ϕ and Y as input variables, supplies the quantity

$$S = \int Y\,d\phi \qquad (11\text{-}6)$$

The variable S is then introduced to the second integrator as a shaft rotation of the disk, and X is introduced as the lead-screw rotation to the wheel carriage, so that the result

$$R = \int XY\,d\phi \qquad (11\text{-}7)$$

is generated by the output of the second integrator.

For further details on special operations, such as the use of the wheel and disk as an *inverse integrator*, refer to Bush and Caldwell (pp. 316–318).[9]

Division. Division is performed either by the slide multiplier[32] or by the integrator.[9] If the slide multiplier is used, the inputs X and Y are interchanged to give

$$X = k_1\left(\frac{Y}{Z}\right) \qquad (11\text{-}8)$$

Division may also be performed with two integrators by taking the reciprocal of one of the variables, Y, for example, and using this reciprocal as the input to the Y (lead-screw) shaft in the first integrator and to the X (disk) shaft in the second integrator:

$$\frac{X}{Y} = \int X\,d\left(\frac{1}{Y}\right) + \int \frac{1}{Y}\,dX \qquad (11\text{-}9)$$

11-2-2. Differentiation and Integration. Both differentiation and integration are performed with the Kelvin wheel-and-disk integrator. The method of its use is described in Sec. 11-2-1 above under Multiplication.

11-2-3. Input and Output. Numbers and orders to a differential analyzer may be introduced by means of coded punched tape.[8] The orders for machine setup include those relating to the shaft connections and to the gear ratios required for the solution of a given problem; numbers representing the initial conditions may also be introduced on punched tape, or in the case of earlier units, these may be set into the machine manually.

Where input of a function in graphical form is required, an *input table*[9,22,32] is used. This is usually a flat platform on which the required function is plotted in cartesian coordinates. The input table is equipped with a pointer for following the plotted function; if $f(x)$ is plotted along the ordinate and x along the abscissa, the ordinate position is often controlled manually by a feed-screw mechanism, while the traverse along the abscissa is driven by a motor. The x drive on the input table is taken from the basic x drive of the machine and must always be in synchronism with it so that the correct value of $f(x)$ is introduced to the other elements (adders, multipliers, integrators, etc.) in the proper phase. Automatic curve-following devices have been reported by *MIT*[18] and the University of California at Los Angeles.[32]

The output of a differential analyzer may be punched on paper tape or printed in tabular form by an electrical typewriter. Frequently, it is convenient to plot the output graphically. An *output table*,[9,22,32] similar in design to the input table described above, is used for this type of output and consists of a flat table equipped with a frame for support and manipulation of a stylus. The function $f(x)$, or y, is plotted automatically on the ordinate and the independent variable, x, on the abscissa.

11-2-4. Generation of Functions. Functions in graphical form may be introduced to the machine through the input table. However, when certain simple analytic functions are required as elements in the solution of a differential equation, they are frequently generated within the analyzer itself.

The square of a function may be generated by one integrator. Since $y = x^2$, y may also be expressed as

$$y = \int 2x\,dx \qquad (11\text{-}10)$$

positioning, counting of shaft rotations, and for temporary storage.

Tables 11-1 and 11-2 summarize the equipment in the new differential analyzer at MIT. These data were presented by Bush and Caldwell (p. 326).[9]

11-4. The General Electric Differential Analyzer

An improved type of differential analyzer has also been designed and built by the General Electric Company.[22] One model of this analyzer is in current use at the University of California at Los Angeles.[32]

The general mechanical features of the components of this analyzer differ in several respects from those in the new differential analyzer at MIT. For example, multiplication and division are performed generally with a mechanical multiplier by using similar-triangle relations (see Multiplication in Sec. 11-2-1), although the two-integrator method for obtaining products may still be involved.

The integrator output shaft is not in physical contact with the integrating wheel but is controlled by a photoelectric follow-up system. Since it is not connected to the output shaft, the wheel experiences no reaction from the torque drive of that shaft; the only torque on the wheel itself is that imposed upon its own bearings. The integrator wheel is a Polaroid disk with a steel rim and hub; the wheel itself forms part of the optical system. A split beam of light passes normally through the wheel face, one beam traversing one of two additional disks and the other beam traversing the second disk. The two disks are set with their planes of polarization at 90° to one another; they are mounted on a common shaft geared to a follow-up motor. The shaft of this motor simulates the rotation of the integrator wheel.

After passing through the integrator wheel and one of the polarized disks, each beam falls upon a photocell. As the integrator wheel rotates, the illumination on the two photocells is varied. The photocells form two arms of a d-c bridge; the output, or current difference between the photocells, causes one thyratron to conduct more and another thyratron to conduct less than its previous current output. The thyratrons both supply current to a pair of field coils on a split-series servomotor. The

sign of the difference in the current outputs controls the direction of rotation of the motor. Therefore, an unbalance in the light striking each photocell is transmitted by thyratron control so as to correct this unbalance by changing the angular position of the polarized disks. This method of balancing is used to keep the shaft of the servomotor at the same angular position as the shaft on the integrator wheel.

Table 11-3 gives a list of components in the differential analyzer at the University of California at Los Angeles; this material is presented in Reference 32 (p. 2). For the design characteristics

TABLE 11-3. COMPONENTS OF THE DIFFERENTIAL ANALYZER[9] AT THE UNIVERSITY OF CALIFORNIA AT LOS ANGELES

Adders (also subtract)	20
Multipliers (also divide)	3
Integrators	14
Input tables	4
Independent variable motors	2
Output tables	2

of other differential analyzers, see articles by Beard,[2] Hartree,[13,16] Hartree and Porter,[14] Lennard-Jones, Wilkes, and Bratt,[23] Massey, Wylie, and Buckingham,[25] and Murray.[26]

11-5. Electronic Differential Analyzer

Recently attention has been given to performing differentiation, integration, and arithmetic operations with electronic circuits.[19,21] Analogous mechanical methods have been outlined previously, and these preceded the development of electronic units for differential analysis. The basis for all the electrical analog computers is the mathematical relation between currents and voltages in electric circuits. For example, the voltage drop across a resistor is proportional to the algebraic sum of the currents which are introduced from various sources. Thus, Kirchhoff's law is the basis for an electrical adder.

In general, the order of magnitude of error in an analog computing device is closely associated with the ratio of output energy produced to input energy required. For this reason, the electron tube is a valuable aid to electrical computing because it requires very little input energy. The use of voltage or current amplifiers[29] in an electrical computer is somewhat analogous to the use of torque amplifiers in mechanical computers.