ONE - YEAR EXPLORATION - TRIP
EARTH - MARS - VENUS - EARTH

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ABSTRACT

The possibility of a unitary exploration trip to Mars and Venus having a total duration of about one year is examined. First of all, the case with no planetary perturbations is taken into consideration, and a possible ideal solution is determined. Subsequently, the perturbation due to Mars by passing at a short distance from it is introduced, and the delay attained thereto in the trip time is computed. Then, the perturbation due to Venus is examined, and requirements of flights at a short distance are determined, capable of correcting the perturbation due to Mars. A numerical example is developed. At last, the correlations among the proposed astronautical point of contacts and the respective astronautical combinations of the three planets are discussed. According to calculations of the Brera's observatory, a favourable chance will occurr on June 1971.

PART I

WITHOUT PERTURBATIONS.

While admiring the bold calculators, who have extended the interplanetary nautics to the whole solar system, I limit myself in this paper only to our nearest planets.

First of all, I am considering Mars. Its transfer ellipse, cotangential to the two Earth's and Mars' orbits, that is calculated according to the criterion of the minimum propellant consumption, requires on the average — according to Clarke — 259 days for going to, plus 455 days for awaiting on a centrifugal orbit around Mars the favourable conditions for the return; and other 259 days for coming back. Hence, we have a total of almost three years, if considering the time necessary for the departure and arrival requirements. Stuhlinger's nuclear space ship requires a total time of two years.

In a recent paper of mine, where the major axis of the transfer ellipse was assumed to be along the Earth-Mars line of nodes, the travel for going to lasted a bit less; without changing, however, the total endurance in a sensible way.

I now wish to examine the propellant consumption, calculated with the known logarithmic relations of the mass ratio, which usually is taken as a function of the so called characteristic velocity for the task into consideration, and which may be chosen as a conventional term of comparison. In the previous case, D.F. Lawden calculates a characteristic velocity of 17.3 kilometers per second, by starting from the ground of the Earth.

I prefer to refer myself, on the contrary, to the necessary specific impulse, expressed in seconds of time, and which is obtained by dividing the above characteristic velocity for the gravitational acceleration at the ground, g_0, as it is common for calculating the available specific impulse of rocket engines (*).

Thus, we obtain for the transfer journey Earth-Mars and return the necessary impulse of 1760 seconds, always by starting from the Earth's ground. Recent calculators compare this impulse to an available impulse of 400 seconds. This belongs somehow to the future, nowadays; however, it will be soon reached, even without using the nuclear energy.

Of the above necessary 1760 seconds, about 580 seconds are required for establishing the space missile in a satellite orbit around Mars and for the subsequent approach. In such a way, the minimum amount required only for the going transfer — that is, 1140 seconds — is therefore aggravated by over a half.

I have therefore thought of taking into consideration the opportunity of a circular reconnaissance travel with no waiting dwell around the planet; that is, by limiting the purpose to documentary observations obtained during the flight performed at a short distance from the planet. And I have drawn the attention on an interesting astronomical phenomenon occuring during the travel, and on some requirements concerning the observation (**).

However, the problem of the return takes no advantage from this. On the contrary, three years are not sufficient to find again the coincidence with the Earth, unless carrying out manoeuvers requiring remarkable reserves of propellant.

In such a kind of reconnaissance travel, I have therefore convinced myself to abandon the idea of the cotangential orbit, which peculiar advantage of minimum consumption becomes illusive, owing to the burden of provisions necessary for the life of the crew during the long journey; and to find out, on the contrary, a

(*) Daily paper « Il Tempo », April 4th, 1954.
(**) See note on page 11.

way for reducing the endurance to a minimum, for instance only one year instead of three years.

First of all, I show the generical possibility of this in the ideal case of no planetary perturbations during the whole trip.

It is obvious that, in order to obtain the evolution time of one year, that is the same time employed by the Earth, the transfer orbit is required to have the identical major axis. In fact, Kepler's equation gives:

$$T = K\, a^{2/3}, \qquad (1)$$

where, T is the lifetime of one evolution, and a is the semi-major axis of any orbit having the Sun as one of its foci. K is a constant which, by expressing T in days and a in megakilometers, equals about 4.98 (*).

Trasferta TERRA MARTE nel giro esplorativo di un anno senza perturbazioni

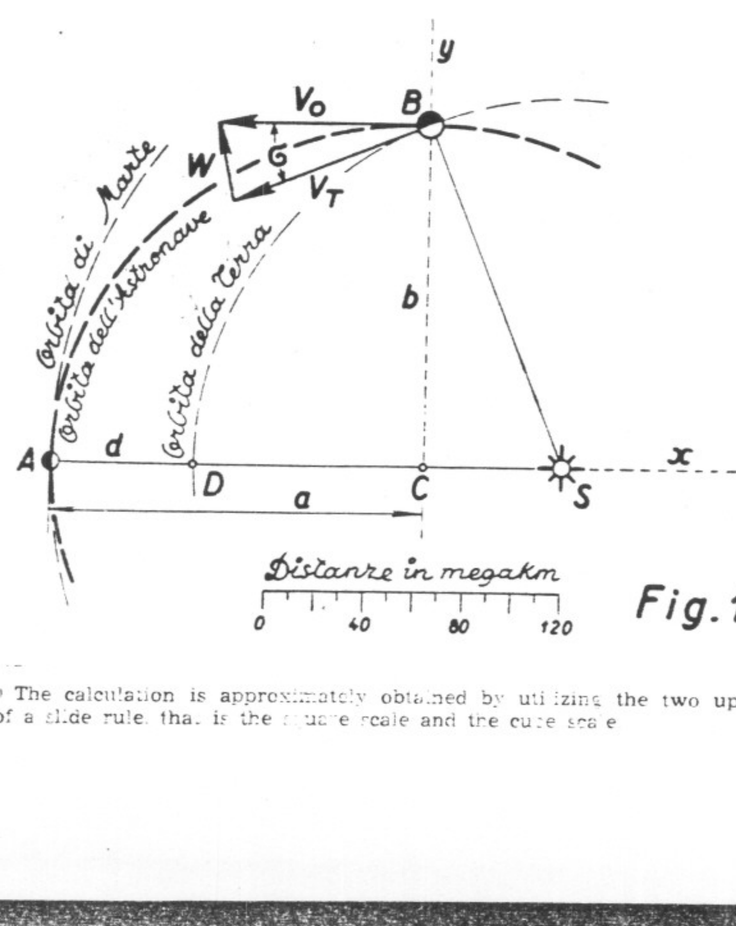

Fig. 1

(*) The calculation is approximately obtained by utilizing the two upper scales of a slide rule, that is the square scale and the cube scale.

If now, for simplicity, we neglect the eccentricity of the Earth's orbit, by assuming it to be a circle of radius s_o around the Sun, S, it follows — for the new orbit we are seeking — that: $a=s_o$.

Let us assume, further, as a first approximation, the planet Mars to revolve along an orbit coplanar with the ecliptic; and that at point A of Fig. 1 Mars is to be found at the minimum distance, d, from the Earth's orbit. It follow: $AD=d$; $DS=s_o$.

At last, assume the point A to be the aphelion of the missile orbit we want to determine. The center C of such an orbit is to be distant from point A of the quantity $AC=s_o$. As a result, we have therefore $CS=d$.

In such a way, the position of the center is located, to which corresponds point B, so that BC results equal to the semi-minor axis, *b*, of the new orbit. The focal triangle S A B results determinated too; providing the eccentricity of the orbit to be found out

$$e = d/a = d/s_o = \text{sen}\ \sigma \qquad (2)$$

If the ship, intended to travel along said orbit, starts from the Earth, according to this scheme, exactly at point B its velocity is to result in a direction parallel to the axis S A, because B corresponds to the mid-point, C, of the orbit to be followed. Simultaneously, such a speed is to result of the same value as the one which the Earth has in that point, having to meet the relation (3), with $r=s_o$, μ=Newton constant;

$$V_o^2 = \bar{\mu}\left[\frac{2}{r} - \frac{1}{a}\right] = \bar{\mu}\left[\frac{2}{s_o} - \frac{1}{s_o}\right] = V_T^2 \qquad (3)$$

However, the velocity of the Earth at point B has a direction at right angles with the side B S of the focal triangle. Therefore, it is necessary the vector « velocity of the Earth », V_T, to be rotated, in the plane S B A, of an angle equal to the angle at the apex B of the focal triangle. This angle has been indicated by the symbol σ.

Therefore, it is necessary to compound vectorially, in the plane S B A, the velocity V_T with a velocity W having a direction along the base side of the isosceles triangle V_o V_T W of Fig. 1: its value is then equal to

$$W / V_T = 2\ \text{sen}\ \sigma/2 \cong \sqrt{1+e} - \sqrt{1-e} \qquad (4)$$

as deduced from (2).

We have thus determined, always schematically, the point where the Earth is to be found at the moment of the launching, as well as the value and direction of vector W required for deviating the velocity from V_T to V_o, in the coplanar case taken into consideration. The orbit of the missile will result tangential to the orbit of the planet at point A corresponding to the aphelion of the orbit in question; that is, after having travelled a quarter of the orbit.

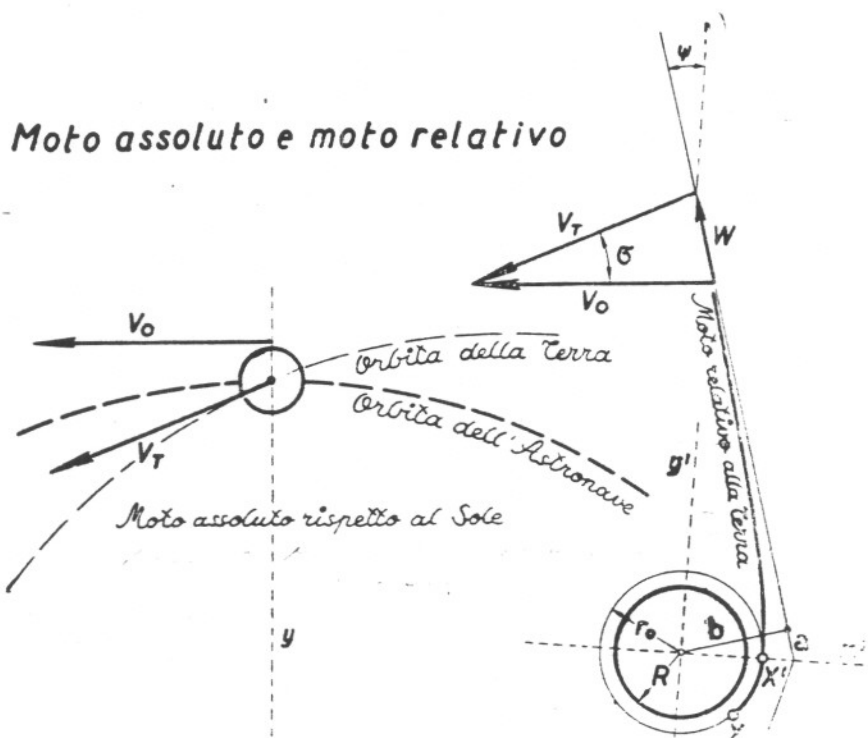

Fig. 2

I have still to consider the possibility of obtaining the impulse corresponding to W and the requirements thereto. In Fig. 2, the ship at the moment of departure is already circling, just like a satellite, in an orbit of radius r_o around the Earth. Let R be the average radius of the Earth. I further assume, for simplicity, the acceleration required to escape from gravitation and to obtain, at the end of escape, the residual velocity W to be operated tangentially; that is, at the moment of departure the radial increment of r_o to be technically negligible. Hence, if V_s is the pre-existing velocity of the satellite path, we have $V_E^2 = 2\ V_s^2$, the square of the escape velocity. Therefore, the launching velocity V_L required to insure the aforesaid residual velocity W, upon completion of the escape, results from the equation of the work:

$$V_L^2 = W^2 + V_E^2 ; \qquad (5)$$

The increment of velocity necessary to calculate the propellant consumption is given by: $V_L - V_s$.

At last, in order to establish the exact direction of vector W, always in the supposition of the coplanar scheme, it is possibile to define the launching hyperbola, referred to a stationary Earth, where W represents the final velocity on the asymptote. Such a hyperbola is drawn for a short initial arc in figure 2 and the orientation of the asymptote is compared with the orientation of W, corresponding to the angle σ as deduced from (2).

Having thus determined the parameters of the hyperbola, by calculating W as a velocity to infinity on the asymptote, we therefore obtain:

$$a = \frac{9_0 R^2}{W^2} ; \quad \psi = \text{arc sen} \frac{a}{r_0 + a} ;$$

where, a is the semi-major axis, Ψ is the asymptote angle to the axis y′ of the hyperbola. The latter is so oriented as to result in the asymptote direction to be parallel to the predetermined direction of vector W with respect to the axis y of the ship orbit. Therefore, it is necessary to calculate the angle $\psi - \sigma/2$, by which the axis x′ of the hyperbola has to rotate with respect to the axis x of the ship orbit, in order to determine the exact position of the launching point X on the satellite orbit, and to deduce the firing point X_o, after having determined from the set of technical data the duration of the jet firing.

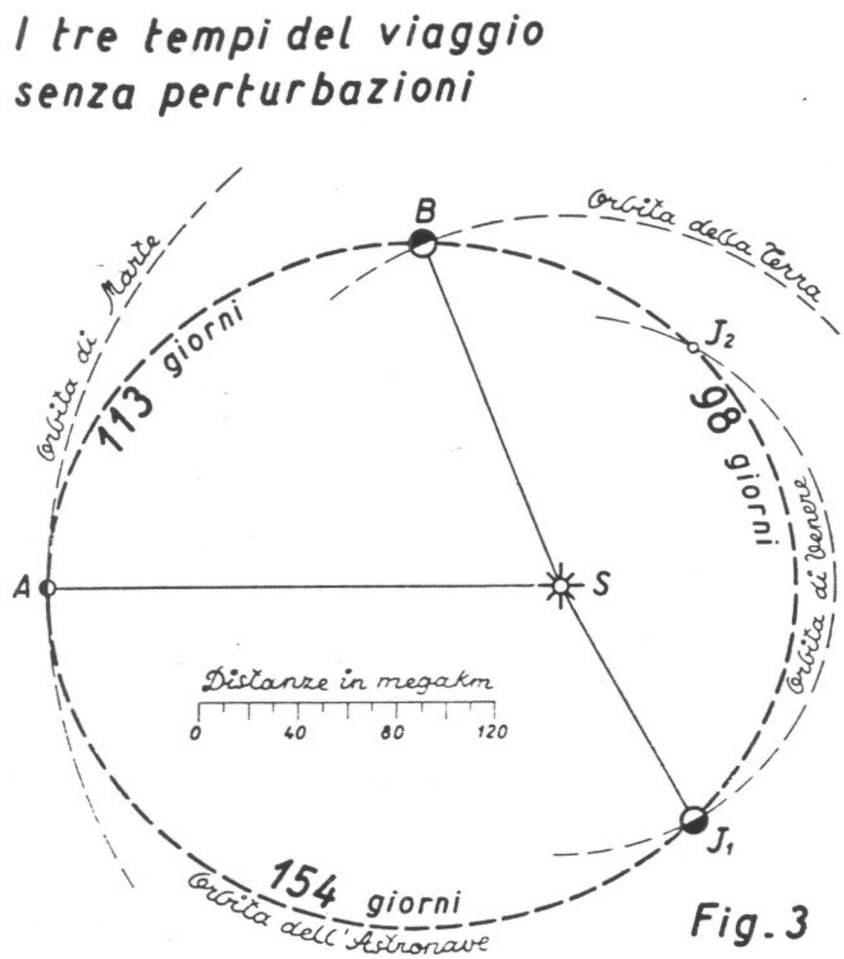

Fig. 3

This being established, and always as a first approximation, let us extend also to the planet Venus the conventional concepts by which its orbit is considered as circular and coplanar with the terrestrial orbit.

As a result, we may see at once in Fig. 3 that the orbit of the spaceship contacts twice the orbit of Venus; namely, at points J_1 and J_2. Therefore, it is sufficient to change of some degrees, as specified in the following, the plane of the ship orbit by adequately modifying the direction of velocity at the point of contact with Mars to obtain an actual point of contact, where the actual position of the planet may be presumed. Such a coincidence is here assumed to be in correspondence with the point of contact J_1, with a determined interval of terrestrial days from point A, where the coincidence with Mars has been assumed. On its turn, Mars is to be at a determined interval from point B, where the Earth is assumed to be starting off. The compatibility of the above three conditions will be discussed later on. In Figure 3, the data assumed for such a scheme and the times elapsing among the three coincidences are traced; that is, 113 terrestrial days from the departure point B until the point of contact with Mars; other 154 days until grazing Venus, and at last other 98 days until finding again the Earth, after one year, at point B of its solar orbit. The final interval is covered more rapidly, because the ship takes advantage of its perihelion velocity.

Fig. 4 shows the astronomical positions of Mars and Venus at the moment when the Earth is at point B.

Posizione astronomica dei tre pianeti alla partenza

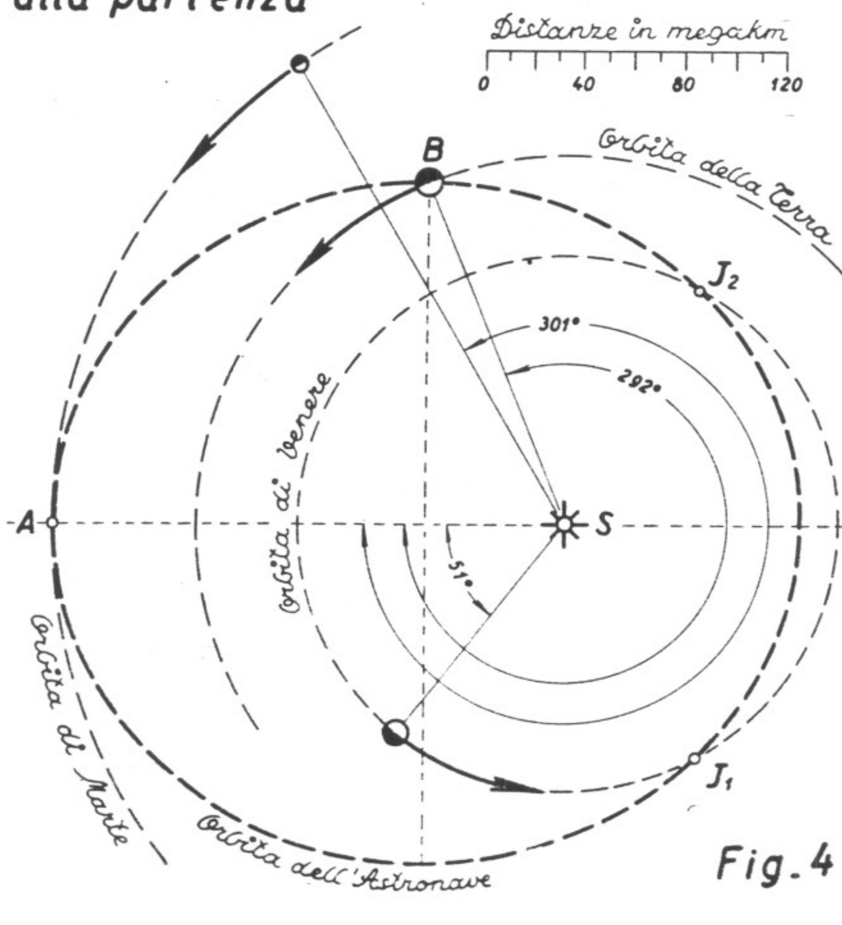

Fig. 4

As it may be seen, the above scheme is attractive, and by comparison with the cotangential scheme a highly favourable result is derived. On that end we calculate the specific impulse necessary for starting by means of the data inserted into the figure determining the launching velocity V_L, assumed as calculated by starting from the Earth's ground, as it has been made for the cotangential case, that is by putting into equation (5) $W=11.7$; $V_E=11.2$. One finds out the value $V_L \simeq 16.2$ kms per second, corresponding to a necessary impulse of 1650 seconds; that is an impulse lower than the 1760 seconds indicated by Lawden for the minimum consumption transfer. Moreover, and here is the best prerogative, the travel lasts one year instead of three years. Therefore, if we assume in case of one year an equipment payload of 2 tons, including 3 observers, by adding as an average other 2 tons for support provisions, at the departure we will get a total payload of 4 tons. The above calculation in case of the three-year travel gives $2+6=8$ tons; that is, exactly a double value. The propellant consumption following by comparison of the specific impulses requested for the two schemes is therefore, for the one-year travel, less than half of the respective amount for the three-year travel.

Still a less amount would result if we conceive a similar one-year scheme for grazing separately upon the planet Venus. In fact, it would suffice a necessary specific impulse of 1435 seconds, always conventionally calculated by starting from the Earth's ground; whilst Lawden gives 2260 seconds for the cotangential case.

The scheme of a long reconnaissance travel is, however, to be considered also from the point of view of the observers to remain into a narrow interior space, where they have also to alternate during their rest time and the exercises necessary for their health. Even from this viewpoint the one-year scheme is incomparably superior than the three-year travel.

I now come to consider the criteria according to which we may pass from the ideal scheme of first approximation to the executive scheme, letting aside the simplifying assumption, and keeping the endurance characteristic of one year only as an order of magnitude.

I mean by this it should be necessary to give up the rigorous observance of such a duration, because in the problem we have to face the conditions overcome in number the variables which we dispose of. Hence, we have eventually to be content with some month less or more, and to keep the duration of one year only for the title, as an attraction to read our paper; (title appeal). Therefore, the return point on the Earth will be however different from the departure point B.

For similar reasons it shall be necessary to give up definitively the condition for the point of contact with Mars to be at the minimum distance from the terrestrial orbit. It is very difficult this may coincide with the proper position the point of contact with Venus must have. We have therefore to be content with a position of the point of contact with Mars not very far from the minimum distance position.

Hence, also the point of contact with Mars will be different from point A in the scheme of first approximation.

Some obviously approximative positions are then to be rectified, such as that of the simultaneous presence of the spaceship and the Earth at point B on the moment of departure. This assumption disagrees with the condition that at point B the spaceship has reached the escape from the terrestrial gravity. Practically, in our previous studies (*) we have been of the opinion to assume as technically acceptable a distance of the spaceship from the Earth equal to 407 terrestrial radii, at which distance the attraction of the planet has become the hundredth part of the Sun's attraction. In other terms, the departing curve of the spaceship from the Earth is to be calculated by taking into consideration the Sun's attraction, and therefore it is not a hyperbola, as we have assumed as a first approximation. Such a curve is asymptotic with the elliptical orbit we want to determine, rather than with the asymptote of the first approximation hyperbola.

At last, also the point of contact with Venus will result different from the point marked in the first approximation scheme.

Therefore, definitively, by giving up the convenient assumption of coplanarity, the effective positions of the Earth and the planets may be represented as in figure 5, which is only a schematic plot.

(*) Formulazioni di Meccanica Astronautica. Proceedings Accademia Lincei July-August 1955.

Posizione spaziale dei tre pianeti per l'angolazione dei piani orbitali

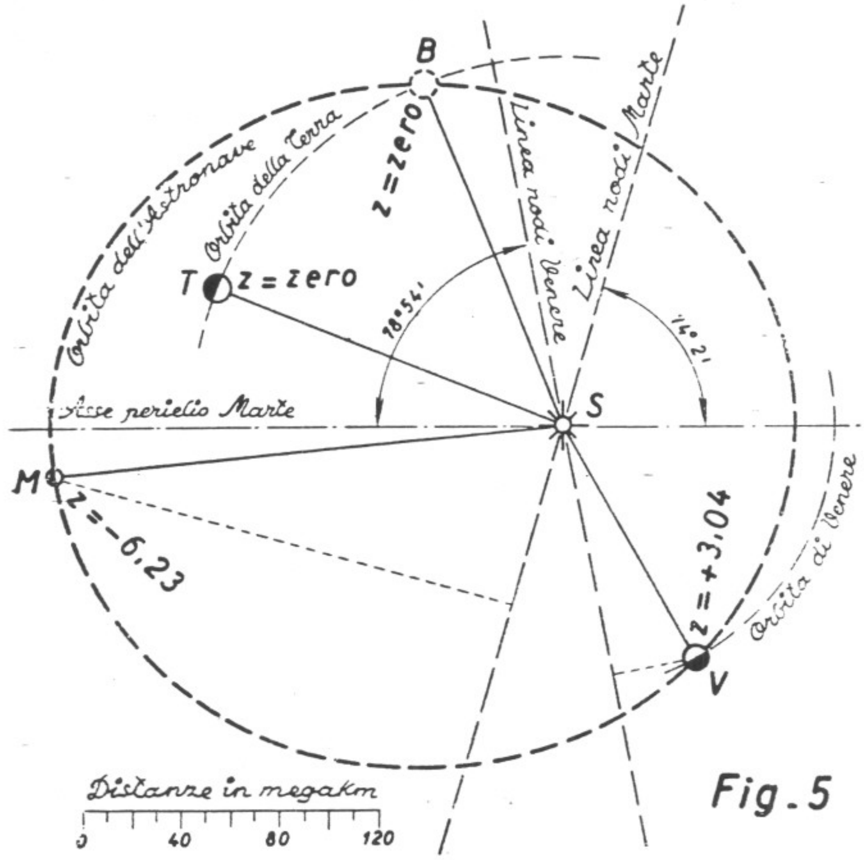

Fig. 5

The plane of the sheet, assumed as transparent, coincides with the plane of the eccliptic and contains the Sun S and two positions of the Earth, that is: B, point of departure; and T, point of arrival. Underneath the sheet, we see the point M of Mars at a negative altitude $Z_m = -6.40$ megakilometers; whilst the point V of Venus remains above the sheet at a positive altitude $Z_v = +1.96$ megakilometers. The above two altitudes are computed by estimating same on the basis of the distances of points M and V from the respective lines of nodes.

As resulting from the figure, the orbital plane of the spaceship will vary three times. At the departure, that is, the ship will have to abandon the plane of the ecliptic in order to follow the plane of the three points B S M leading it to the effective point of contact with Mars. Once arrived here, it should undergo an angular correction of the velocity vector leading it to follow the new plane M S V, crossing Venus. At last, here it should still correct the angle of the velocity vector until determining the new plane V S T, which contains the point T of final contact with the Earth, in general different from the departure point B.

The generic examination of this scheme makes us believe that the first correction will be introduced into the very problem of departure, by suitably deviating of some degree the velocity vector V_0 from the plane of the ecliptic. The second correction is in general of a slight nature and could also be operated with the manoeuvre propellant existing on board the spaceship.

This is not the case for the third correction, which could be of a remarkable nature if we do not keep it into consideration in the predisposed generical laying out of the points of contact with respect to the lines of nodes.

It is useful to point out how such a laying out has a fundamental importance in all problems of interplanetary transfer. Thus, for instance, in the ellipses of cotangential transfer or, anyway, in the ellipses where the point of arrival on the planet is diametrically opposed to the departure point from the Earth, it may be observed that, when the above diametrical position is predisposed on the line of nodes (*), the orbital plane of the spaceship is indeterminate and may be chosen according to convenience. If, on the other hand, the diametrical position should be at right angles with the line of nodes, the plane of the three points earth-sun-planet should rotate exactly 90° with respect to the plane of the ecliptic!

This points out in our case the necessity for a presumable laying out of the programming or for a final variation of same, which lead to acceptable values for the three corrections of the flight plane of the spaceship.

The more so as the corrections of the flight plane are then geometrically connected with the corrections in the flight plane,

(*) As I have assumed in the paper cited in the previous note.

which are necessarily introduced by the planetary perturbations, so far deliberately not considered.

As a matter of fact, the perturbations caused by the contact with the two planets, subject of the programmed exploration trip, are unavoidable. To avoid same it would mean to pass at least at a distance of four hundred radii: and this would frustrate the exploration scope of the trip.

Passing at a short distance from each planet, between zero and some radius, is on the contrary intimately connected with the task of the spaceship, which will be equipped with a telescope of moderate aperture and will have to obtain a magnifying power of images such as to reveal and distinguish the natural accidentalities of the planet from artificial construction, marking the presence of intelligent baings, in the event of any being observed. It will be necessary to approach close also to Venus in order to be able of probing the riddle which is concealed by her thick atmosphere.

On the other hand, it is a great fortune for astronautics that matters stand so, because perturbations can constitute for the pilot exceptional chances of free manoeuvres, that is without consumption of propellant; as F. Lawden has sharply illustrated and codified (*), in case of isolated perturbations.

In our executive scheme we consider on the contrary two successive perturbations, and it is logical to think of the second one in order to compensate the first one for the best, thus achieving definitively an acceptable point of contact with the Earth. The contact with Venus, which in our first approximation scheme is optional, becomes hence an integral part of the executive scheme.

PART II

WITH PERTURBATIONS.

The perturbations which act upon the orbit of a spaceship due to the contact with a planet depend first of all by the relative velocity the spaceship overtakes the planet with, or by which it is overtaken by the planet; and, moreover, by the minimum distance at which the spaceship is flying above the ground of the

(*) F. LAWDEN: « Perturbation manoeuvres », BIS Nov. 1954.

planet, as well as by the orientation of the plane of such a flight with respect to the orbital plane.

The relative velocity of flight is given, on its turn, by the vectorial difference between the velocity of the spaceship and the velocity of the planet. Therefore, it is not only dependent by their numerical value, but also by the angle included between the two velocities.

In laying out the problem, it is however convenient as a first examination to neglect the orientation of the plane of flight with respect to the orbital plane: that is, to take the above two velocities as being both on the orbital plane. In such a case the perturbation acts only in the orbital plane, and remains — ceteris paribus — as function of only one variable. On the contrary, in case the plane of flight with respect to the orbital plane has an orientation different than zero, the perturbation acts simultaneously upon the plane and on its orientation, thus becoming a function of two variables.

This being established, we shall call V_A the own velocity of spaceship, and V_M, V_V the velocity of the planet, respectively Mars and Venus. We shall indicate by σ the angle included, and by u_σ the vector of relative velocity. All of the above three velocities lie on the orbital planes of the respective terns. However, due to the smallness of the angles with respect to the ecliptic, and due to the purpose of demonstration which we propose ourselves, we will assume said velocities to be, as said before, all on the plane of the sheet.

Moreover, the case for Venus is different than the case for Mars, because for the first at the moment of the contact the velocity of the spaceship is higher than the velocity of the planet, while in the case of Mars it is lower. Hence, u_σ, in the case of Venus, has a positive sign, and it is therefore a grazing velocity in an aeronautical sense, that is the spaceship goes towards the planet and overtakes it. On the contrary, in the case of Mars, it is the planet which overtakes the spaceship, by coming upon the latter from the side opposite to the motion; because the relative velocity u_σ has a negative sign.

During the overtaking period, then, in whatever direction, the orbit of the spaceship under the attraction of the planet undergoes an inflection and a variation of its relative velocity, which, with respect to the planet assumed as stationary, results into a coplanar approaching hyperbola similar to that already considered in Fig. 2

I due tipi di passaggio su MARTE

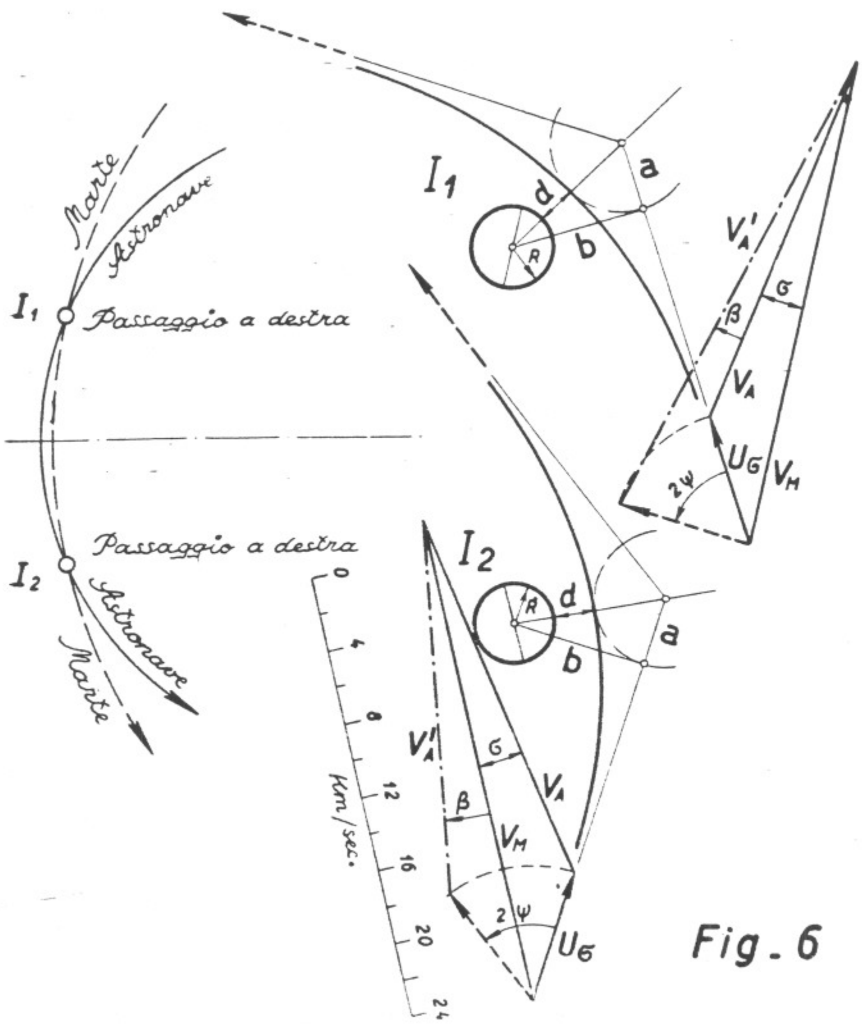

Fig. 6

of this paper. The relative velocity u_G is therefore ideally the velocity to infinity on the inlet asymptote of this hyperbola, while on the outlet asymptote it results to have, at infinity, the same value of the inlet deviated by the angle 2ψ between the asymptotes. At mid-way, it reaches a minimum approach, d, to the ground of the planet.

Practically, to the concept of infinity at inlet and outlet it will be possible to substitute, so long as a greater rigour is not necessary, the concept of a suitable finite distance from the center of the planet, as it has been outlined in the first part of this paper. That is, a distance at which it is possible to assume as conventionally negligible the attraction of the planet, interesting the relative motion, with respect to the Sun's attraction, which prevails over the configuration of the orbit, considered like absolute motion.

We have elsewhere proposed for the Earth a distance of 407 terrestrial radii, which reduces the above ratio to one hundredth. It will be almost equal for Venus, and remarkably lower for Mars, that is about 320 Martian radii.

Let us sketch with such criteria in Fig. 6, on the left, the case of contact with Mars occurring in one of the two points I_1 or I_2, different from A, for which let σ be the angle included between the velocity V_M of the planet and the velocity V_A of the spaceship.

We shall call the above contact as « secant », in order to distinguish it from the contact at A, which will be denoted as « tangent », and which has been not taken into consideration in the following study because it has proved unfavourable.

The two velocities V_M and V_A will be calculated from the known equations of celestial mechanics

$$V_M^2 = \frac{2\,\overline{\mu}}{S_M} - \frac{\overline{\mu}}{a_M} \; ; \; V_A^2 = \frac{2\,\overline{\mu}}{S_M} - \frac{\overline{\mu}}{a_A} \; ; \qquad (6)$$

where μ is the gravitational constant of the Sun; S_M and a_M the radius vector and the semi-major axis of Mars; a_A the semi-major axis of the spaceship orbit.

On the right diagram of the figure, Mars is now presented into the two alternate positions I_1 or I_2, to a scale more than 2000 times greater of the scale of the orbits. To the above diagram we refer the relative velocity u_σ, which is deduced from the velocity triangle, of which we know two sides and the angle included. It remains to establish the minimum approach, d, in order to compute the hyperbola axis a and b, and the rotation 2ϑ of the asymptotes. The own velocity u_σ is to be settled at the predetermined conventional distance. However, in the figure it is practically carried at a distance more suitable for the drawing.

Meanwhile, if R is the radius of the planet, we shall have, as it is known

$$\operatorname{tang} \psi = \frac{a}{b} \quad ; \quad \operatorname{sen} \psi = \frac{a}{R + d + a} \quad ; \tag{7}$$

where a and b are the two axis of the hyperbola, d is the minimum approach of the flight, and 2ψ is the angle by which the outlet asymptote rotates with respect to the inlet asymptote.

Moreover, if g_0 is the gravitation at the ground of the planet, we shall have

$$\frac{a}{R} = \frac{g_0 R}{u_\sigma{}^2} = \frac{V_E{}^2}{2 u_\sigma{}^2} \quad ; \tag{8}$$

where V_E represents the escape velocity. We get the generic calculation equation

$$\operatorname{sen} \psi = \frac{V_E{}^2}{V_E{}^2 + 2\left(1 + \frac{d}{R}\right) u_\sigma{}^2} \quad ; \tag{9}$$

which will be used hereinafter in order to evaluate the relationship between the rotation 2ψ of the asymptote and the minimum approach d.

Anyway, we note at once how equation (9) is not useful for the definition of the piloting manoeuvre intended to obtain the predetermined minimum approach d, owing to the fact that the pilot cannot previously identify d. He can only identify, through the assistance of a suitable optical instrumentation, the yaw b of his relative course from the center of the planet, so to obtain in time that it corresponds with the inlet asymptote of the hyperbola at the conventionally defined moment.

To such a purpose, we will determine from (7) the equation of the ratio

$$\frac{b}{R} = \sqrt{\left(1 + \frac{d}{R}\right)\left(1 + \frac{d}{R} + \frac{V_E{}^2}{u_\sigma{}^2}\right)} \quad ; \tag{10}$$

denoting the yaw of the inlet asymptote necessary to cause the minimum approach d.

Thus, for instance, in the case represented in figure 6 for the two kinds of contact with Mars, for which we have $V_E = 5$ kilometers per second, if assuming $u_\sigma = 6.78$ kms per second, and if desiring to get $\frac{d}{R} = 1$, it is necessary to have a yaw of the inlet asymptote $\frac{b}{R}=2.26$. Similarly, to obtain d/R=zero it is necessary to have a yaw of the asymptote $\frac{b}{R}=1.59$.

Let us now compare the two kinds represented in figure 6.

First of all, we note that, as already mentioned, the spaceship in that case is going before the planet towards the point of contact along its own motion, and it may be in every way orientated at the moment of starting the approaching manoeuvre. Anyway, it is obvious that, when that moment has arrived, the pilot will first of all point against Mars the optical axis of his sight, *facing the target*, as it is customary to say in artillery; and the more so as in this case the target is coming upon him. If the own velocity of the spaceship was correct and sighted, the relative velocity of the pilot will also be correct and sighted; which will thus be automatically referred to the planet, assumed stationary. The correct yaw of the relative course could be checked by means of suitable optical collimation instruments.

The unavoidable errors from what has been foreseen could be corrected in time by means of the double-impulse method, through the propellant reserve and the jet devices intended for manoeuvring purposes. Summing up, the pilot will enter the hyperbola on the predetermined asymptote and will have the feeling of going and overtaking the planet on his right or his left-hand, according to what has been prescribed, like on an invisible motorcar racecourse.

This being established, let us revert to fig. 6, where the overtaking subjectively occurs on the right-hand, since the orbit of the spaceship has been assumed to be inside the orbit of Mars, on the hemisphere lighted by the Sun.

We note at once how in both cases the vector V_A of the own velocity at the hyperbola inlet undergoes an angular deflection β in the same sense, shifting to V'_A, in correspondence with the rotation 2ϑ undergone by the relative velocity u_r. However, the value of said own velocity differs in a remarkable way for the two cases. In fact, in the case I_1 the value of V_A increases when shifting to V'_A; whilst it decreases in the case I_2.

This leads immediately to the exclusion of case I_1, since the increase in velocity causes an increment of the major axis of the perturbed orbit, as it may be calculated by the second equation of (6) by introducing V'_A in place of V_A. From such an increment, a remarkable increase of the periodic time T of the spaceship is accrued, which is given with a rough approximation by the equation

$$5\,T = a_A^{3/2}\,; \qquad (11)$$

where T is computed in terrestrial days and a in Mkms. As a result, a noticeable delay arises in the duration of the travel.

Therefore, we shall choose hereinafter the case corresponding to the point I_2, where the periodic time is decreased.

The new angle of the own velocity V'_A, which we have called β with respect to V_A, introduces then into the perturbation caused by Mars a rotation φ of the orbital major axis of the spaceship, which results in an anticlockwise sense, namely in the same sense where the spaceship orbits. It follows a remarkable deviation of its points of contact with the orbit of Venus. The way for computing the rotation φ is geometrically easy, but analitically it is somewhat difficult. We shall briefly indicate it with reference to Mars (Fig. 8).

First of all, the position of the planet is to be determined, which we call here M, in correspondence with the point of contact, for instance I_2. Then, its solar radius vector S M is to be defined, and the velocity of the planet V_M and its direction are calculated. Being then known the center of the spaceship's orbit, and hence its second focus S_1, the bisecting line of the focal angle S M S_1 is plotted, that is the perpendicular n to the own velocity V_A of the spaceship at the point M of contact. We thus obtain the angle σ between V_M and V_A, giving the relative velocity u_σ at the moment of the contact on the hyperbola inlet asymptote. Having then fixed the minimum approach d to be realized, the corresponding rotation 2ψ is calculated as per Fig. 8. This yields the deviation β of the own velocity from V_A to V'_A. Such a deviation is carried starting from the previous perpendicular n in order to obtain the new perpendicular n', which is the bisecting line of the new focal angle $S\,M\,S_1''$. At last, the new semi-major axis a' of the perturbed orbit is calculated, thus determining the new radius vector $M\,S_1'' = 2\,a' - M\,S$, completing the focal triangle $S\,M\,S_1''$, and hence giving finally the wanted rotation φ of the major axis of the perturbed orbit, and therefore its angle Φ with respect to the perihelion axis of Mars.

In Fig. 8, the triangulation has been plotted for the position M

I due tipi di passaggio su VENERE

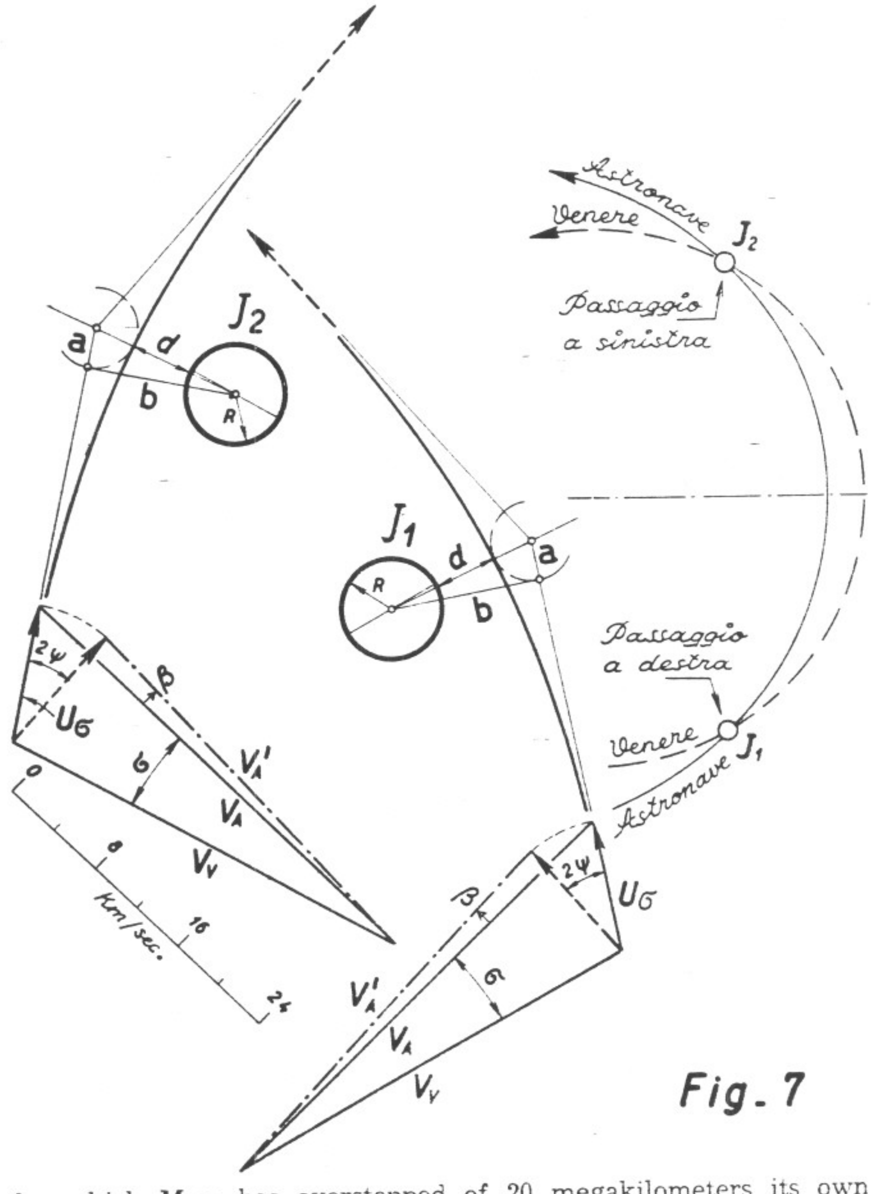

Fig. 7

by which Mars has overstepped of 20 megakilometers its own perihelion. If the spaceship follows a grazing flight, that is $d=0$, the angle Φ of the major axis is $13°30'$. For $d=1$, by which we have $\Phi=7°30'$, the triangulation has not been plotted.

We could now proceed further in settling the perturbation of ı VENERE

Fig. 7

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grazing flight, that is
30′. For d = 1, by which
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ling the perturbation of

Ritardo dovuto alla sola perturbazione di MARTE

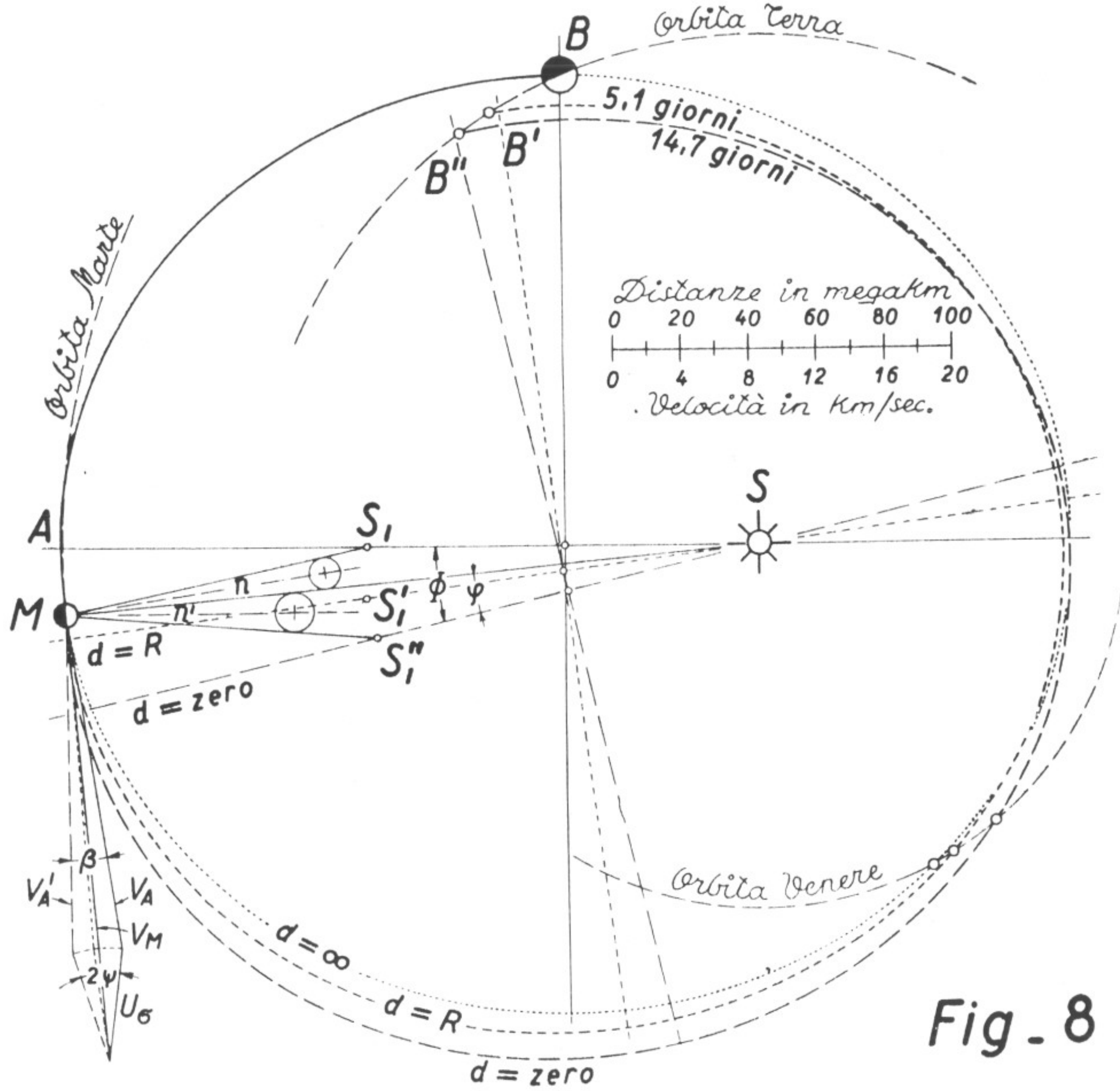

Fig. 8

Ritardi corretti dalla perturbazione di VENERE (per passaggio su MARTE a quota d = 0)

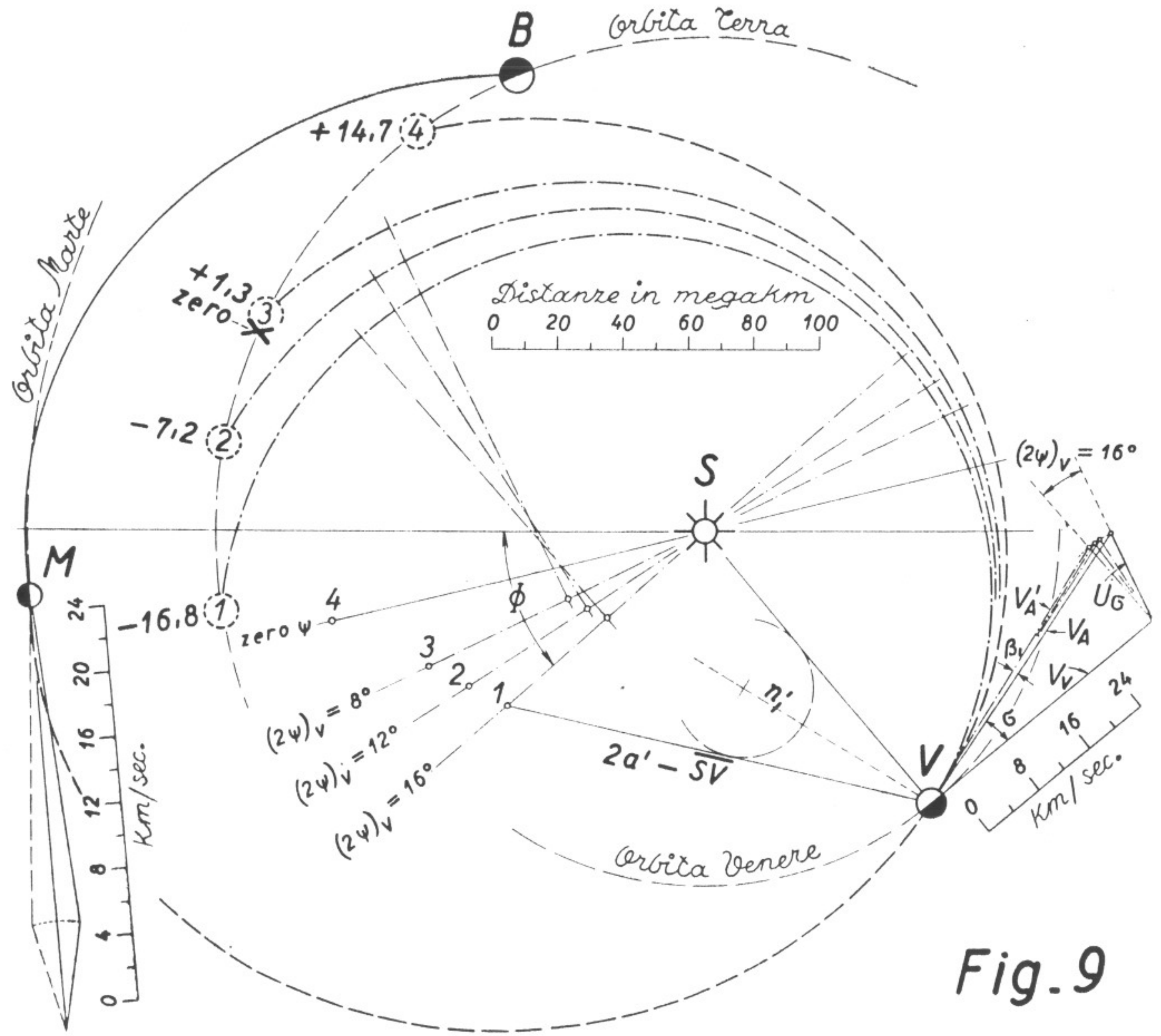

Fig. 9

Ritardi corretti dalla perturbazione di VENERE (per passaggio su MARTE d = R)

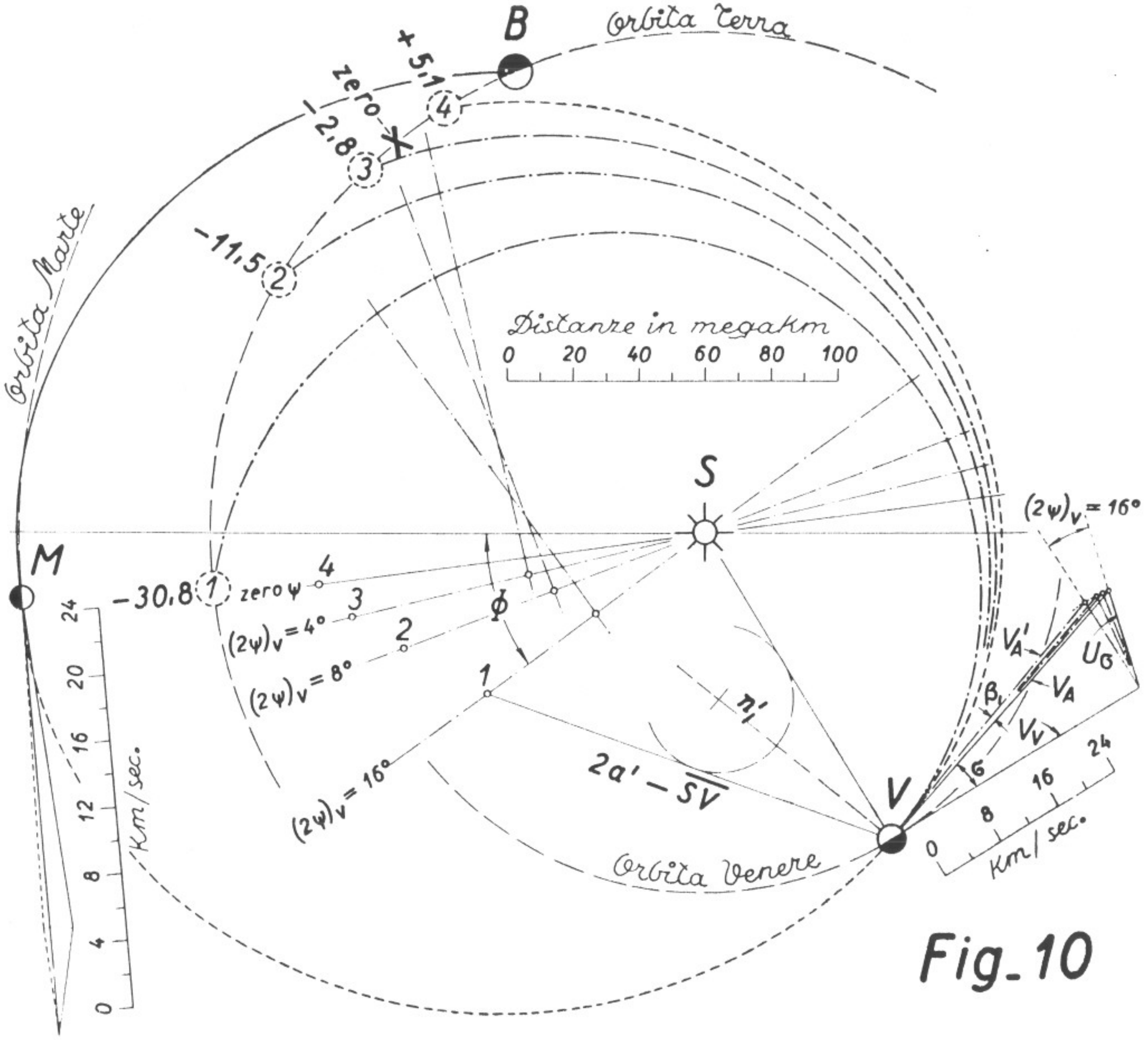

Fig_10

Mars on the remaining of the travel, assumed free from other perturbations. However, we prefer first to take into consideration the requirements of the perturbation of Venus, to point out its different kind, with respect to Mars.

Therefore, we refer to figure 7, where the two orbits intersecting at the two generic points J_1 and J_2 are similarly shown at the right-hand. The two points J_1 and J_2 are assumed, for simplicity, to be symmetrical with respect to the major axis of Mars; whilst in the definitive reality, after the perturbation of the latter, they will be symmetrical with respect to the new major axis of the perturbed orbit.

We have then plotted, at the left-hand and in the middle, to a scale more than 2000 times greater of the scale of the orbits, the planet Venus in way of the two alternate points of contact J_1 and J_2, which therefore do not appear in an evident correlation with the points marked on the right-hand orbit. This for typographic reasons and to avoid the superimposition of the two triangles. For the same reasons, we have further changed the scale of velocities, which helps by sight the typical comparison between this case and the case for Mars. Here, in fact, not only the spaceship is faster than the planet, but the two own velocities V_V and V_A are remarkably less sloping with respect to the orbital major axis of the spaceship, so that we may say the ship to fly over the planet *by side*, that is on an asymptote almost directed towards the Sun. Once arrived in the middle of the hyperbola, the spaceship in its relative motion passes perceptibly from the sun-lit hemisphere to the shadow hemisphere, or vice-versa.

From the above, we derive the existence of two requirements of flight for each of the points J_1 and J_2, which may then be examined, from the corrective point of view of Mars' perturbation, without prejudice on the exploration purpose of the trip, which on the contrary may result optically favoured from a better visibility through the atmosphere of Venus.

Of the four manoeuvring possibilities resulting from this remark, we have however chosen in the figure the two chances promising a decrease in the value of the own velocity, and we have plotted the two corresponding triangles and the two corresponding hyperbolas, which ,assuming to face the planet, must lead the pilot to overtake it subjectively on the *right-hand* for point J_1 and on the *left-hand* for point J_2.

The figure thus confirms for both cases and for a given minimum approach, d, a remarkable decrease in the own velocity of the spaceship when passing from V_A to V'_A, and a simultaneous deviation β in its direction.

The rotation 2ψ of the relative velocity $u_{,,}$ from the inlet asymptote to the outlet asymptote, is also here computed by using the already cited equation (9), putting for V_E the value of the escape velocity from the ground of Venus instead than from the ground of Mars.

Therefore, both cases are interesting. Anyway, for the moment we have had to limit ourselves and to consider only the case of the contact defined by point J_1, and specified in the middle of figure 7.

The perturbation produced on the orbit of the spaceship by the approached contact with Mars is specified in Fig. 8 for the case of the grazing flight d=0. It will be also indicated, but without specification, for the case d=R. The perturbation of Venus is assumed null.

Therefore, let us assume the point M of contact, projected upon the ecliptic, at 20 megakilometers from the Martian perihelion, and let us take M S=206.9 Mkms, putting the semi-major axis of Mars=228 Mkms, and the one of the Earth=149.5 Mkms. Let us evaluate consequently the time t, which the spaceship takes to cover the arc of ellipse B M, having as semi-axis a=149.5 and b=137.8, calculating the sector Ω delimited between the arc B M and the Sun, and introducing it into equation (12)

$$\frac{t}{T} = \frac{\Omega}{\pi\, ab} \quad ; \qquad (12)$$

where T is the periodic time, equal to the time of the Earth, assigned to the ellipse of axis a and b. It is to be remarked that, being the two ratios dimensionless, it will be possible to a fair accuracy to measure Ω by means of a suitable planimeter and to measure a and b to the scale of the drawing.

In such a way, and by checking analitically the result when necessary, we have determined the times which will be quoted hereinafter, getting meanwhile t=125.42 for T=365 terrestrial days.

To evaluate now the successive time t" necessary to cover the remaining of the orbit from point M to point B" of contact with the Earth's orbit after the perturbation caused by Mars, but without any perturbation from Venus, assumed far enough from the point of contact between her orbit and the orbit of the spaceship, we shall proceed as follows. First of all, we shall calculate at point M the velocity of Mars $V_M=26.46$, and the velocity of the spaceship $V_A=19.87$, as well as the angle σ between the two velocities, getting the relative velocity $u_\sigma=6.78$. From the latter, putting as escape velocity from the ground of Mars $V_E=5$ kms per second, by means of equation (9) we obtain the value of $2\psi=24°42'$. We shall then calculate the value of the deviated own velocity V'_A, and from the latter by means of equation (6) the new values a^* and b^* of the perturbed orbit, and the ensuing angle φ of the deviated major axis. Computing at last the new sector Ω^*, having the Sun as its apex and the perturbed trajectory M B″ as its arc, represented by long dashes and corresponding to $d=0$, we have the new additional time t^*.

We obtain, in round figures

$$a^*=149.8 \qquad b^*=138 \qquad t^*=268;$$

so that we get the total time $t+t^*=393.4$, whilst the Earth to be again after one revolution at point B″ requires only $t_T=378.7$ days. Therefore, the spaceship will arrive at point B″ of contact with the terrestrial orbit with a *delay* of 14.7 days. Such a delay could be assumed as a measure for the perturbation of Mars with $d=0$.

For $d=R=$radius of Mars, we similarly get

$$t^*=254.1; \qquad t+t^*=379.5; \qquad t_T=374.4$$

and the measure of perturbation is reduced to 5.1 days of delay, thus lacking also in this case the contact with the Earth. Only for $d=\infty$, that is without perturbations neither from Mars nor from Venus, the contact with the departure planet will be allowed to happen.

We now show how the perturbation of Mars may be corrected by the perturbation of Venus, and we point out the requirements thereto.

As it was shown in Fig. 7 and as it may be seen in Fig. 9, where the maximum perturbation of Mars is taken into consideration, for $d=0$, the contact of the spaceship with the trajectory of Venus, which results displaced by the perturbation of Mars as may be noted in the previous figure 8, appears as a secant contact, having a large angle σ. For the case d=0, by means of equation (6) we calculate the velocity of Venus $V_V=35.04$, and the velocity of the spaceship at the point of contact $V_A=39.63$. The relative velocity $u_\sigma=11.8$ is then deduced, which is remarkably oriented into the direction of the Sun.

To see now if a point of contact, X, exists between the Earth's orbit and the orbit of the spaceship perturbed by Mars and then corrected by Venus, coinciding with the arrival of the Earth at the same point, we have tentatively proceeded into the two figures for a Martian perturbation corresponding respectively to d=zero and d=R. We will specify Fig. 9, assuming the flight over Venus to occur on the right-hand as described in the case J_1 of Fig. 7. Each trial presumes an arbitrary angle $(2\psi)_v$, of which the corresponding minimum approach d_v is then defined.

We then calculate in Fig. 9, from the trigonometric relations to be deduced from the velocity polygon, the value of the velocity perturbed by the flight on the right-hand $V_A=36.8$, and from this the axis of the new orbit $a'_1=120.4$. From the point of contact V, resulting from the illustration of Fig. 8 for d=0, we then plot the perpendicular n_I' to the velocity V'_A deviated of β_1 from the pre-existing value V_A. We have to take into account how said perpendicular bisects the angle between the two focal radii outgoing from point V, while their arithmetical sum must be equal to $2a_1'$. From this we get the second focus of the spaceship orbital ellipse, denoted by number 1, which determines the orientation of the new major axis, having an angle $\Phi=41°30'$ from the perihelion axis of Mars, as per figure.

We may thus define the new path of the spaceship between the point of contact with Venus and the point of contact with the Earth's orbit, denoted by number 1, and we may deduce by means of equation (12) a duration $T_I=264.2$ terrestrial days. Evaluating then the area Ω_I of the concave sector V S 1, we obtain the additional time t_1, required by the spaceship in such a new path for going from V to 1, from the relation

$$\frac{t_1}{T_1} = \frac{\Omega_1}{\pi\, a_1'\, b_1'} , \qquad (13)$$

We obtain $t_1 = 129.6$, whiih must be added to the 296.4 required for the distance Earth-Mars-Venus, and previously calculated. Summing up, we thus get a total time of 426 days, whilst the time of the Earth is evaluated, until point 1, to be 442.9 days. The spaceship hence meets the terrestrial orbit at point 1 with an *advance* of 16.9 terrestrial days.

This shows first of all the corrective power of the perturbation of Venus, since the rotation of the asymptotes of 16 deg, taken as an example, corresponds to a minimum approach to the surface of the planet $d_M = 1.4\ R_v$, calculated by means of equation (4), where for V_E the value of 10.4 kms per second has been put. Therefore, an ample margin exists for additional corrections.

Moreover, the remarkable advance of 16.9 days, compared with the identical delay caused by the only perturbation of Mars for $d = 0$, not corrected by Venus and carried to point 4 of figure 9, corresponding to point B″ of Fig. 8, confirms at first sight the existence at mid-way or thereabout between points 1 and 4 of the wanted point X of coincidence between spaceship and Earth.

The two successive trials, denoted by numbers 2 and 3, for values of $(2\psi)_v$ equal to 12° deg and 8° deg, allow to locate by means of a graph and to a rough approximation the position of X. To this position corresponds, always for $d = 0$ on Mars, the minimum approach $d_v = 4\ R_v$ on Venus. The duration of the whole trip results to be about *thirteen months and a half.*

Fig. 10 presents the similar research for a neutral point X in the case of the minimum approach $d = R$ on Mars. Only the result is indicated, placing in the graph the solutions obtained from the three trials 1, 2, 3 and from the position 4, where only the perturbation of Mars is operating.

We thus get the position of X, plotted in the figure, to which a corrective minimum approach on Venus $d_v = 71\ R_v$ and a duration of the whole trip equal to about *twelve months and a half* are corresponding.

Summing up, we have outlined in this paper the theoretical possibility of an exploration trip Earth-Mars-Venus-Earth having a duration of about one year, showing how the perturbation caused by the approach to Mars may be neutralized, to the purpose of the final contact with the Earth, by the perturbation caused from the approach to Venus.

Obviuosly, the research is to be considered like a laying out of the problem and not like a solution. Too many questions are still to be examined in order to make a positive possibility.

First of all, the existence and the recurrence of a favourable astronomical chance of the three planets, as required by the proposed astronautical contacts, are to be verified. From such a chance, a new check of the astronautical calculation will be originated, and from this a new astronomical investigation, until the most acceptable solution is reached.

Moreover, it will be necessary to verify every chance from the viewpoint of the angular corrections of the orbital plane for each point of contact. These corrections have been so far assumed as being of a slight nature, consistent with the corrective availability of the very perturbations, whilst some of the necessary corrections could result to be implicitly excessive if compared with the means which we dispose of in order to obtain them.

It is further necessary to realize a flight instrumentation suitable for preparing in time to the due accuracy the planetary contacts, as well as a manoeuvring instrumentation able of locating and obtaining the correct asymptote inlets.

At last, it is not possible to neglect the cases we have set aside without examination, some of which lets foresee solutions even more favourable than those discussed. The same initial laying out of an orbit of the spaceship having its semi-major axis equal to the mean terrestrial radius should be dived deep in our search for the best.

Finally, I have the pleasure of pointing out the effective assistance given by the main Chief Draughtsman Spartaco Migani for having faced and carried out with passion and competence all the graphical and analytical calculations of this research as well as the illustrative drawings: and of expressing my gratitude to Capt. Glauco Partel for his valuable co-operation and for having accepted to translate into English language the present paper, filled up with peculiar modes of speech of personal character.

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