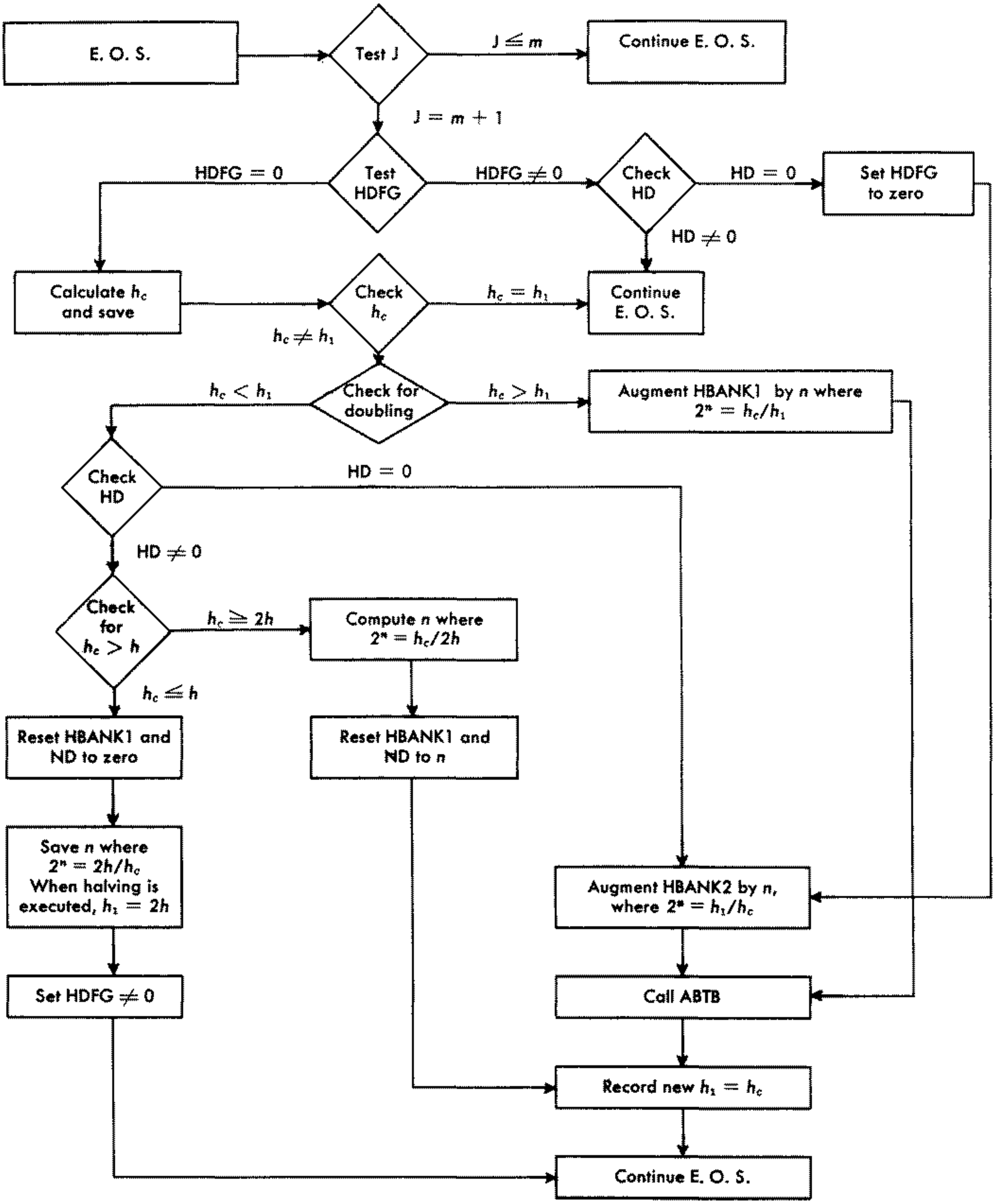
3. Flow Chart for the Step-size Control



VI. DESCRIPTION OF THE OUTPUT FOR THE SPACE TRAJECTORIES PROGRAM WITH INTERPRETATION OF THE MNEMONIC CODES

A. Output Philosophy

The output of the Space Trajectories Program displays for each trajectory the fundamental astronomical constants used in the calculation, the injection conditions which serve as a starting point for the trajectory, and desired output groups which are requested principally as a function of time. The selection of the groups and the print times is phase-dependent as described in Section IVE-2. The start of the phase in which powered flight is used is heralded by the powered-flight header.

To facilitate identification of the output quantities, a lettered mnemonic code precedes the floating-point representation of the quantity printed; each output group consists of an array of pairs and falls into one of the classifications: geocentric, geocentric conic, heliocentric, heliocentric conic, spacecraft and powered flight, target, and target conic. Each output group is further identified by a header which gives the reference body for the group and the class of output, and which further identifies the group in addition to the mnemonic codes.

As a further class of output, each tracking station has for identification a unique name which appears in its output group; all station output is of the same format except for the station name which therefore functions as an identifying header.

B. Explanation of Output and Mnemonic Codes

A sample output of the Space Trajectories Program is given in Exhibit A, followed by explanations of related output groups and interpretation of the mnemonic codes.

Exhibit A. Sample of space trajectories output

```
CASE 1                                         SPACE TRAJECTORIES                                               1

                                      LUNAR TRAJECTORY FOR DISPLAY OF OUTPUT

  GME  .39860320 06      J  .16234500-02      H -.57499999-05      D  .78749999-05     RE  .63781650 04    REM  .63781650 04
    G  .66709998-19      A  .88745998 29      B  .88763998 29      C  .88800998 29    OME  .41780741-02     AU  .14959900 09
  GMM  .49007589 04    GMS  .13271544 12    GMV  .32476950 06    GMA  .42977799 05    GMB  .00000000 00    GMJ  .12671060 09

INJECTICN CONDITIONS               MOON                JULIAN DATE 2437605.46008102                 NOV. 1,1961   23 02 31.000

GEOCENTRIC      RAD .66111676 04 LAT-.13312895 02 LON .35185650 03  VR .10531770 02 PTR .53912348 01 AZR .12183937 03
EARTH-FIXED     GMC .10000000 02 SGC .23500000 02  TO .82951000 05 GHA .26615809 02 GHO .40040394 02

   0 DAYS  0 HRS.  0 MIN.  0.000 SEC.              JULIAN DATE 2437605.46008102                 NOV. 1,1961   23 02 31.000

GEOCENTRIC                                                                                          EQUATORIAL COORDINATES

    X  .61020315 04      Y  .20384328 04      Z -.15223453 04     DX -.32657006 01     DY  .87950401 01     DZ -.56105608 01
    R  .66111673 04    DEC -.13312894 02     RA  .18472312 02      V  .10931419 02    PTH  .51935801 01     AZ  .12053672 03
    R  .66111671 04    LAT -.13312894 02    LON  .35185650 03     VE  .10531768 02    PTE  .53912356 01    AZE  .12183937 03
   XS -.11487518 09     YS -.86241661 08     ZS -.37395627 08    DXS  .19345540 02    DYS -.21062717 02    DZS -.91319481 01
   XM -.33705844 06     YM  .20458179 06     ZM  .90244289 05    DXM -.53328919 00    DYM -.76933039 00    DZM -.24058118 00
   XT -.33705844 06     YT  .20458179 06     ZT  .90244289 05    DXT -.53328919 00    DYT -.76933039 00    DZT -.24058118 00
   RS  .14843302 09     VS  .30021333 02     RM  .40448255 06     VM  .96651224 00     RT  .40448255 06     VT  .96651224 00
  GED -.13400356 02    ALT  .23410565 03    LOS  .19028130 03    RAS  .21689711 03    RAM  .14874382 03    LOM  .12212801 03
  DUT  .34000000 02     DT  .15000000 02     DR  .98952285 00    SHA  .36332697 04    DES -.14592105 02    DEM  .12891791 02

GEOCENTRIC                                               CONIC       ORBITAL B.T AND B.R            EQUATORIAL COORDINATES

EPOCH OF PERICENTER PASSAGE                        JULIAN DATE 2437605.45880927                 NOV. 1,1961   23 00 41.122
  SMA  .36606209 06    ECC  .98208910 00    INC  .33053889 02    LAN  .17714929 03    APF  .19449016 03    RCA  .65565008 04
   VH  .99195071-01     C3 -.10888951 01     C1  .71972740 05    SLR  .12995569 05    APO  .72556767 06    TFP  .10987826 03
   TA  .10482147 02     EA  .99919346 00     MA  .17946206-01    DAO -.78438361 01    RAO  .93712649 01    MTA  .18000000 03
   WX  .27126051-01     WY  .54475266 00     WZ  .83815793 00     PX  .97742251 00     PY  .16130761 00     PZ -.13647353 00
   QX -.20954557 00     QY  .82293641 00     QZ -.52807789 00     RX -.18851423 00     RY  .37401758 00     RZ -.90806012 00
  SXO  .97742251 00    SYO  .16130761 00    SZO -.13647353 00     TX  .95433517-01     TY -.91328567 00     TZ -.39598201 00
   BX  .20954558 00     BY -.82293645 00     BZ  .52807791 00     MX -.38387022 00     MY  .77985642 00     MZ -.49443663 00
  B.T  .38794487 05    B.R -.57027818 05      B  .68972345 05    PER  .36735872 05    CMD  .69215423-02    NOD -.46179063-02

   2 DAYS 17 HRS. 49 MIN.  3.027 SEC.              JULIAN DATE 2437608.20247716                 NOV. 4,1961   16 51 34.028

GEOCENTRIC                                                                                          EQUATORIAL COORDINATES

    X -.39863348 06      Y -.29945887 04      Z  .20618383 05     DX -.19910060 01     DY  .60147274 00     DZ  .61111473 00
    R  .39917757 06    DEC  .29607674 01     RA  .18043040 03      V  .21677951 01    PTH  .68384241 02     AZ  .30947590 03
    R  .39917755 06    LAT  .29607674 01    LON  .24384899 03     VE  .29758704 02    PTE  .38832075 01    AZE  .27097978 03
   XS -.11016135 09     YS -.91132499 08     ZS -.39516117 08    DXS  .20435530 02    DYS -.20210708 02    DZS -.87627225 01
   XM -.40001575 06     YM -.20879216 04     ZM  .21155252 05    DXM  .19506438-01    DYM -.92545351 00    DZM -.32619497 00
   XT -.40001575 06     YT -.20879216 04     ZT  .21155252 05    DXT  .19506438-01    DYT -.92545351 00    DZT -.32619497 00
   RS  .14833131 09     VS  .30047777 02     RM  .40058021 06     VM  .98145191 00     RT  .40058021 06     VT  .98145191 00
  GED  .29809082 01    ALT  .39279941 06    LOS  .28301831 03    RAS  .21959972 03    RAM  .18029905 03    LOM  .24371764 03
  DUT  .34000000 02     DT  .59999999 02     DR  .20153453 01    SHA -.27174378 06    DES -.15450419 02    DEM  .30272849 01
```

LUNAR TRAJECTORY FOR DISPLAY OF CUTPUT

GEOCENTRIC CONIC ORBITAL B.T AND B.R EQUATORIAL COORDINATES

EPOCH OF PERICENTER PASSAGE JULIAN DATE 2437606.26008714 NOV. 2,1961 18 14 31.530

SMA	-.14750990 06	ECC	.16517328 01	INC	.14043133 03	LAN	.18401888 03	APF	.26201373 03	RCA	.96137042 05
VH	.16438410 01	C3	.27022131 01	CI	.31877235 06	SLR	.25492974 06	APO	.00000000 00	TFP	.16782250 06
TA	.10263726 03	EA	.82934364 02	MA	.10715475 03	DAO	.18148300 02	RAO	.16064909 03	MTA	.12725941 03
WX	-.44644500-01	WY	.63543611 00	WZ	-.77C86173 00	PX	.19209591 00	PY	-.75177096 00	PZ	-.63082445 00
QX	-.98036015 00	QY	-.17624225 00	QZ	-.88502251-01	RX	-.14580064 00	RY	.45424170 00	RZ	-.87886664 00
SXO	-.89657073 00	SYO	.31486914 CO	SZO	.31147761 00	TX	.41821413 00	TY	.83337980 00	TZ	.36135171 00
BX	-.44064475 00	BY	-.70503785 00	BZ	-.55565624 00	MX	.27038722-01	MY	.77211700 00	MZ	.63490490 00
B.T	-.18861269 06	B.R	.45054496 05	B	.19391920 06	PER	.00000000 00	GMD	.55164090-04	NOD	.43146510-04

HELIOCENTRIC ECLIPTIC COORDINATES

X	.10976271 09	Y	.99336496 08	Z	.19986000 05	DX	-.22426536 02	DY	.22823516 02	DZ	.32056320 00
R	.14803916 09	LAT	.77352060-02	LON	.42145452 02	V	.31999456 02	PTH	-.23517084 01	AZ	.89425214 02
XE	.11016135 09	YE	.99331041 08	ZE	-.12200000 03	DXE	-.20435530 02	DYE	.22028572 02	DZE	-.83124636-03
XT	.10976133 09	YT	.99337542 08	ZT	.201175C0 05	DXT	-.20416024 02	DYT	.21049736 02	DZT	.68C64510-01
LTE	-.47124811-04	LOE	.42040567 02	LTT	.77861176-02	LOT	.42146111 02	RST	.14803884 09	VST	.29324223 02
EPS	.13699183 03	ESP	.10537696 00	SEP	.42902996 02	EPM	.14373027 03	EMP	.36122655 02	MEP	.14704286 00
MPS	.79277553 02	MSP	.98911702-02	SMP	.10072178 03	SEM	.43050066 02	EPS	.13684409 03	ESM	.10607100 00
EPT	.14373027 03	ETP	.36122655 02	TEP	.14704286 CC	TPS	.79277553 02	TSP	.98911702-02	STP	.10072178 03
SET	.43050066 02	STE	.13684409 03	EST	.10607100 00						

SPACECRAFT ATTITUCE AND POWERED FLIGHT SELENOCENTRIC EQUATORIAL COORDINATES

CX	.23309914 00	CY	.49189188 CO	CZ	.83887249 CC	CR	-.33032792 00	CPH	.77472363 00	CTH	.53915358 00
CW	.35443289 00	CV	.39685103 00	CGM	-.84669147 CO	CPE	.78863795 02	CPS	.13440009 03	CPM	.70711318 02
CPC	.11261770 03	CPT	.70711318 02			PHI	.21371750 03	PSI	.32737112 02	THA	.32737112 02
F	.00000000 00	M	.00000000 00	AC	.0000C000 00	INA	.00000000 00	IAS	.00000000 00		

SELENOCENTRIC EQUATORIAL COORDINATES

X	.13822747 04	Y	-.90666703 C3	Z	-.53686891 03	DX	-.20105124 01	DY	.15269262 01	DZ	.93730970 00
R	.17380900 04	DEC	-.17992012 02	RA	.32673817 03	V	.26929934 01	PTH	-.85574033 02	AZ	.56891172 02
R	.17380897 04	LAT	-.43402778 01	LON	.32124455 03	VR	.26926489 01	PTR	-.85669645 02	AZR	.77664858 02
LTS	.15159357 01	LNS	.22060488 03	LTE	-.38043675 C1	LNE	.35745796 03				
ALT	.89981078-01	SHA	.17077466 04	ALP	.20321495 02	DR	-.26849626 01	DP	.68507379-02	ASD	.89416961 02
HGE	.22300817 03	SVL	-.18324544 00	HNG	.79277496 02	SIA	.54313308 02				

SELENOCENTRIC CONIC CRBITAL B.T AND B.R EQUATORIAL COORDINATES

EPOCH OF PERICENTER PASSAGE JULIAN DATE 2437608.20775949 NOV. 4,1961 16 59 10.421

SMA	-.30383508 04	ECC	.10043716 01	INC	.37186323 02	LAN	.35208359 03	APF	.13790257 03	RCA	.13282400 02
VH	.12700263 01	C3	.16129668 01	CI	.36120943 03	SLR	.26622866 02	APC	.00000000 00	TFP	-.45639272 03
TA	-.16863648 03	EA	-.58361501 02	MA	-.10930402 02	DAI	.21200383 02	RAI	.14133522 03	MTA	.17465230 03
WX	-.83244152-01	WY	-.59864965 00	WZ	.79667421 00	PX	-.66137613 00	PY	.63119017 00	PZ	.40519207 00
QX	-.74542095 00	QY	-.49317148 00	QZ	-.44847548 00	RX	-.13273064 00	RY	.39774799 00	RZ	-.90784312 00
SXO	.58902467 00	SYC	-.67440617 00	SZO	-.44522611 00	DAO	-.26437806 02	RAO	.31113388 03	TF	.65944282 02
SXI	-.72797026 00	SYI	.58247959 00	SZI	.36163081 00	TX	.67263801 00	TY	.70888228 00	TZ	.21223561 00
BX	.68053634 00	BY	.54985127 00	BZ	.48428696 00	MX	.79667437 00	MY	-.73559021-11	MZ	.83244170-01
B.T	.27028028 03	B.R	-.88531979 02	B	.28441052 03	PER	.10435501 02				

(Identification as input to cells 100–109)

GME $\mu_{\oplus}$	J J	H H	D D	RE a	REM $a_{\oplus}$
G G	A A	B B	C C	OME ω	AU $A_{\odot}$
GMM $\mu_{☾}$	GMS $\mu_{\odot}$	GMV $\mu_{♀}$	GMA $\mu_{♂}$	GMB μ_B	GMJ $\mu_{♃}$

$\mu_{\oplus}$ gravitational coefficient for the Earth in km^3/sec^2

J coefficient of the second harmonic in Earth's oblateness

H coefficient of the third harmonic in Earth's oblateness

D coefficient of the fourth harmonic in Earth's oblateness

a Earth radius to be used in the Earth's oblate potential, km

$a_{\oplus}$ Earth radius to convert lunar ephemeris to km

G universal gravitational constant for lunar oblateness, km^3/sec^2-kg

A, B, C moments of inertia for the Moon to be used in the lunar oblate potential; units are kg-km^2

ω rotation rate of the Earth in deg/sec

$A_{\odot}$ Astronomical Unit to convert planetary ephemerides to km

$\mu_{☾}$ gravitational coefficient for the Moon in km^3/sec^2

$\mu_{\odot}$ gravitational coefficient for the Sun in km^3/sec^2

$\mu_{♀}$ gravitational coefficient for Venus in km^3/sec^2

$\mu_{♂}$ gravitational coefficient for Mars in km^3/sec^2

μ_B gravitational coefficient for Barycenter in km^3/sec^2

$\mu_{♃}$ gravitational coefficient for Jupiter in km^3/sec^2

INJECTION CONDITIONS (EQUINOX) (TARGET) (JULIAN DATE) (CALENDAR DATE)

(Central Body)	*X0 X_0	Y0 Y_0	Z0 Z_0	DX0 $\dot{X}_0$	DY0 $\dot{Y}_0$	DZ0 $\dot{Z}_0$
(Type)	GMC γ_c	SGC σ_c	TO t_0	GHA $\Upsilon(T_0)$	GHO $\Upsilon(T_M)$	(Ref. plane)**

X_0, Y_0, Z_0 vernal equinox Cartesian position, km

$\dot{X}_0$, $\dot{Y}_0$, $\dot{Z}_0$ vernal equinox Cartesian velocity, km/sec

γ_c elevation angle of reference vector for powered flight, deg

σ_c azimuth angle of reference vector, deg

t_0 seconds past midnight of injection time, sec

$\Upsilon(T_0)$ Greenwich hour angle of vernal equinox at injection epoch, deg

$\Upsilon(T_M)$ Greenwich hour angle of vernal equinox at previous midnight, deg

* If type is spherical inertial, then the line appears as:

RAD R DEC Φ RA Θ V V PTI Γ AZI Σ

If Earth-fixed or selenographic, the line appears as:

RAD r LAT ϕ LON θ VR v PTR γ AZR σ

If energy-asymptote, the line is modified to read:

AZL Σ_L RAD R PTH Γ C3 c_3 DAO Φ_S RAO Θ_S

R radius, km

Φ declination, deg

Θ right ascension, deg

V velocity, km/sec

Γ path angle, deg

Σ azimuth angle, deg

r radius, km

ϕ latitude, deg

θ longitude, deg

v velocity relative to rotating coordinate system, km/sec

γ path angle relative to rotating coordinate system, deg

σ azimuth angle relative to rotating coordinate system, deg

Σ_L azimuth at launch site, deg

R radius, km

Γ path angle, deg

c_3 "energy" or *vis viva* integral, km²/sec²

Φ_S declination of ascending asymptote, deg

Θ_S right ascension of ascending asymptote, deg

** If ecliptic coordinates are input, then ECLIPTIC is printed; otherwise space is left blank.

POWERED-FLIGHT PARAMETERS THRUST F FLOW $\dot{m}$ MASS m_0 BURN t_B

F thrust, lb force

$\dot{m}$ mass flow rate, lb mass/sec

m_0 initial mass, lb

t_B burning interval, sec

The powered-flight header appears only at the start of the powered-flight phase

Format for time at print epoch:

(SEXAGESIMAL INTERVAL PAST INJECTION) (EQUINOX) (JULIAN DATE) (CALENDAR DATE)

GEOCENTRIC GROUP

GEOCENTRIC (COORDINATE PLANE)

X	X	Y	Y	Z	Z	DX	$\dot{X}$	DY	$\dot{Y}$	DZ	$\dot{Z}$
R	R	DEC	Φ	RA	Θ	V	V	PTH	Γ	AZ	Σ
R	r	LAT	ϕ	LON	θ	VE	υ	PTE	γ	AZE	σ
XS	X_S	YS	Y_S	ZS	Z_S	DXS	$\dot{X}_S$	DYS	$\dot{Y}_S$	DZS	$\dot{Z}_S$
XM	X_M	YM	Y_M	ZM	Z_M	DXM	$\dot{X}_M$	DYM	$\dot{Y}_M$	DZM	$\dot{Z}_M$
XT	X_T	YT	Y_T	ZT	Z_T	DXT	$\dot{X}_T$	DYT	$\dot{Y}_T$	DZT	$\dot{Z}_T$
RS	R_S	VS	V_S	RM	R_M	VM	V_M	RT	R_T	VT	V_T
GED	ϕ'	ALT	h_E	LOS	θ_S	RAS	Θ_S	RAM	Θ_M	LOM	θ_M
DUT	ΔT	DT	h	DR	$\dot{R}$	SHA	d	DES	Φ_S	DEM	Φ_M

X, Y, Z vernal equinox Cartesian position, km

$\dot{X}, \dot{Y}, \dot{Z}$ vernal equinox Cartesian velocity, km/sec

R radius, km

Φ declination, deg

Θ right ascension, deg

V inertial speed, km/sec

Γ path angle, deg

Σ azimuth angle, deg

r radius, km

ϕ geocentric latitude, deg

θ longitude, deg

υ Earth-fixed speed, km/sec

γ Earth-fixed path angle, deg

σ Earth-fixed azimuth angle, deg

X_T, Y_T, Z_T the geocentric position of the target body, km

$\dot{X}_T, \dot{Y}_T, \dot{Z}_T$ the geocentric velocity of the target body, km/sec

R_S Earth-Sun distance, km

V_S the geocentric speed of the Sun, km/sec

R_M Earth-Moon distance, km

V_M speed of Moon, km/sec

R_T Earth-Target distance, km

V_T speed of Target body, km/sec

ϕ' geodetic latitude

h_E altitude above the Earth's surface, km

θ_S longitude of Sun, deg

Θ_S right ascension of Sun, deg

Θ_M right ascension of Moon, deg

θ_M longitude of Moon, deg

$\left.\begin{matrix} X_S \\ Y_S \\ Z_S \end{matrix}\right\}$ the geocentric position of the Sun, km

$\left.\begin{matrix} \dot{X}_S \\ \dot{Y}_S \\ \dot{Z}_S \end{matrix}\right\}$ the geocentric velocity of the Sun, km/sec

$\left.\begin{matrix} X_M \\ Y_M \\ Z_M \end{matrix}\right\}$ the geocentric position of the Moon, km

$\left.\begin{matrix} \dot{X}_M \\ \dot{Y}_M \\ \dot{Z}_M \end{matrix}\right\}$ the geocentric velocity of the Moon, km/sec

ΔT Ephemeris Time minus Universal Time, sec

h Adams-Moulton step size, sec

$\dot{R}$ radial speed, km/sec

d Sun shadow parameter, km

$$d = \frac{-\,|\mathbf{R}_{\oplus P} \times \mathbf{R}_{\oplus \odot}|}{R_{\oplus \odot}} \operatorname{sgn}(\mathbf{R}_{\oplus P} \cdot \mathbf{R}_{\oplus \odot})$$

Φ_S declination of the Sun, deg

Φ_M declination of the Moon, deg

TRACKING STATIONS

(STATION NAME)		HA α_i	DEC δ_i	ELE γ_i	AZI σ_i
POL p_i	LKA λ_i	DHA $\dot{\alpha}_i$	DDE $\dot{\delta}_i$	DEL $\dot{\gamma}_i$	DAZ $\dot{\sigma}_i$
XIP X_{ip}	YIP Y_{ip}	ZIP Z_{ip}	RGE r_{ip}	DRG $\dot{r}_{ip}$	DDR $\ddot{r}_{ip}$
RDI r_i	PHI ϕ_i	THI θ_i	FBI f_{B_i}	FCI f_{C_i}	FRQ f_i

α_i local hour angle of probe, deg

δ_i local declination of probe, deg

γ_i north azimuth of probe, deg

σ_i elevation angle of probe, deg

p_i polarization angle, deg

λ_i look angle, deg

$\dot{\alpha}_i$ hour-angle rate, deg/hr

$\dot{\delta}_i$ declination rate, deg/hr

$\dot{\gamma}_i$ azimuth rate, deg/hr

$\dot{\sigma}_i$ elevation rate, deg/hr

$\left.\begin{matrix} X_{ip} \\ Y_{ip} \\ Z_{ip} \end{matrix}\right\}$ Cartesian coordinates of the probe centered at the station and axes parallel to those of the true equator and equinox of date, km

r_{ip} slant range of probe, km

$\dot{r}_{ip}$ slant-range rate, km/sec

$\ddot{r}_{ip}$ rate of the slant-range rate, km/sec^2

r_i radius of the station, km

ϕ_i north geocentric latitude of the station, deg

θ_i east longitude of the station, deg

f_{B_i} additive constant for doppler, cps

f_{C_i} multiplicative constant for doppler, cps/km/sec

f_i doppler: $f_i = f_{B_i} - f_{C_i}\,\dot{r}_{ip}$, cps

OSCULATING CONICS

(CENTRAL BODY) CONIC (TYPE OF **B·T** AND **B·R**) (COORDINATE PLANE)

EPOCH OF PERICENTER PASSAGE (JULIAN DATE) (CALENDAR DATE)

SMA	a	ECC	ε	INC	i	LAN	Ω	APF	ω	RCA	q
VH	V_H	C3	c_3	C1	c_1	SLR	p	APO	q_2	TFP	Δt
TA	υ	EA	E	MA	M	DAI[a]	Φ_I	RAI[a]	Θ_I	MTA	υ_{max}
WX	W_x	WY	W_y	WZ	W_z	PX	P_x	PY	P_y	PZ	P_z
QX	Q_x	QY	Q_y	QZ	Q_z	RX	R_x	RY	R_y	RZ	R_z
SXI[a]	S_{xI}	SYI[a]	S_{yI}	SZI[a]	S_{zI}	TX	T_x	TY	T_y	TZ	T_z
BX	B'_x	BY	B'_y	BZ	B'_z	MX	M_x	MY	M_y	MZ	M_z
B.T	$\mathbf{B\cdot T}$	B.R	$\mathbf{B\cdot R}$	B	b	PER	P	OMD[b]	$\dot{\omega}$	NOD[b]	$\dot{\Omega}$

a semimajor or semitransverse axis; $a < 0$ for hyperbola, km

ε eccentricity, rad

i inclination, deg

Ω longitude or right ascension of ascending node, deg

ω argument of pericenter, deg

q closest approach distance, km

V_H hyperbolic excess speed (velocity at apogee for ellipse), km/sec

c_3 twice the total energy per unit mass or *vis viva* integral, km²/sec²

c_1 angular momentum, km²/sec

p semilatus rectum, km

q_2 apocenter distance, km

Δt time from pericenter passage, sec

υ true anomaly, deg

E eccentric anomaly, deg

M mean anomaly, deg

Φ_I declination or latitude of incoming asymptote, deg

Θ_I right ascension or longitude of incoming asymptote, deg

υ_{max} maximum true anomaly, deg

W_x X-component of, W_y Y-component of, W_z Z-component of $\mathbf{W} = \mathbf{P} \times \mathbf{Q}$

P_x X-component of, P_y Y-component of, P_z Z-component of **P**

Q_x X-component of, Q_y Y-component of, Q_z Z-component of **Q**

R_x X-component of, R_y Y-component of, R_z Z-component of **R**

S_{xI} X-component of, S_{yI} Y-component of, S_{zI} Z-component of $\mathbf{S}_I$, incoming asymptote

T_x X-component of, T_y Y-component of, T_z Z-component of $\mathbf{T} = \mathbf{R} \times \mathbf{S}$

B'_x X-component of unit vector **B′**, B'_y Y-component of unit vector **B′**, B'_z Z-component of unit vector **B′** $\mathbf{B}' = \dfrac{\mathbf{B}}{b}$

M_x X-component of M, M_y Y-component of M, M_z Z-component of M $\mathbf{M} = \mathbf{W} \times \dfrac{\mathbf{R}_0}{R_0}$

B·T T-component of **B**, km

B·R R-component of **B**, km

b magnitude of **B**, km

P period, days if heliocentric, otherwise min; if $\varepsilon \geqq 1$, P is replaced by

$$\Delta T_f = \frac{|a|^{3/2}}{\sqrt{\mu}} \sinh^{-1}\left(\frac{\varepsilon^2 - 1}{2\varepsilon}\right),$$

the linearized time-of-flight correction, target conic only, sec

$\dot{\omega}$ rate of change of argument of perigee, deg/day

$\dot{\Omega}$ rate of change of right ascension of the ascending node, deg/day

---

[a]Values are printed in the heliocentric and target-centered conics. In addition, the target-centered conic prints an extra line marked SXO SYO SZO DAO RAO TF where O is for outgoing asymptote. This line is printed just above the line marked SXI SYI . . . . The geocentric conic prints SXO SYO SZO in place of SXI SYI SZI in line six and prints DAO and RAO in place of DAI and RAI in line three. *TF*, the time of flight, is in hr for Moon or Earth target, in days otherwise.

[b]$\dot{\omega}$ and $\dot{\Omega}$ are printed only in geocentric conic.

HELIOCENTRIC GROUP

HELIOCENTRIC (COORDINATE PLANE)

X	X	Y	Y	Z	Z	DX	$\dot{X}$	DY	$\dot{Y}$	DZ	$\dot{Z}$
R	R	LAT	β	LON	λ	V	V	PTH	Γ	AZ	Σ
XE	X_E	YE	Y_E	ZE	Z_E	DXE	$\dot{X}_E$	DYE	$\dot{Y}_E$	DZE	$\dot{Z}_E$
XT	X_T	YT	Y_T	ZT	Z_T	DXT	$\dot{X}_T$	DYT	$\dot{Y}_T$	DZT	$\dot{Z}_T$
LTE	$\beta_\oplus$	LOE	$\lambda_\oplus$	LTT	β_T	LOT	λ_T	RST	R_{ST}	VST	V_{ST}
EPS	$\angle(\mathbf{R}_{EP}, \mathbf{R}_{SP})$	ESP	$\angle(\mathbf{R}_{ES}, \mathbf{R}_{PS})$	SEP	$\angle(\mathbf{R}_{SE}, \mathbf{R}_{PE})$	EPM	$\angle(\mathbf{R}_{EP}, \mathbf{R}_{MP})$	EMP	$\angle(\mathbf{R}_{EM}, \mathbf{R}_{PM})$	MEP	$\angle(\mathbf{R}_{ME}, \mathbf{R}_{PE})$
MPS	$\angle(\mathbf{R}_{MP}, \mathbf{R}_{SP})$	MSP	$\angle(\mathbf{R}_{MS}, \mathbf{R}_{PS})$	SMP	$\angle(\mathbf{R}_{SM}, \mathbf{R}_{PM})$	SEM	$\angle(\mathbf{R}_{SE}, \mathbf{R}_{ME})$	EMS	$\angle(\mathbf{R}_{EM}, \mathbf{R}_{SM})$	ESM	$\angle(\mathbf{R}_{ES}, \mathbf{R}_{MS})$
EPT	$\angle(\mathbf{R}_{EP}, \mathbf{R}_{TP})$	ETP	$\angle(\mathbf{R}_{ET}, \mathbf{R}_{PT})$	TEP	$\angle(\mathbf{R}_{TE}, \mathbf{R}_{PE})$	TPS	$\angle(\mathbf{R}_{TP}, \mathbf{R}_{SP})$	TSP	$\angle(\mathbf{R}_{TS}, \mathbf{R}_{PS})$	STP	$\angle(\mathbf{R}_{ST}, \mathbf{R}_{PT})$
SET	$\angle(\mathbf{R}_{SE}, \mathbf{R}_{TE})$	STE	$\angle(\mathbf{R}_{ST}, \mathbf{R}_{ET})$	EST	$\angle(\mathbf{R}_{ES}, \mathbf{R}_{TS})$						

- X, Y, Z — vernal equinox Cartesian position, km
- $\dot{X}, \dot{Y}, \dot{Z}$ — vernal equinox Cartesian velocity, km/sec
- R — Sun-Probe radius, km
- β — celestial latitude (or Φ, the declination if equatorial), deg
- λ — celestial longitude (or Θ, the right ascension if equatorial), deg
- V — speed, km/sec
- Γ — path angle, deg
- Σ — azimuth angle, deg
- X_E, Y_E, Z_E — heliocentric position of the Earth, km
- $\dot{X}_E, \dot{Y}_E, \dot{Z}_E$ — heliocentric velocity of the Earth, km/sec
- X_T, Y_T, Z_T — heliocentric position of the target, km
- $\dot{X}_T, \dot{Y}_T, \dot{Z}_T$ — heliocentric velocity of the target, km/sec
- $\beta_\oplus$ — celestial latitude (or declination) of the Earth, deg
- $\lambda_\oplus$ — celestial longitude (or right ascension) of the Earth, deg
- β_T — celestial latitude (or declination) of the target, deg
- λ_T — celestial longitude (or right ascension) of the target, deg
- R_{ST} — distance of the target from the Sun, km
- V_{ST} — speed of the target with respect to the Sun, km/sec
- $\angle(\mathbf{R}_{EP}, \mathbf{R}_{SP})$ — Earth-Probe-Sun angle, deg
- $\angle(\mathbf{R}_{ES}, \mathbf{R}_{PS})$ — Earth-Sun-Probe angle, deg
- $\angle(\mathbf{R}_{SE}, \mathbf{R}_{PE})$ — Sun-Earth-Probe angle, deg
- $\angle(\mathbf{R}_{EP}, \mathbf{R}_{MP})$ — Earth-Probe-Moon angle, deg
- $\angle(\mathbf{R}_{EM}, \mathbf{R}_{PM})$ — Earth-Moon-Probe angle, deg
- $\angle(\mathbf{R}_{ME}, \mathbf{R}_{PE})$ — Moon-Earth-Probe angle, deg
- $\angle(\mathbf{R}_{MP}, \mathbf{R}_{SP})$ — Moon-Probe-Sun angle, deg
- $\angle(\mathbf{R}_{MS}, \mathbf{R}_{PS})$ — Moon-Sun-Probe angle, deg
- $\angle(\mathbf{R}_{SM}, \mathbf{R}_{PM})$ — Sun-Moon-Probe angle, deg
- $\angle(\mathbf{R}_{SE}, \mathbf{R}_{ME})$ — Sun-Earth-Moon angle, deg
- $\angle(\mathbf{R}_{EM}, \mathbf{R}_{SM})$ — Earth-Moon-Sun angle, deg
- $\angle(\mathbf{R}_{ES}, \mathbf{R}_{MS})$ — Earth-Sun-Moon angle, deg
- $\angle(\mathbf{R}_{EP}, \mathbf{R}_{TP})$ — Earth-Probe-Target angle, deg
- $\angle(\mathbf{R}_{ET}, \mathbf{R}_{PT})$ — Earth-Target-Probe angle, deg
- $\angle(\mathbf{R}_{TE}, \mathbf{R}_{PE})$ — Target-Earth-Probe angle, deg
- $\angle(\mathbf{R}_{TP}, \mathbf{R}_{SP})$ — Target-Probe-Sun angle, deg
- $\angle(\mathbf{R}_{TS}, \mathbf{R}_{PS})$ — Target-Sun-Probe angle, deg
- $\angle(\mathbf{R}_{ST}, \mathbf{R}_{PT})$ — Sun-Target-Probe angle, deg
- $\angle(\mathbf{R}_{SE}, \mathbf{R}_{TE})$ — Sun-Earth-Target angle, deg
- $\angle(\mathbf{R}_{ST}, \mathbf{R}_{ET})$ — Sun-Target-Earth angle, deg
- $\angle(\mathbf{R}_{ES}, \mathbf{R}_{TS})$ — Earth-Sun-Target angle, deg

SPACECRAFT GROUP

SPACECRAFT ATTITUDE AND POWERED FLIGHT (CENTRAL BODY) (COORDINATE PLANE)

CX C_x	CY C_y	CZ C_z	CR $\mathbf{C}\cdot\mathbf{R}'$	CPH $\mathbf{C}\cdot\mathbf{\Phi}'$	CTH $\mathbf{C}\cdot\mathbf{\Theta}'$
CW $\mathbf{C}\cdot\mathbf{W}$	CV $\mathbf{C}\cdot\mathbf{V}'$	CGM $\mathbf{C}\cdot(\mathbf{W}\times\mathbf{V}')$	CPE $\angle(\mathbf{C}, -\mathbf{R}_{EP})$	CPS $\angle(\mathbf{C}, -\mathbf{R}_{SP})$	CPM $\angle(\mathbf{C}, -\mathbf{R}_{MP})$
CPC $\angle(\mathbf{C}, \mathbf{C}_{CAN})$	CPT $\angle(\mathbf{C}, -\mathbf{R}_{TP})$		PHI ϕ	PSI ψ	THA θ
F F	M m	AC a	INA $\int a$	IAS $\int a^2$	

C_x X-component of $\mathbf{C}$
C_y Y-component of $\mathbf{C}$
C_z Z-component of $\mathbf{C}$
} $\mathbf{C}$ is given by the input quantities γ_C, σ_C

$\mathbf{C}\cdot\mathbf{R}'$ $\cos\angle(\mathbf{C}, \mathbf{R}')$
$\mathbf{C}\cdot\mathbf{\Phi}'$ $\cos\angle(\mathbf{C}, \mathbf{\Phi}')$
$\mathbf{C}\cdot\mathbf{\Theta}'$ $\cos\angle(\mathbf{C}, \mathbf{\Theta}')$
} $\mathbf{R}' = \dfrac{\mathbf{R}}{R}$

$$\cos\Phi\,\mathbf{\Phi}' + \sin\Phi\,\mathbf{R}' = (0, 0, 1)$$

where

$$\sin\Phi = \frac{Z}{R} \qquad \mathbf{\Theta}' = \mathbf{\Phi}' \times \mathbf{R}'$$

$\mathbf{C}\cdot\mathbf{W}$ $\cos\angle(\mathbf{C}, \mathbf{W})$
$\mathbf{C}\cdot\mathbf{V}'$ $\cos\angle(\mathbf{C}, \mathbf{V}')$
$\mathbf{C}\cdot(\mathbf{W}\times\mathbf{V}')$ $\cos\angle(\mathbf{C}, (\mathbf{W}\times\mathbf{V}'))$
} $\mathbf{W} = \dfrac{\mathbf{R}\times\mathbf{V}}{|\mathbf{R}\times\mathbf{V}|}$ $\quad \mathbf{V}' = \dfrac{\mathbf{V}}{V}$

$\angle(\mathbf{C}, -\mathbf{R}_{EP})$ Axis-Probe-Earth angle, deg
$\angle(\mathbf{C}, -\mathbf{R}_{SP})$ Axis-Probe-Sun angle, deg
$\angle(\mathbf{C}, -\mathbf{R}_{MP})$ Axis-Probe-Moon angle, deg

$\angle(\mathbf{C}, \mathbf{C}_{CAN})$ Axis-Probe-Canopus angle, deg
$\angle(\mathbf{C}, -\mathbf{R}_{TP})$ Axis-Probe-Target angle, deg

$$\begin{cases} \cos\phi = \mathbf{S}_1\cdot\mathbf{C} \\ \sin\phi = (\mathbf{S}\times\mathbf{S}_1)\cdot\mathbf{C} \end{cases} \quad 0 \le \phi < 360^\circ$$

$$\psi = \sin^{-1}\mathbf{S}\cdot\mathbf{C} \quad -90^\circ \le \psi \le 90^\circ$$

$$\theta = |\psi|$$

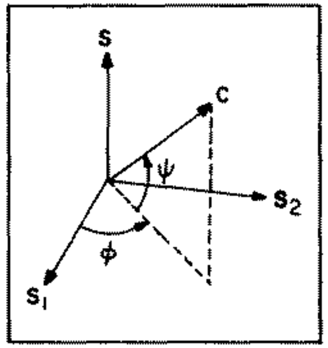
F thrust, lb
m mass or weight, lb
a acceleration, km/sec²
$\int a$ integral of a, km/sec
$\int a^2$ integral of a, km²/sec³

$$a = \frac{F}{m}$$

$$m = m_0 - \dot{m}\,(T - T_I)$$

$$\int a = \frac{F}{\dot{m}} \ln\frac{m_0}{m}$$

$$\int a^2 = \frac{F^2}{\dot{m}}\left[\frac{1}{m} - \frac{1}{m_0}\right]$$



Sketch 6. Spacecraft coordinates

$$\mathbf{S}_1 = \frac{\mathbf{R}_{P\odot}}{R_{P\odot}}$$

$$\mathbf{S}_2 = \frac{\mathbf{R}_{P\oplus}}{R_{P\oplus}}$$

$$\mathbf{S} = \frac{\mathbf{S}_1\times\mathbf{S}_2}{|\mathbf{S}_1\times\mathbf{S}_2|}$$

TARGET GROUP

(TARGET) CENTRIC (COORDINATE PLANE)

X X	Y Y	Z Z	DX $\dot{X}$	DY $\dot{Y}$	DZ $\dot{Z}$
R R	DEC Φ	RA Θ	V V	PTH Γ	AZ Σ
R r	LAT ϕ	LON θ	VR v	PTR γ	AZR σ
LTS $\beta_{☾\odot}$	LNS $\lambda_{☾\odot}$	LTE $\beta_{☾\oplus}$	LNE $\lambda_{☾\oplus}$		
ALT h_T	SHA d	ALP α	DR $\dot{R}$	DP $\dot{\psi}$	ASD δ
HGE Θ_{SE}	SVL Φ_{ST}	HNG Θ_{ST}	SIA δ_S		

$\left.\begin{matrix}X\\Y\\Z\end{matrix}\right\}$ target-centered vernal equinox position, km

$\left.\begin{matrix}\dot{X}\\\dot{Y}\\\dot{Z}\end{matrix}\right\}$ target-centered vernal equinox velocity, km/sec

R radius from target center, km

Φ declination (or celestial latitude), deg

Θ right ascension (or celestial longitude), deg

V speed relative to the target, km/sec

Γ target-body path angle, deg

Σ target-body azimuth angle, deg

(for Moon only):

r radius from target center, km

ϕ target-centered latitude, deg

θ target-centered longitude, deg

v speed relative to the rotating target, km/sec

γ rotating target-body path angle, deg

σ rotating target-body azimuth angle, deg

(for Moon only):

$\beta_{☾\odot}$ selenographic latitude of the Sun, deg

$\lambda_{☾\odot}$ selenographic longitude of the Sun, deg

$\beta_{☾\oplus}$ selenographic latitude of the Earth, deg

$\lambda_{☾\oplus}$ selenographic longitude of the Earth, deg

h_T altitude above the target body's surface, km

d Sun's shadow parameter, km

$$d = \frac{-|\mathbf{R}_{TP} \times \mathbf{R}_{T\odot}|}{R_{T\odot}} \operatorname{sgn}(\mathbf{R}_{TP} \cdot \mathbf{R}_{T\odot})$$

α illuminated crescent orientation viewing angle, deg

$\dot{R}$ radial rate, km/sec

$\dot{\psi}$ transverse angular velocity, deg/sec

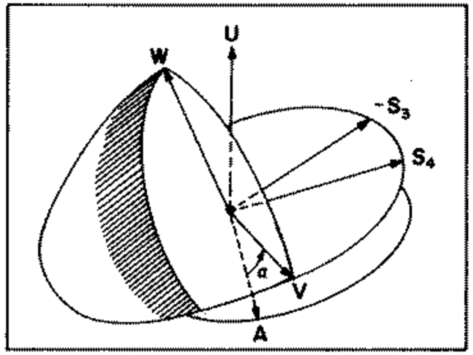
δ angular semidiameter, deg

Θ_{SE} right ascension of Earth in spacecraft coordinate system, deg

Φ_{ST} declination of target in spacecraft coordinate system, deg

Θ_{ST} right ascension of target in spacecraft coordinate system, deg

δ_E $\angle(\mathbf{R}_{TP}, \mathbf{R}_{EP}) - \delta$, deg



Sketch 7. Illuminated crescent orientation viewing angle

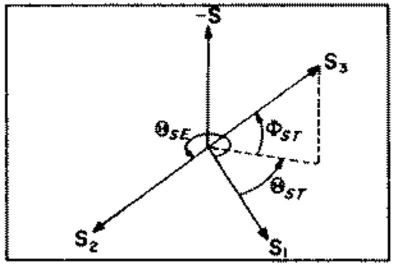
$$-\mathbf{S}_3 = \frac{\mathbf{R}_{TP}}{R_{TP}} \qquad \mathbf{U} = (0, 0, 1)$$

$$\mathbf{S}_4 = \frac{\mathbf{R}_{T\odot}}{R_{T\odot}} \qquad \mathbf{V} = \mathbf{W} \times \mathbf{S}_3$$

$$\mathbf{A} = \frac{\mathbf{U} \times \mathbf{S}_3}{|\mathbf{U} \times \mathbf{S}_3|}$$

$$\mathbf{W} = \frac{\mathbf{S}_3 \times \mathbf{S}_4}{|\mathbf{S}_3 \times \mathbf{S}_4|} \qquad \cos\alpha = \mathbf{A} \cdot \mathbf{V}$$

Sketch 8. Hinge and swivel angles



Earth hinge angle $\begin{cases}\cos\Theta_{SE} = \mathbf{S}_2 \cdot \mathbf{S}_1 \\ \sin\Theta_{SE} = \mathbf{S}_2 \cdot (\mathbf{S}_1 \times \mathbf{S})\end{cases}$ $\quad 0 \leq \Theta_{SE} < 360°$

Target swivel angle $\{\sin\Phi_{ST} = -\mathbf{S}_3 \cdot \mathbf{S}$ $\quad -90° \leq \Phi_{ST} \leq 90°$

Target hinge angle $\begin{cases}\cos\Theta_{ST} = \mathbf{S}_3 \cdot \mathbf{S}_1 \\ \sin\Theta_{ST} = \mathbf{S}_3 \cdot (\mathbf{S}_1 \times \mathbf{S})\end{cases}$ $\quad 0 \leq \Theta_{ST} < 360°$

$$\mathbf{S}_1 = \frac{\mathbf{R}_{P\odot}}{R_{P\odot}} \qquad \mathbf{S} = \frac{\mathbf{S}_1 \times \mathbf{S}_2}{|\mathbf{S}_1 \times \mathbf{S}_2|}$$

$$\mathbf{S}_2 = \frac{\mathbf{R}_{P\oplus}}{R_{P\oplus}} \qquad \mathbf{S}_3 = \frac{\mathbf{R}_{PT}}{R_{PT}}$$

APPENDIX

Description of Major Subroutines

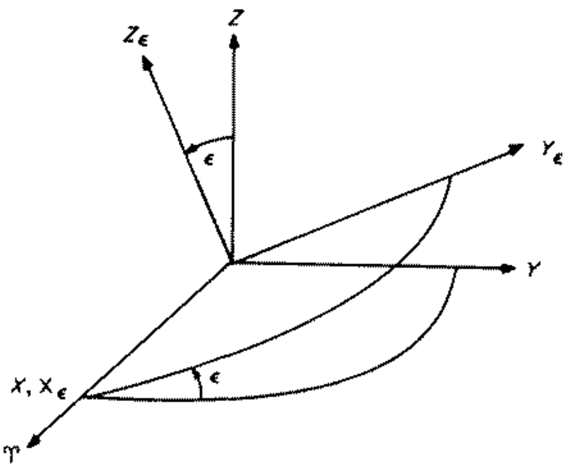
INDEX

INDEX (Cont'd)

1. Input-Output Routines

ECLIP

The ecliptic plane is characterized by its inclination to the equator, ϵ, the obliquity of the ecliptic, and its ascending node on the equator, the vernal equinox.



Sketch A-1. Relation between ecliptic and equatorial planes

In Sketch A-1, X, Y, Z is the equatorial frame; $X_\epsilon, Y_\epsilon, Z_\epsilon$ the ecliptic. ♈ is the vernal equinox. The coordinates are related by

$$\begin{pmatrix} X_\epsilon \\ Y_\epsilon \\ Z_\epsilon \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\epsilon & \sin\epsilon \\ 0 & -\sin\epsilon & \cos\epsilon \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

The calling sequence is given by

CALL ECLIP

(OP) X,,Y

X − 3, X − 2, X − 1 contain the input vector; Y − 3, Y − 2, Y − 1 contain the output vector; X = Y is permitted. OP = PZE assumes equatorial input to be rotated to ecliptic; OP = MZE regards input as ecliptic and rotates to equatorial.

Normally X, Y, Z is regarded as the true equator and equinox of date and ϵ the true obliquity; however, for some applications it is necessary to rotate between the mean equator and equinox of 1950.0 and the ecliptic of 1950.0; for the latter purpose $\bar{\epsilon}_{1950.0}$, the mean obliquity of 1950.0, is used. To provide for this flexibility, ECLIP assumes that the desired obliquity has been placed in the COMMON location ET.

The subroutine uses nine cells of erasable storage starting at COMMON.

GHA

For purposes of calculating ♈(T), the Greenwich hour angle of the vernal equinox at epoch T, the following mean value is assumed:

$$\Upsilon_M(T) \equiv 100^\circ.07554260 + 0^\circ.9856473460d + (2^\circ.9015)10^{-13}d^2 + \omega t \pmod{360^\circ}$$

$$0 \leqq \Upsilon_M(T) < 360^\circ$$

where T is the epoch under consideration in U.T.; d is integer days past 0^h January 1, 1950; t is seconds past 0^h of the epoch T. ω, the Earth's rotation rate, is assumed to be a function of time:

$$\omega = \frac{0.00417807417}{1 + (5.21)10^{-13}d} \text{ deg/sec}$$

Given $\delta\alpha$, the nutation in right ascension, the true value of the hour angle is computed:

$$\Upsilon(T) = \Upsilon_M(T) + \delta\alpha$$

The calling sequence consists of

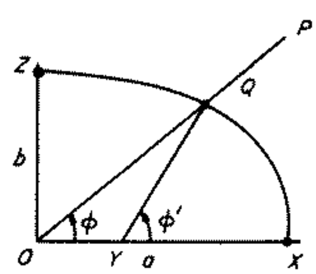
CALL GHA,

where it is assumed that the U.T. epoch appears in double-precision seconds past 0^h January 1, 1950, in the COMMON cells T, T + 1, and that $\delta\alpha$ has been computed and appears in NUTRA. ♈(T) is stored in the COMMON location GHA(T), while ω is placed in OMEGA and ω in rad/sec is stored in LOMEGA.

The subroutine uses seven cells of erasable storage starting at COMMON.

GEDLAT

To obtain an accurate numerical expression for the small difference between the geodetic latitude ϕ' and the geocentric latitude ϕ, a Fourier series expansion is resorted to. The geometry appears in Sketch A-2:



Sketch A-2. Geodetic and geocentric latitudes

Consider a point P above the Earth and extend a line to the center of the Earth O. If a spheroidal Earth is assumed, then let OZ be the spin axis of the Earth and the plane ZOX contain the line OP with Q the intersection of OP with the surface; OX lies in the equatorial plane. Then the angle ϕ, the geocentric latitude, is the angle between the lines OQ and OX. If the normal YQ to the surface is constructed at Q to intersect OX at Y, then ϕ', the geodetic latitude, is the angle between the lines YQ and YX. The ellipse of cross section is characterized by a, the semimajor axis, and b, the semiminor axis. It is convenient to introduce $\varepsilon^2 = 1 - b^2/a^2$ to describe $\phi - \phi'$ by a Fourier series.

As the defining relation, $\tan \phi = (1 - \varepsilon^2) \tan \phi'$ is adopted which leads to the series in $2\phi'$ for $\phi - \phi'$:

$$\phi - \phi' = \sum_{j=1}^{\infty} a_j \sin 2 j\phi'$$

where

$$a_j = \frac{(-1)^j}{j}\left(\frac{\varepsilon^2}{2 - \varepsilon^2}\right)^j$$

Alternatively, $\phi' - \phi$ may be expanded as a Fourier series in 2ϕ:

$$\phi' - \phi = \sum_{j=1}^{\infty} b_j \sin 2 j \phi$$

where the b_j are obtained by replacing $1 - \varepsilon^2$ by $1/(1 - \varepsilon^2)$ in the expression for the a_j. Incidentally, $b_j = (-1)^j a_j$ is obtained by performing the substitution.

Using the Clarke spheroid of 1866 with $a = 6378.2064$ km, $b = 6356.5838$ km, and the derived value $\varepsilon^2 = 0.006768657997$, the following numerical formula results:

$$\phi' - \phi = b_1 \sin 2\phi + b_2 \sin 4\phi + b_3 \sin 6\phi$$

where

$$b_1 = 0^\circ\!.19456624$$

$$b_2 = 0^\circ\!.00033036$$

$$b_3 = 0^\circ\!.00000075$$

An auxiliary problem is the determination of the altitude of P above the spheroid. An approximate solution is obtained by regarding $\overline{QP} = h$ as the desired altitude. If $R = \overline{OP}$ is given, then if $\rho = \overline{OQ}$ is calculated, h would be given by $h = R - \rho$.

The arc of the ellipse may be described by the parameter ψ, where $x = a \cos \psi$, $y = b \sin \psi$ for $Q(x, y)$. Then the expression for ρ is

$$\rho = a\sqrt{1 - \varepsilon^2 \sin^2 \psi}$$

Actually, the formula programmed for ρ differs in that ϕ was used for ψ:

$$\rho' = a\sqrt{1 - \varepsilon^2 \sin^2 \phi}$$

The numerical difference between the two formulas may be assessed by expanding ρ and ρ' in power series in ε^2 and using the relation

$$\sin^2 \phi = \frac{(1 - \varepsilon^2) \sin^2 \psi}{1 - \varepsilon^2 \sin^2 \psi};$$

$$\frac{\rho}{a} = 1 - \frac{1}{2}\varepsilon^2 \sin^2 \psi - \frac{1}{8}\varepsilon^4 \sin^4 \psi - \frac{1}{16}\varepsilon^6 \sin^6 \psi + O(\varepsilon^8)$$

$$\frac{\rho'}{a} = 1 - \frac{1}{2}\varepsilon^2 \sin^2 \psi$$

$$+ \varepsilon^4\left\{-\frac{1}{2}\sin^2 \psi (\sin^2 \psi - 1) - \frac{1}{8}\sin^4 \psi\right\}$$

$$+ \varepsilon^6\left\{-\frac{3}{4}\sin^4 \psi (\sin^2 \psi - 1) - \frac{1}{16}\sin^6 \psi\right\} + O(\varepsilon^8)$$

so

$$\rho' - \rho = a\left\{\frac{1}{8}\varepsilon^4 \sin^2 2\psi + \frac{3}{16}\varepsilon^6 \sin^2 \psi \sin^2 2\psi + O(\varepsilon^8)\right\}$$

Thus the maximum difference, occurring near $\psi = 45^\circ$, should be about $a\varepsilon^4/8 \approx 0.06$ km.

The calling sequence is given by

(AC) = ϕ

CALL GEDLAT

and upon return

(AC) = ϕ', (MQ) = ρ'

The subroutine uses 10 words of erasable storage starting at COMMON.

JEKYL

JEKYL is the subroutine which is used to generate orbital elements to be used either as input to the subroutines CLASS and SPECL or for printed output. The equations used are similar in most respects to those described in the discussion of CONIC (Section 4, Appendix) and are listed here for comparison.

$$p = \frac{R^2 V^2 - (R\dot{R})^2}{\mu}, \text{ the semilatus rectum,}$$

where

$$R\dot{R} = \mathbf{R}\cdot\mathbf{V},$$

$$c_1 = \sqrt{R^2V^2 - (R\dot{R})^2}\ \text{, the angular momentum}$$

$$\frac{1}{a} = \frac{2\mu - RV^2}{R\mu}$$

$$c_3 = -\frac{\mu}{a}\text{, the "energy" or } \textit{vis viva} \text{ integral}$$

At this point a test is made with the help of the I.D. input to determine whether or not a is an acceptable parameter. a^* is defined by

$$a^* = \begin{cases} 10^{10}\ \text{km for the planets} \\ 10^{9}\ \text{km for the Sun} \\ 10^{12}\ \text{km for the Moon} \end{cases}$$

The motion is considered parabolic and c_3 is set to zero whenever $|a| > a^*$.

$$1 - \varepsilon^2 = \frac{p}{a}$$

$$\varepsilon = \sqrt{1 - (1 - \varepsilon^2)}\ \text{, the eccentricity}$$

$$\begin{cases} \cos v = \dfrac{p - R}{\varepsilon R} \\ \sin v = \dfrac{\dot{R}}{\varepsilon}\sqrt{\dfrac{p}{\mu}} \end{cases}\text{, true anomaly}$$

$$q = \frac{p}{1+\varepsilon}\text{, closest approach distance}$$

$$\mathbf{W} = \frac{\mathbf{R}\times\mathbf{V}}{c_1}\text{, unit angular momentum vector}$$

$$\mathbf{U}_1 = \frac{\mathbf{R}}{R}$$

$$\mathbf{V}_1 = \frac{R}{c_1}\mathbf{V} - \frac{\dot{R}}{c_1}\mathbf{R}$$

$$\mathbf{P} = \cos v\,\mathbf{U}_1 - \sin v\,\mathbf{V}_1$$

$$\mathbf{Q} = \sin v\,\mathbf{U}_1 + \cos v\,\mathbf{V}_1$$

If $c_3 \neq 0$, $T - T_p$ is computed from Kepler's equation according to the sign of a:

If $a > 0$:

$$\begin{cases} \cos E = \dfrac{R}{p}(\cos v + \varepsilon) \\ \sin E = \dfrac{R}{p}\sqrt{1-\varepsilon^2}\sin v \end{cases}$$

$$M = E - \varepsilon\sin E \quad \text{if } 1 - \varepsilon > 0.1 \text{ or if } 1 - \varepsilon \leqq 0.1 \text{ and } |\sin E| > 0.1$$

$$M = (1-\varepsilon)\sin E + \left(\frac{\sin^3 E}{6} + \frac{3\sin^5 E}{40}\right) \quad \text{if } 1 - \varepsilon \leqq 0.1 \text{ and } \cos E > 0, |\sin E| \leqq 0.1$$

$$M = n(T - T_p) \text{ where } n = \sqrt{\mu}\,a^{-3/2}$$

If $a < 0$:

$$\sinh F = \frac{R\dot{R}}{\varepsilon\sqrt{\mu|a|}}$$

$$M = \varepsilon\sinh F - F \quad \text{if } \varepsilon - 1 > 0.1 \text{ or if } \varepsilon - 1 \leqq 0.1 \text{ and } |\sinh F| > 0.1$$

$$M = (\varepsilon - 1)\sinh F - \left(\frac{3\sinh^5 F}{40} - \frac{\sinh^3 F}{6}\right) \quad \text{if } \varepsilon - 1 \leqq 0.1 \text{ and } |\sinh F| \leqq 0.1$$

$$M = n(T - T_p) \text{ where } n = \sqrt{\mu}\,|a|^{-3/2}$$

If $c_3 = 0$, the formula for the parabola is used:

$$M = \sqrt{\mu}\,(T - T_p) = qD + \frac{1}{6}D^3$$

where $D = R\dot{R}/\sqrt{\mu} = \sqrt{2q}\tan v/2$

JEKYL may be called by the sequence

```
CALL  JEKYL
PZE   0,,A
PZE   B,,C
PZE   D,,0
PZE   E,,F
PZE   G
(ERROR RETURN)
```

The locations A, A + 1 contain for input μ and an I.D. number:

0 = planets

1 = Moon

2 = Sun

The cells B, B + 1, B + 2 contain the input position vector **R**, and the locations C, C + 1, C + 2 contain the input velocity vector **V**; the vectors **P**, **Q**, and **W** are output to the locations D, . . . , D + 8. The single-precision epoch T is input to location E, while the single-precision epoch of closest approach T_p is output to location F. Finally, the locations G, . . . , G + 2 are used to output the quantities $\Delta T = T - T_p$, c_1, and c_3.

Additional quantities are stored at the COMMON cations

ECCEN	ε
1MINE	$1 - \varepsilon$
AVAL	a
PVAL	p
NORB	n
NU	v
JECAN	E (or F)
MENAN	M

The subroutine uses 15 words of erasable storage starting at COMMON.

SPECL

The subroutine SPECL is used to calculate the auxiliary impact parameters $\mathbf{B} \cdot \mathbf{T}$ and $\mathbf{B} \cdot \mathbf{R}$ along with reference unit vectors $\mathbf{R}$, $\mathbf{S}$, $\mathbf{T}$ and also $\mathbf{B}$ itself. Two cases arise according to the value of ε:

(1) $\varepsilon \geqq 1$, the hyperbolic case with $a < 0$

$$\mathbf{S} = \begin{cases} \dfrac{1}{\varepsilon}\mathbf{P} + \dfrac{\sqrt{\varepsilon^2 - 1}}{\varepsilon}\mathbf{Q} & \text{for the incoming asymptote} \\ \dfrac{-1}{\varepsilon}\mathbf{P} + \dfrac{\sqrt{\varepsilon^2 - 1}}{\varepsilon}\mathbf{Q} & \text{for the outgoing asymptote} \end{cases}$$

$$\mathbf{B} = \begin{cases} \dfrac{|a|(\varepsilon^2 - 1)}{\varepsilon}\mathbf{P} - \dfrac{|a|\sqrt{\varepsilon^2 - 1}}{\varepsilon}\mathbf{Q} & \text{for the incoming asymptote} \\ \dfrac{|a|(\varepsilon^2 - 1)}{\varepsilon}\mathbf{P} + \dfrac{|a|\sqrt{\varepsilon^2 - 1}}{\varepsilon}\mathbf{Q} & \text{for the outgoing asymptote} \end{cases}$$

(2) $\varepsilon < 1$, the elliptic case with $a > 0$

$$\left.\begin{aligned} \mathbf{S} &= \mathbf{P} \\ \mathbf{B} &= a\sqrt{|\varepsilon^2 - 1|}\,\mathbf{Q} \end{aligned}\right\} \text{for both the incoming and outgoing asymptote options}$$

The remaining two reference vectors $\mathbf{T}$ and $\mathbf{R}$ are given in either the hyperbolic or elliptic case by

$$\mathbf{T} = \left(\frac{S_y}{\sqrt{S_x^2 + S_y^2}}, \frac{-S_x}{\sqrt{S_x^2 + S_y^2}}, 0\right)$$

$$\mathbf{R} = \mathbf{S} \times \mathbf{T}$$

SPECL is called according to the sequence

(AC) $= a$, $a < 0$ for hyperbola

(MQ) $= \varepsilon$

CALL SPECL

PZE A, , n

PZE B

(ERROR RETURN)

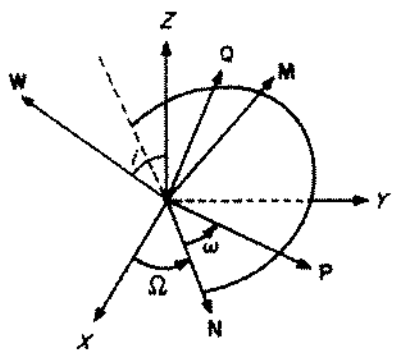
The locations A, . . . , A + 8 contain the vectors $\mathbf{P}$, $\mathbf{Q}$ $\mathbf{W}$; $n = 0$ is a flag for output to be referenced to ar incoming asymptote while $n = 1$ references the output t an outgoing asymptote. The output is placed in the tabl B, . . . , B + 14 where the assignment is in sequenc $\mathbf{B} \cdot \mathbf{T}$, $\mathbf{B} \cdot \mathbf{R}$, $\mathbf{S}$, $\mathbf{B}$, $\mathbf{T}$, $\mathbf{R}$.

The error return will only be used in the case that $|$ is so large that a^2 exceeds the machine capacity, an eve which may happen only for wild trajectories resulti from an input error.

The subroutine uses four words of erasable stora beginning at COMMON.

CLASS

CLASS was written as a subroutine to calculate a tional orbital elements from those provided by JEK



Sketch A-3. Description of the Euler angles for the orbital plane

The formulas that may be deduced from Sket are as follows:

$i = \cos^{-1} W_z$, where $0 \leqq i \leqq 180°$ for the inclin

$$\begin{cases} \sin\Omega = \dfrac{W_x}{\sin i} \\ \cos\Omega = \dfrac{-W_y}{\sin i}, \end{cases}$$ where $0 \leqq \Omega < 360°$ for the ascension of the ascending

$$\begin{cases} \sin\omega = \dfrac{P_z}{\sin i} \\ \cos\omega = \dfrac{Q_z}{\sin i}, \quad \text{where } 0 \leqq \omega \leqq 360° \text{ for the argument of the pericenter} \end{cases}$$

The formulas for Ω may be derived by constructing the unit vector **N** at the ascending node:

$$\mathbf{N} = \frac{\mathbf{U} \times \mathbf{W}}{|\mathbf{U} \times \mathbf{W}|}$$

where $\mathbf{U} = (0, 0, 1)$ and $\sin i = |\mathbf{U} \times \mathbf{W}|$. **N** is then projected onto the X and Y axes to give the formulas for the cosine and the sine.

Next, the auxiliary unit vector $\mathbf{M} = \mathbf{W} \times \mathbf{N}$ is constructed so that ω is given by

$$\begin{cases} \sin\omega = \mathbf{P}\cdot\mathbf{M} = \mathbf{P}\cdot(\mathbf{W}\times\mathbf{N}) = -\mathbf{N}\cdot(\mathbf{W}\times\mathbf{P}) = -\mathbf{N}\cdot\mathbf{Q} \\ \cos\omega = \mathbf{P}\cdot\mathbf{N} \end{cases}$$

The conic parameters are given by the standard formulas for $c_1 \neq 0$:

$q = \dfrac{p}{1+\varepsilon}$, the closest approach distance

$V_p = \dfrac{\mu(1+\varepsilon)}{c_1}$, the velocity at closest approach

$V_a = \dfrac{\mu(1-\varepsilon)}{c_1}$, velocity at farthest departure ($c_3 < 0$)

$V_h = \sqrt{c_3}$, hyperbolic excess velocity ($c_3 > 0$)

$q_2 = a(1+\varepsilon)$, farthest departure distance ($c_3 < 0$)

$P = \dfrac{2\pi}{n}$, the period

For an Earth satellite, the quantities $\dot{\omega}$ and $\dot{\Omega}$ are also computed:

$$\dot{\omega} = \frac{nJa_\oplus^2}{p^2}\left(2 - \frac{5}{2}\sin^2 i\right)$$

$$\dot{\Omega} = \frac{-nJa_\oplus^2}{p^2}\cos i$$

where J is the coefficient of the second harmonic in the Earth's oblateness and $a_\oplus$ is the value of the Earth radius in km. The subroutine assumes that n has been given in rad/sec and p in km so that $\dot{\omega}$ and $\dot{\Omega}$ may be converted to deg/day for output.

The subroutine is called according to the sequence

```
CALL   CLASS
PZE    A, , B
PZE    C
(ERROR RETURN)
(ERROR RETURN FOR PARABOLA)
```

Input locations A, . . . , A + 8 contain the vectors **P**, **Q**, **W**, while the table composed of c_1, c_3, μ, ε, $1-\varepsilon$, a, p, and n is used as input from the cells B, . . . , B + 7. The output is stored in the cells C, . . . , C + 9 forming the table

i

Ω

ω

q

V_p

V_a (or V_h if $c_3 > 0$)

q_2 (or zero if $c_3 > 0$)

P (or zero if $c_3 > 0$)

$\dot{\omega}$

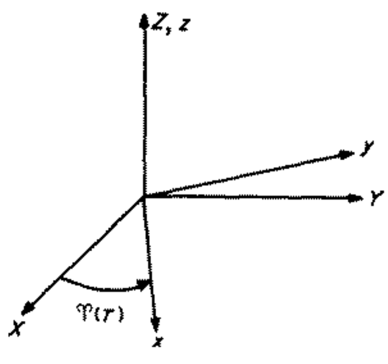
$\dot{\Omega}$

In the event $c_3 = 0$ at entry, the parabola error return is given.

The subroutine uses four cells of erasable storage starting at COMMON.

EARTH, SPACE

At the epoch T a "space-fixed" Cartesian coordinate system is defined, centered at the Earth with the $X-Y$ plane the equator, the X axis the direction of the vernal equinox, and the Z axis the spin axis of the Earth. The "Earth-fixed" frame is obtained from the space-fixed by rotating about the Z axis by an angle $\Upsilon(T)$, the Greenwich hour angle of the vernal equinox, to bring the x axis in coincidence with the Greenwich meridian (Sketch A-4).



Sketch A-4. Earth-fixed equatorial coordinate system

The coordinates are then related by

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \cos \Upsilon\,(T) & \sin \Upsilon\,(T) \\ -\sin \Upsilon\,(T) & \cos \Upsilon\,(T) \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix}$$

$$z = Z,$$

and

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} \cos \Upsilon\,(T) & \sin \Upsilon\,(T) \\ -\sin \Upsilon\,(T) & \cos \Upsilon\,(T) \end{pmatrix} \begin{pmatrix} \dot{X} \\ \dot{Y} \end{pmatrix} + \omega \begin{pmatrix} -\sin \Upsilon\,(T) & \cos \Upsilon\,(T) \\ -\cos \Upsilon\,(T) & -\sin \Upsilon\,(T) \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix}$$

$$\dot{z} = \dot{Z},$$

where ω is the rotation rate of the Earth.

The coordinates may be inverted to give

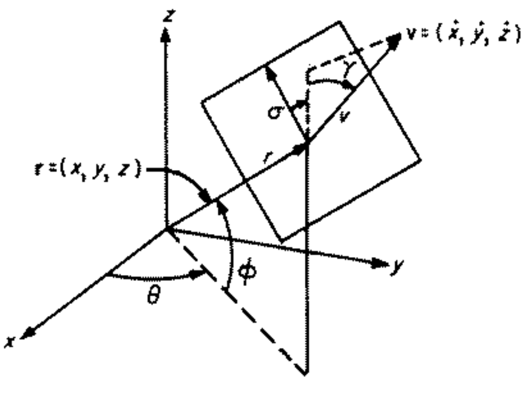
$$\begin{pmatrix} X \\ Y \end{pmatrix} = \begin{pmatrix} \cos \Upsilon\,(T) & -\sin \Upsilon\,(T) \\ \sin \Upsilon\,(T) & \cos \Upsilon\,(T) \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$Z = z$$

and

$$\begin{pmatrix} \dot{X} \\ \dot{Y} \end{pmatrix} = \begin{pmatrix} \cos \Upsilon\,(T) & -\sin \Upsilon\,(T) \\ \sin \Upsilon\,(T) & \cos \Upsilon\,(T) \end{pmatrix} \begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} + \omega \begin{pmatrix} -\sin \Upsilon\,(T) & -\cos \Upsilon\,(T) \\ \cos \Upsilon\,(T) & -\sin \Upsilon\,(T) \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\dot{Z} = \dot{z}$$



Sketch A-5. An Earth-fixed spherical set of coordinate system

In Sketch A-5, r is the radius, ϕ the north latitude, and θ the east longitude of the Earth-fixed position vector. It is convenient to translate the Earth-fixed velocity vector **v** to the end of the position vector and project it on the local horizontal, a plane perpendicular to **r**. v is the magnitude, γ the path angle or the elevation angle above the local horizontal, and σ the azimuth from north of the velocity vector. The transformation between spherical and Cartesian coordinates, and the inverse, are described in the discussions of subroutines RVIN and RVOUT, respectively, which follow.

EARTH is the subroutine which makes the transformation from Earth-fixed spherical to Earth-fixed Cartesian via RVIN and then rotates to space-fixed Cartesian. SPACE manages the inverse transformation by first rotating from space-fixed Cartesian to Earth-fixed Cartesian and obtaining the spherical set with the aid of RVOUT. Both EARTH and SPACE assume that the subroutine GHA has been called and that the COMMON locations GHA(T) and LOMEGA contain, respectively, $\Upsilon\,(T)$ in deg and ω in rad/sec.

The calling sequence for EARTH is

```
CALL   EARTH
PZE    A
PZE    B,,C
```

A, . . . , A + 5 contain the spherical set r, ϕ, θ, v, γ, σ. X, Y, Z are placed in the cells B, B + 1, B + 2; $\dot{X}$, $\dot{Y}$, $\dot{Z}$ are placed in the cells C, C + 1, C + 2.

The calling sequence for SPACE is

```
CALL   SPACE
PZE    A,,B
PZE    C,,D
```

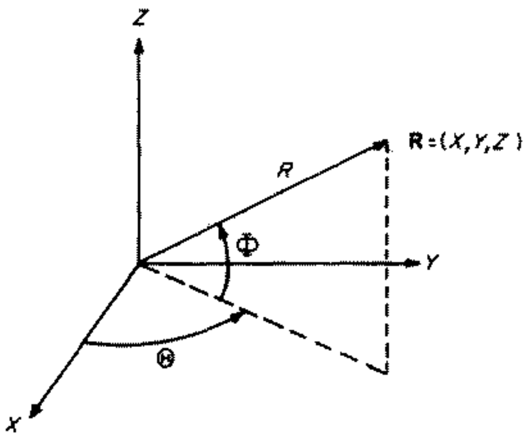
A, A + 1, A + 2 contain X, Y, Z; B, B + 1, B + 2 contain $\dot{X}$, $\dot{Y}$, $\dot{Z}$.

The Earth-fixed spherical set r, ϕ, θ, v, γ, σ is deposited in the cells C, . . . , C + 5, while the Earth-fixed Cartesian set x, y, z, $\dot{x}$, $\dot{y}$, $\dot{z}$ is placed in the locations D, . . . , D + 5.

The subroutines use four words of erasable storage starting at COMMON.

RVIN, RVOUT

Transformations between Cartesian position and velocity **R** and **V** and the spherical set (R, Φ, Θ, V, Γ, Σ) are provided for by RVOUT, while the inverse transformation from spherical to Cartesian is obtained with RVIN.



Sketch A-6. Inertial spherical position coordinates

Projecting **R** on the $X-Y$ plane, Θ is the angle from the X axis to the projection measured counterclockwise. Φ is the elevation of **R** above the $X-Y$ plane (Sketch A-6). The formulas are

$$\mathbf{R} = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} R\cos\Phi\cos\Theta \\ R\cos\Phi\sin\Theta \\ R\sin\Phi \end{pmatrix}$$

and inversely,

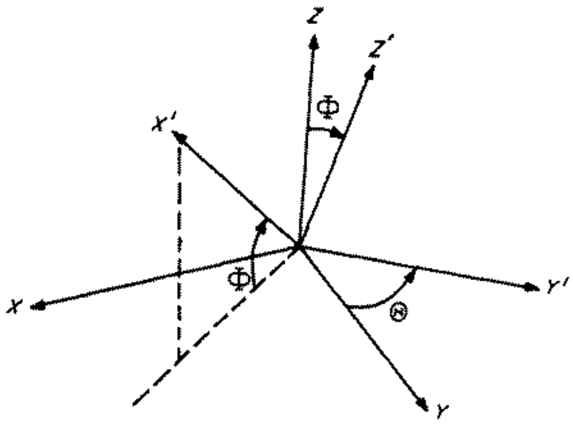
$$R = \sqrt{X^2+Y^2+Z^2}$$

$$\Phi = \sin^{-1}\frac{Z}{R}, \quad -90° \leqq \Phi \leqq 90°$$

$$\Theta = \arg(X,Y), \quad 0 \leqq \Theta < 360°$$

$$\arg(x,y) = \begin{cases} \tan^{-1}\dfrac{y}{x} & \text{if } x > 0 \\ \tan^{-1}\dfrac{y}{x} + 180° & \text{if } x \leqq -0 \end{cases}$$

To describe the spherical coordinates for the velocity vector **V**, it is convenient to construct a new reference frame obtained by first rotating about the Z axis by an amount Θ so that the new X axis lies along the projection of **R** on the $X-Y$ plane; a subsequent rotation about the intermediate Y axis by the angle Φ completes the coordinate change. The resultant X' axis lies along **R**, the Z' axis lies in the plane formed by the Z axis and **R**, and the Y' axis completes the right-handed system and thus remains in the $X-Y$ plane (Sketch A-7).

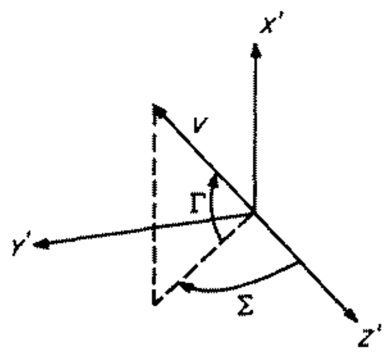


Sketch A-7. Rotation to the local plane

Evidently

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} \cos\Phi\cos\Theta & -\sin\Theta & -\sin\Phi\cos\Theta \\ \cos\Phi\sin\Theta & \cos\Theta & -\sin\Phi\sin\Theta \\ \sin\Phi & 0 & \cos\Phi \end{pmatrix} \begin{pmatrix} X' \\ Y' \\ Z' \end{pmatrix}$$

Representing the velocity vector **V** in the X', Y', Z' system, the path angle Γ is the elevation of **V** above the $Y'-Z'$ plane, positive in the radial outward or X' direction; the azimuth Σ is the angle measured clockwise from the Z' axis to the projection of **V** on the $Y'-Z'$ plane. The geometry appears in Sketch A-8.



Sketch A-8. Inertial velocity vector in the local horizontal plane

Regarding the X', Y', Z' frame as nonrotating, **V** may be expressed as

$$\mathbf{V} = \begin{pmatrix} \dot{X}' \\ \dot{Y}' \\ \dot{Z}' \end{pmatrix} = \begin{pmatrix} V\sin\Gamma \\ V\cos\Gamma\sin\Sigma \\ V\cos\Gamma\cos\Sigma \end{pmatrix}$$

and rotate to the original frame to obtain $\dot{X}$, $\dot{Y}$, $\dot{Z}$.

Inversion may be obtained as follows:

$$V = \sqrt{\dot{X}^2 + \dot{Y}^2 + \dot{Z}^2}$$

$$\Gamma = \sin^{-1} \frac{\dot{X}'}{V}, \quad -90° \leqq \Gamma \leqq 90°$$

$$\Sigma = \arg (\dot{Z}', \dot{Y}'), \quad 0 \leqq \Sigma < 360°$$

Of course **V** expressed in the X', Y', Z' system is given by

$$\begin{pmatrix} \dot{X}' \\ \dot{Y}' \\ \dot{Z}' \end{pmatrix} = \begin{pmatrix} \cos\Phi\cos\Theta & \cos\Phi\sin\Theta & \sin\Phi \\ -\sin\Theta & \cos\Theta & 0 \\ -\sin\Phi\cos\Theta & -\sin\Phi\sin\Theta & \cos\Phi \end{pmatrix} \begin{pmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{pmatrix}$$

The calling sequence for RVIN is

CALL RVIN

PZE „A

PZE „B

PZE „C

A, . . . , A + 5 contain the spherical coordinates R, Φ, Θ, V, Γ, Σ; X, Y, Z are placed in the locations B, B + 1, B + 2, while the Cartesian velocity components $\dot{X}$, $\dot{Y}$, $\dot{Z}$ are stored in the cells C, C + 1, C + 2.

For RVOUT, the calling sequence is

CALL RVOUT

PZE 1„A

PZE 1„B

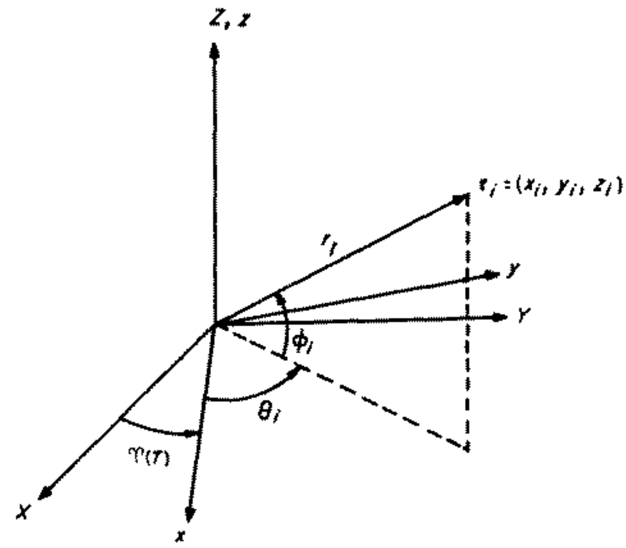
PZE 1„C

X, Y, Z are contained in the cells A, A + 1, A + 2, while the locations B, B + 1, B + 2 contain $\dot{X}$, $\dot{Y}$, $\dot{Z}$. The spherical set R, Φ, Θ, V, Γ, Σ is placed in the cells C, . . . , C + 5 as output.

The subroutines use four words of erasable storage starting at COMMON.

LOOP

Let $\mathbf{R} = (X, Y, Z)$ and $\mathbf{V} = (\dot{X}, \dot{Y}, \dot{Z})$ be the Earth-centered "space-fixed" Cartesian coordinates of the probe referenced to the true equator and equinox of date. For a given station with Earth-fixed spherical coordinates (r_i, ϕ_i, θ_i), it is desired to compute a number of topocentric quantities as given below. The basic coordinate systems are shown in Sketch A-9.



Sketch A-9. Earth-fixed station coordinates

$♈(T)$ is the Greenwich hour angle of vernal equinox at epoch T or alternatively, the right ascension of the Greenwich meridian. It is assumed that GHA has computed $♈(T)$ and the correct value appears in the COMMON location GHA(T). r_i is the distance of the station from the center of the Earth, ϕ_i is the geocentric north latitude, and θ_i is the east longitude.

The Earth-fixed Cartesian coordinates of the station are

$$x_i = r_i \cos\phi_i \cos\theta_i$$

$$y_i = r_i \cos\phi_i \sin\theta_i$$

$$z_i = r_i \sin\phi_i$$

Those for the probe are

$$x = X\cos ♈(T) + Y\sin ♈(T)$$

$$y = -X\sin ♈(T) + Y\cos ♈(T)$$

$$z = Z$$

$$\dot{x} = \dot{X}\cos ♈(T) + \dot{Y}\sin ♈(T) + \omega y$$

$$\dot{y} = -\dot{X}\sin ♈(T) + \dot{Y}\cos ♈(T) - \omega x$$

$$\dot{z} = \dot{Z}$$

$$\dot{\mathbf{r}} = (\dot{x}, \dot{y}, \dot{z})$$

where ω is the rotation rate of the Earth.

Thus the topocentric Cartesian coordinates of the probe are

$$\mathbf{r}_{ip} = (x - x_i, y - y_i, z - z_i)$$

$$\dot{\mathbf{r}}_{ip} = (\dot{x}, \dot{y}, \dot{z}) = \dot{\mathbf{r}}$$

The slant range r_{ip} is then given by $|\mathbf{r}_{ip}|$, while the slant-range rate $\dot{r}_{ip}$ may be obtained from the formula

$$2r_{ip}\dot{r}_{ip} = \frac{d(r_{ip}^2)}{dt} = \frac{d(\mathbf{r}_{ip}\cdot\mathbf{r}_{ip})}{dt} = 2\mathbf{r}_{ip}\cdot\dot{\mathbf{r}}_{ip}$$

Provisions have been made to compute $\ddot{r}_{ip}$, the slant-range acceleration, when the Earth is the central body. The pertinent formulas may be developed as follows:

$$\begin{pmatrix}\dot{x}\\ \dot{y}\end{pmatrix} = \begin{pmatrix}\cos \Upsilon\,(T) & \sin \Upsilon\,(T)\\ -\sin \Upsilon\,(T) & \cos \Upsilon\,(T)\end{pmatrix}\begin{pmatrix}\dot{X}\\ \dot{Y}\end{pmatrix} + \omega\begin{pmatrix}-\sin \Upsilon\,(T) & \cos \Upsilon\,(T)\\ -\cos \Upsilon\,(T) & -\sin \Upsilon\,(T)\end{pmatrix}\begin{pmatrix}X\\ Y\end{pmatrix}$$

$$\dot{z} = \dot{Z}$$

$$\begin{pmatrix}\ddot{x}\\ \ddot{y}\end{pmatrix} = \begin{pmatrix}\cos \Upsilon\,(T) & \sin \Upsilon\,(T)\\ -\sin \Upsilon\,(T) & \cos \Upsilon\,(T)\end{pmatrix} \times \left\{\begin{pmatrix}\ddot{X}\\ \ddot{Y}\end{pmatrix} + 2\omega\begin{pmatrix}\dot{Y}\\ -\dot{X}\end{pmatrix} - \omega^2\begin{pmatrix}X\\ Y\end{pmatrix}\right\}$$

$$\ddot{z} = \ddot{Z}$$

From

$$r_{ip}\dot{r}_{ip} = \mathbf{r}_{ip}\cdot\dot{\mathbf{r}}_{ip} = \mathbf{r}_{ip}\cdot\dot{\mathbf{r}}$$

obtain

$$r_{ip}\ddot{r}_{ip} + \dot{r}_{ip}^2 = \mathbf{r}_{ip}\cdot\ddot{\mathbf{r}} + \dot{\mathbf{r}}_{ip}\cdot\dot{\mathbf{r}}_{ip}$$

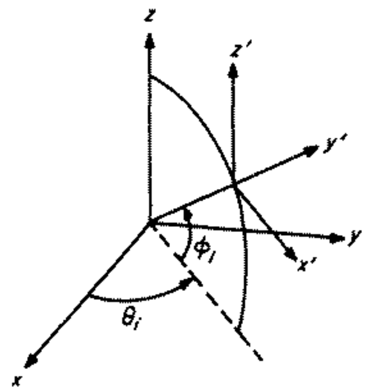
or

$$\ddot{r}_{ip} = \frac{1}{r_{ip}}\left\{\mathbf{r}_{ip}\cdot\ddot{\mathbf{r}} + v^2 - \dot{r}_{ip}^2\right\}$$

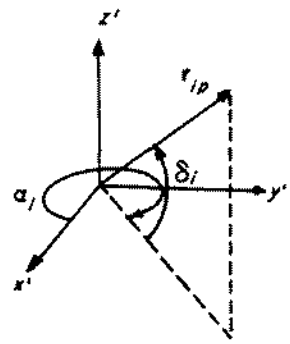
where $v = |\dot{\mathbf{r}}|, \ddot{\mathbf{r}} = (\ddot{x}, \ddot{y}, \ddot{z})$

Contributions to $\ddot{\mathbf{R}}$ are obtained from COMMON locations where they have been deposited by DOT and are only valid for the Earth as a central body.

The topocentric hour-angle declination system is described in Sketches A-10 and A-11.



Sketch A-10. Rotation to the station meridian



Sketch A-11. Local hour-angle declination coordinate system

The $x-y$ plane has been translated to the station and rotated through the angle θ_i so that x' lies along the meridian; the z' axis remains parallel to the z axis. The declination δ_i is given by

$$\delta_i = \sin^{-1}\frac{z_{ip}}{r_{ip}}\ ; \quad -90° \leqq \delta_i \leqq 90°$$

and the hour angle may be computed from

$$\alpha_i \equiv \theta_i - \arg\,(x_{ip}, y_{ip})\,(\mathrm{mod}\,360°), \quad 0 \leqq \alpha_i < 360°$$

where

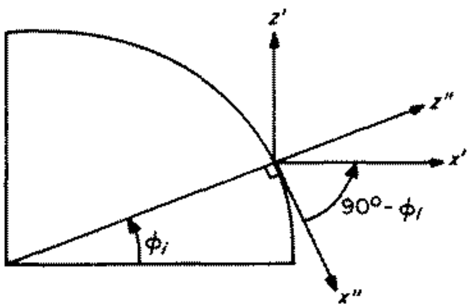
$$\arg\,(x, y) = \begin{cases}\tan^{-1}\dfrac{y}{x} & \text{if } x > 0, \quad -90° \leqq \tan^{-1} u \leqq 90°\\ \tan^{-1}\dfrac{y}{x} + 180° & \text{otherwise}\end{cases}$$

From the above formulas, the angular rates follow:

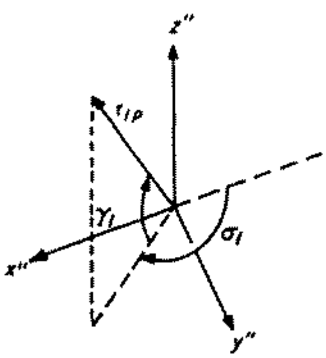
$$\dot{\delta}_i = \frac{\dot{z} - \dot{r}_{ip}\sin\delta_i}{r_{ip}\cos\delta_i}$$

$$\dot{\alpha}_i = \frac{\dot{x}y_{ip} - \dot{y}x_{ip}}{x^2_{ip} + y^2_{ip}}$$

To construct the azimuth-elevation topocentric coordinate system, rotate the x' and z' axes about the y' axis so that the resultant $x''-y''$ plane is perpendicular to $\mathbf{r}_i$ and the z'' axis points to the zenith; the $x''-z''$ plane is still the meridian plane as illustrated in Sketches A-12 and A-13.



Sketch A-12. Rotation to station latitude



Sketch A-13. Azimuth elevation coordinate system

The elevation angle γ_i may be obtained immediately by

$$\sin\gamma_i = \frac{\mathbf{r}_i \cdot \mathbf{r}_{ip}}{r_i\, r_{ip}}, \quad -90° \leqq \gamma_i \leqq 90°$$

The component of $\mathbf{r}_{ip}$ which lies in the $x''-y''$ plane is $r_{ip}\cos\gamma_i$ so that the azimuth σ_i is given by

$$\begin{cases} \cos\sigma_i = \dfrac{-x''_{ip}}{r_{ip}\cos\gamma_i} \\ \sin\sigma_i = \dfrac{y''_{ip}}{r_{ip}\cos\gamma_i} \end{cases}$$

By performing the rotations to transform the coordinate systems, $\mathbf{r}_{ip}$ may be determined in the $x''-y''-z''$ reference:

$$x''_{ip} = x_{ip}\sin\phi_i\cos\theta_i + y_{ip}\sin\phi_i\sin\theta_i - z_{ip}\cos\phi_i$$

$$y''_{ip} = -x_{ip}\sin\theta_i + y_{ip}\cos\theta_i$$

$$z''_{ip} = x_{ip}\cos\phi_i\cos\theta_i + y_{ip}\cos\phi_i\sin\theta_i + z_{ip}\sin\phi_i$$

The program uses an inverse function defined for $0 \leqq \cos^{-1} u \leqq 180°$ so that

$$\sigma_i = \begin{cases} \cos^{-1}\left(\dfrac{-x''_{ip}}{r_{ip}\cos\gamma_i}\right) & \text{if } \sin\sigma_i \geqq 0 \\ 360° - \cos^{-1}\left(\dfrac{-x''_{ip}}{r_{ip}\cos\gamma_i}\right) & \text{otherwise} \end{cases}$$

Thus $0 \leqq \sigma_i \leqq 360°$.

The angular rates are calculated from the formulas

$$\dot{\gamma}_i = \frac{\mathbf{r}_i \cdot \dot{\mathbf{r}} - r_i\,\dot{r}_{ip}\sin\gamma_i}{r_i\, r_{ip}\cos\gamma_i}$$

$$\dot{\sigma}_i = \frac{\dot{x}''_{ip} + \cos\sigma_i\,(\dot{r}_{ip}\cos\gamma_i - r_{ip}\,\dot{\gamma}_i\sin\gamma_i)}{r_{ip}\cos\gamma_i\sin\sigma_i}$$

where $\dot{x}''_{ip} = \dot{x}\sin\phi_i\cos\theta_i + \dot{y}\sin\phi_i\sin\theta_i - \dot{z}\cos\phi_i$.

The look angle λ_i is the angle between the spacecraft attitude vector $\mathbf{C}$ and the slant-range vector where $\mathbf{C}$ is specified by the calling sequence and is a unit vector expressed in the true equator and equinox of date. It is convenient to construct $\mathbf{R}_{ip}$ in a topocentric system parallel to the X, Y, Z axes:

$$X_i = x_i\cos \Upsilon(T) - y_i\sin \Upsilon(T)$$

$$Y_i = x_i\sin \Upsilon(T) + y_i\cos \Upsilon(T)$$

$$Z_i = z_i$$

$$\mathbf{R}_{ip} = (X - X_i, Y - Y_i, Z - Z_i)$$

Then λ_i is obtained from

$$\lambda_i = \cos^{-1}\left(\frac{\mathbf{R}_{ip}\cdot\mathbf{C}}{R_{ip}}\right), \quad 0 \leqq \lambda_i \leqq 180°$$

The polarization angle p_i is defined as

$$p_i = \cos^{-1}\left\{\frac{\mathbf{R}\times\mathbf{R}_{ip}}{|\mathbf{R}\times\mathbf{R}_{ip}|}\cdot\frac{\mathbf{C}\times\mathbf{R}_{ip}}{|\mathbf{C}\times\mathbf{R}_{ip}|}\right\}, \quad 0 \leqq p_i \leqq 180°$$

An expression for the measured received frequency, including a scaled doppler shift, appears as

$$f = f_{Bi} - f_{Ci}\,\dot{r}_{ip}$$

where f_{Bi} represents a bias frequency in the receiver and f_{Ci} includes the velocity of light and may be adjusted to represent either two-way or normal doppler.

The calling sequence is

```
CALL   LOOP
PZE    X,,Y
OP     B,,C
```

X, X + 1, X + 2 contain **R**; Y, Y + 1, Y + 2 contain **V**.

B contains the binary control word which selects the appropriate stations from among the available 15. The small subroutine CW1 transforms the octal input to the required binary format which permits LOOP to scan the stations from bit 35 to bit 21.

C, C + 1, C + 2 contain the unit vector **C**.

If OP = PZE, LOOP will compute the quantities for each station in turn and will print out whenever $\gamma_i \geqq -10°$. If OP = MZE, γ_i and $\dot{\gamma}_i$, for each station up to a maximum of five, will be stored in a buffer to be used by MARK as dependent variables for the view-period computation.

The parameters describing the stations are stored in the following sequence:

STABCD	+0		4 BCD words for station 1 name
	1		
	2		
	3		
	⋮		
	70		4 BCD words for station 15 name
	71		
	72		
	73		
STACRD	+0	ϕ_1	coordinates for station 1
	1	θ_1	
	2	r_1	
	3	f_{B_1}	frequency parameters for station 1
	4	f_{C_1}	
	⋮		
	106	ϕ_{15}	coordinates for station 15
	107	θ_{15}	
	110	r_{15}	
	111	$f_{B_{15}}$	frequency parameters for station 15
	112	$f_{C_{15}}$	

To describe the view periods for the stations, three other parameters are used:

STACRD	−3	γ_B
	−2	$\dot{\gamma}_B$
	−1	γ_0

The elevation condition is met for rise or set with respect to the station whenever $|\gamma_i - \gamma_0| \leqq \gamma_B$; at this time the station quantities are printed and further testing is suppressed for one integration step. The elevation-rate condition is met for extreme elevation whenever $|\dot{\gamma}_i| \leqq \dot{\gamma}_B$ and $\gamma_i \geqq \gamma_0$. Upon success, the station quantities are printed and the test is suppressed for one integration step.

The subroutine uses 100 words of erasable storage starting at COMMON.

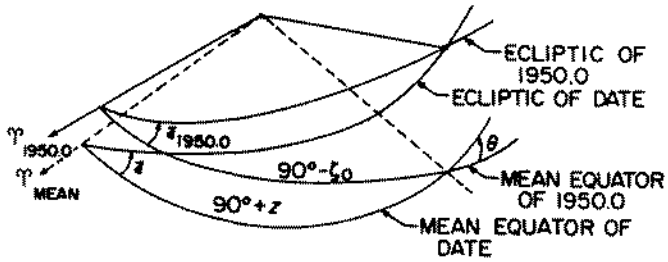
2. Basic Coordinate Transformations

ROTEQ

The general precession of the Earth's equator and the consequent retrograde motion of the equinox on the ecliptic may be represented by the rotation matrix:

$$\begin{pmatrix} X' \\ Y' \\ Z' \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

where X, Y, and Z are expressed in the mean equator and equinox of 1950.0 and X′, Y′, Z′ are the coordinates in the mean equator and equinox of date. The geometry of the precession has been represented by the three small parameters ζ_0, z, and θ in Sketch A-14:



Sketch A-14. Relationship between fundamental reference equators

$\Upsilon_{1950.0}$ is the mean equinox of 1950.0; $\bar{\epsilon}_{1950.0}$ is the mean obliquity of 1950.0; Υ_{mean} is the mean equinox of date; $\bar{\epsilon}$ is the mean obliquity of date. Measured in the mean equator of 1950.0 from the mean equinox of 1950.0, $90° - \zeta_0$ is the right ascension of the ascending node of the mean equator of date on the mean equator of 1950.0. $90° + z$ is the right ascension of the node measured in the mean equator of date from the mean equinox of date. θ is the inclination of the mean equator of date to the mean equator of 1950.0.