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SOME PROBLEMS OF ASTRODYNAMICS AND CELESTIAL MECHANICS

by

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While preparing the structure and contents of a review paper for the Astrodynamics and Celestial Mechanics Section, the authors arrived at the conclusion that it would be much more useful and interesting for the delegates to the Congress to listen to some general considerations on the main problems of the field of contemporary science to which the section is devoted than to hear a monotonous enlistment of numerous names of different scientists and titles of individual contributions.

Really, the number of papers dealing with various problems of motion of artificial celestial bodies which have appeared in recent years in the world periodicals is so great and it grows so rapidly that any review of these papers compiled now and delivered in several months will not satisfactorily characterize the present state of the art.

On the other hand, the number of principal problems around which all separate studies in this field are grouped is not very great, and, a proper description of these problems will make it possible to get a correct understanding both of the problems of the branch of science dealing with space investigations which is of interst to us and its modern status and development.

Besides, the authors deem it necessary to express in the present report their views on the interdependence of different fields of science used in investigations of motions of artificial celestial bodies, and also to consider the question on existing differences of opinion of the names of these fields which sometimes leads to undersirable confusion and misunderstandings.

1. Classical Celestial Mechanics

The term "celestial mechanics" appeared at the very end of the eighteenth century and was coined by one of the main founders of this science, LAPLACE, who also defined its subject. In full agreement with LAPLACE, the following definition can be given: "Celestial mechanics is part of astronomy which deals with the investigation of translational, rotational, and deformational motions of any celestial objects, both natural and artificial, which are under the action of attractive, repulsive, and resistance forces produced by the existence of dust, gaseous or liquid media, electromagnetic forces, forces of radiant pressure, and so on."

Objects whose motion celestial mechanics investigates or should investigate are various material formations moving in interplanetary space from the minute particles of cosmic dust to colossal stars and even star systems. Thus, celestial mechanics deals with the same objects which are studied by astronomy, and difference between celestial mechanics and other parts of astronomy lies mainly in investigation methods.

We have defined celestial mechanics as a science that studies motions of celestial bodies both natural and artificial. However, up to the most recent time (October 4, 1957), the objects of investigations in celestial mechanics were only natural celestial bodies—planets of the solar system, their satellites, asteroids, comets, meteors and meteor streams, star systems, etc.

The characteristic feature of all of these objects is that their existence and motion absolutely do not depend on man's will. Man can only observe celestial bodies from distance, study their nature and properties, and, in particular, watch their motion in the sky. Thus, astronomy of the prespace era and its inseparable part, celestial mechanics, was of observational, one can say, contemplative character, and there was no experimental part up to the most recent time.

Let us briefly describe the main subject of classical celestial mechanics, its aims, methods, and principal tasks which have been formed historically for centuries under the impact of urgent requirements of practical activities of human society.

The most ancient problems of astronomy which belong to celestial mechanics consisted, as is known, in studied of motion of some of the largest or most conspicuous to human eye celestial bodies—the Sun, the Moon, and some of the brightest major planets of the solar system. The development of astronomical instrumentation enabled man to study the motion of satellites of major planets, comets, etc.

The investigation of motion of a celestial body consists in the establishment of general properties of motion, and also in the finding of methods that enable us to determine the position and speed of this body at some moment of time respect to other bodies of the universe. The result of such investigation is, for instance, a compilation of a motion table (ephemerides, astronomical yearbooks, etc.) which make it possible to determine numerical values of coordinates of a heavenly body, and to use the information

obtained for the solution of different practical problems. Methods should also be given for determination of characteristic of motion for sufficiently distant (from the present moment) future and past.

In studies of motions of celestial bodies, their characteristics are used, such as masses of planets and the Sun, mean distances of planets from the Sun, periods of rotation of planets, dimensions and mass of the Earth, etc. These values are known only approximately, and all inaccuracies in their magnitudes necessarily cause corresponding errors in values which determine the positions and velocities of celestial bodies for different moments of time. Comparing the results of calculation with observational results, we get the opportunity of improving the quality of the forementioned characteristics. In such a way, in particular, one can obtain some data on the shape and inner structure of our Earth.

The determination of motions of celestial bodies is reduced to a mathematical problem which consists in integration of some set of ordinary differential equations at given initial conditions and numerical values of parameters. The type and structure of these differential equations depend on the formulation of the problem and on the character of acting forces. It depends on the skill of an investigator in bringing this problem close enough to the actual problem of nature. Various methods have been worked out in classical celestial mechanics for the solution and studies of differential equations.

The aim of analytical methods is to obtain the solution of differential equations of motion in the form of letter formulas which represent sought-for values as explicit functions of time, parameters of the problem, and arbitrary integration constants. If we know numerical values of coordinates from observations, then the same formulas give the possibility of determining parameters of the problem or initial values.

Unfortunately, a strict solution of equations is almost always imposible, and one should be satisfied with an approximate solution which, as a rule, is expressed by means of long and clumsy formulas which represent finite parts of infinite series or are the result of several successive approximations. In addition, such approximate formulas give the necessary solution only for some finite and generally not very large time intervals.

These circumstances have compelled astronomers and mathematicians to find other methods of solution to differential equations of celestial mechanics, and chiefly for this reason numerical methods were introduced which permit one step by step, to obtain separate numerical values of functions satisfying these differential equations, and thus to compile tables that represent the solution of the problem in a purely numerical form.

Beginning from the end of the last century, on the initiative of LYAPUNOV and POINCARÉ, qualitative methods have been developed, the aim of which was a profound investigation of the properties of functions which satisfy differential equations without carrying out integration in an analytical or numerical form.

In some problems of celestial mechanics, examples may be encountered of the use of methods belonging to one of these three groups. For instance, to compile the tables of motion of the Moon, major planets, and some satellites, analytical methods are usually used. In the study of motion of a number of comets and asteroids, mostly numerical methods are used, and for the establishing of general properties of motion, for instance, for the study of periodic solutions and investigation of the motion stability, qualitative methods should be resorted to.

At the end of this section, let us mention that, besides the term "celestial mechanics" adopted by the overwhelming majority of specialists, some other terminology is used. For instance, sometimes science on the motion of celestial bodies is called "theoretical astronomy" or "dynamical astronomy" or otherwise, leaving the term "celestial mechanics" for that part of astronomy which investigates motions only within the limits of the solar system. In our opinion, there are no grounds to change the magnificent term introduced by LAPLACE.

2. Astrodynamics

We have defined celestial mechanics as that part of astronomy dealing with investigation of motion of different celestial bodies, both natural and artificial. The branch of science which long since deals with investigations of natural celestial bodies we have conditionally called "classical celestial mechanics." Now let us turn to consideration of motions of artificial celestial bodies, the first of which, as is well known, appeared on October 4, 1957, and the number of which has grown constantly and progressively.

The investigation and use of motions of these artificial celestial bodies turned, out to be the task of the new branch of celestial mechanics which naturally should be given its own name.

This new name may be introduced by analogy with the names of some sections of classical celestial mechanics, for instance, the theory of the Moon's motion, the theory of motion of major planets, the theory of motion of comets, etc., due to which this new modern part of celestial mechanics may be called the theory of motion of artificial celestial bodies.

Flights of artificial satellites, spaceships, and other artificial objects caused the necessity of considering not only classical but also quite new problems which classical celestial mechanics, due to its "contemplative" character and absolute impossibility of the realization of any experiments, could not even imagine. This branch of science encompassing the totality of dynamic problems of spaceflights should get its own name.

At present, the name "astrodynamics" is widely used. Let us try to describe new problems originating during consideration of motion of artificial celestial bodies. We shall also discuss the problems which are not new but continue to be important.

Let us point out the problems that the new discipline borrowed from classical celestial mechanics. Since the motion of artificial celestial bodies is governed by the same laws as the motion of natural bodies, all methods of classical celestial mechanics regarding the determination of orbits, the calculation of ephemerides, the creation of analytical theories of motion, the determination of the parameters of the Earth and solar system from observed motions, qualitative investigation of the properties of motions, and calculations of disturbances of different kinds are at the same time the problems of astrodynamics.

Let us note that in the majority of cases natural celestial bodies belonging to the solar system (major planets, their satellites, and many minor planets) move in almost circular orbits lying almost in one plane. Methods of classical celestial mechanics have been developed in a considerable degree relative to this situation. Therefore, in the analysis of the motion of artificial celestial bodies, the widely used methods of celestial mechanics are often insufficient and require development and extension by new methods of investigations. Because of causes of such kind, the theory of motion of artificial celestial bodies, considered as part of celestial mechanics, differs from classical science not only by objects of investigation but, to a great extent, by methods of investigation.

In analysis of motions of artificial celestial bodies, new problems often arise which have absolutely no analogies with the corresponding problems of classical celestial mechanics. Really, at present we not only "contemplate" artificial objects in outer space, but we ourselves, by hands of our engineers and technicians, send satellites, rockets, and spaceships beyond the Earth's atmosphere. To conduct such investigations, we should know beforehand how an artificial body will move in outer space, what its orbit will be, now inevitable disturbances will influence its motion, and what regions of outer space it can visit. Such requirements have made us carry out preliminary projection of orbits, and such projection requires the use of entirely new methods.

It should be mentioned that projection of motions with desirable properties set beforehand can be carried out with purely celestial mechanical purposes, for instance, for the determination of a parameter or a constant important for general celestial mechanics or for the verification of some analytical theory experimentally. Thus, old and new sections of science on motion of celestial bodies are interrelated and mutually help each other. It can be also said that, at present, celestial mechanics has an experimental basis that influences considerably the development and use of this splendid science.

At projection of space flights many problems of different character arise, purely technological and mathematical or celestial mechanical. The main direction in this field is preliminary calculation or projection of motions from the point of view of the best satisfaction of the totality of requirements on the flight with a program worked out beforehand. Such calculation should give the initial values of coordinates and components of the velocity of the object launched which correspond to the moment of the beginning of free flight (the moment of discontinuation of the work of rocket engines) and at whose realization the space vehicle will continue to move as is required in order to achieve the objective.

During free flight, the vehicle is influenced by different forces which are known insufficiently exactly and also possibly by a number of forces which are not yet known to us. As a result of this, the vehicle will actually move in an orbit which somewhat differs from the calculated one. Therefore, during free flight of the vehicle it will be necessary at some moments to modify its motion, introducing corresponding corrections into the value and direction of its velocity.

Such circumstances, which involve the consideration of the active influence on a moving celestial body and the control of its flight, sharply distinguish science on the motion of artificial celestial bodies from classical celestial mechanics.

Let us define the subject of astrodynamics. Let us consider that astrodynamics deals with the investigation of motion of uncontrolled and controlled space vehicles, the development and use of methods of orbit projection, and other dynamic problems associated with the accomplishment of space flights.

Astrodynamics is being constantly and intensively developed, using in its development achievements of other sciences—celestial mechanics, analytical mechanics, mathematics, aero- and hydrodynamics, physics, statistics, computer engineering, the theory of control, etc.

3. Celestial Mechanics at the Service of Astrodynamics

After clarification of the term of astrodynamics, as the authors of the present survey understand it, we can naturally turn to discussion of some fundamental problems of this field of science.

The most urgent problems of astrodynamics are the problems of motion of artificial Earth satellites, of the Moon, Venus, and Mars, of interplanetary

travels to the Moon, Venus, Mars and maybe to the regions far away from the plane of ecliptic, near which almost all natural celestial bodies of the solar system move.

Since the main forces that govern, the motions of artificial celestial bodies during free flight beyond the Earth's atmosphere are attractive forces caused by natural celestial bodies, then before one starts calculations of spaceflights and travels, it is necessary to know the character of motion of the planets and satellites most important for us. These data are obtained from astronomers, from classical celestial mechanics, which permits us to calculate very accurately the motions of planets and their satellites either by means of tables published in astronomical yearbooks or by means of methods of numerical integration of different equations of motion or, at last, analytically by means of lettered formulas. These calculations of celestial mechanics are based on numerical values of parameters which are included in equations of motion either directly (for instance, masses of planets) or indirectly, for instance, through gravitational constant, whose numerical value depends on selected units of length, mass, and time.

However, for astrodynamics, numerical values of the main astronomical units and the most important parameters of the Earth and other planets of the solar system in a number of cases turn to be insufficiently exact, and so astrodynamics requires for celestial mechanics more precise determination of some fundamental astronomical constants (the length of the astronomical unit, the Sun's mass, the length of solar day, etc.) which play an essential role in the consideration of the motion of artificial celestial bodies and the calculation of their orbits.

Such problems are, at present, the most important astronomical problems, and astronomers treat them thoroughly, using for this also the information given by astrodynamics about the motion of its objects.

On the other hand, in celestial mechanics itself new problems appear, the results of whose solution aim at increasing the accuracy of the calculation of the motion of planets and satellites by consideration of additional factors, which, up until now, were not taken into account.

The effect of the planet's shape and structure on its translational motion is one such problem. In classical celestial mechanics, the Sun, major planets, and the Moon are considered material points governed by the law of universal gravitation. However, in reality these celestial bodies represent bodies rather complex in shape and physical structure. The attractive forces of bodies are different from those of material points, and so differential equations of motion in celestial mechanics are not exact, which leads to the corresponding inaccuracy in the coordinates and velocities of the planets determined by these differential equations.

The important problem in celestial mechanics is a detailed study of the Earth's gravitational field from the motion of artificial satellites. At present, a detailed analysis of this problem is made only to a first approximation. Additional more precise determination of the parameters of the Earth's gravitational field is of paramount importance.

The study of the structure of the Earth's atmosphere is beyond the scope of celestial mechanics and belongs to geophysics, but investigation of a satellite's motion within the Earth's atmosphere, assuming that the law of drag is known, also is a problem of celestial mechanics for which the corresponding analytical theory may be created. Analysis of satellites' motion permits one to obtain information on the structure of the upper atmosphere.

The study of the Earth's magnetic field which undoubtedly affects the motion of a satellite in the subject of geophysics or physics, but if the laws of this effect are known, then the consideration of the effect of this factor on the satellite's motion is already within the realm of celestial mechanics, and the problem can be set on the creation of the adequate analytical theory and elaboration of methods of investigations of the Earth's magnetic field from the motion of the satellite about the center of mass.

Remarks of this kind can be made regarding some other factors which affect or can affect the motions of satellites, space probes, and spaceships, for instance, solar radiation pressure, the Earth's radiation pressure, etc.

Let us assume that all factors influencing the motion of the given artificial celestial body are revealed and taken into account in adequate terms of differential equations of motion determining some coordinates of the object of interest to us. Thereby some mathematical problem is formulated. We should consider the solution of this problem with utter mathematical rigorousness.

To solve this problem, let us turn primarily to well-known methods of celestial mechanics which permit us to solve similar problems referring to natural celestial bodies.

We have already mentioned that such methods are divided into numerical, qualitative, and analytical ones. Numerical methods, methods of numerical integration of differential equations of motion, have been developed chiefly in classical celestial mechanics (methods by ADAMS, RUNGE, STÖRMER, COWELL, NUMEROV, and others) where numerous examples of their use are known. It should be pointed out that, because of the existence of high-speed computers, numerical methods are widely used in astrodynamics, since they permit us to carry out necessary calculations very rapidly, reliably, confidently, and with the sufficient degree of accuracy. However, numerical methods only are insufficient for investigation of motions of artificial celestial bodies, and astrodynamics should turn to celestial mechanics also for creation of analytical theories of motion.

Such theories, which permit us to obtain lettered formulas representing some coordinates of the object of interest to us, depending on time, initial conditions, and parameters, also designated by letters, do not depend on the degree of accuracy of our knowledge of the parameters or a unit of measurement and, therefore, are sufficiently universal.

Having such analytical theory, we can investigate general properties of the object's motion, for instance, the shape and disposition of its orbit, geometrical properties of the orbit itself, and the law of motion in it. Besides, the analytical theory is very graphic and permits us to reveal the action of individual factors on the motion of the object, orders of individual disturbances, to establish their correlation and interaction, etc. At last, the analytical theory permits us to work out the most simple and economical calculation algorithms by means of which the use of computers permits us to obtain necessary results much more accurately and rapidly than by means of methods of numerical integration of equations of motion.

Moreover, having a lettered analytical theory, we can much more precisely and confidently construct or project orbits having the necessary, predetermined properties, e.g., orbits of travel to neighboring planets.

The construction of any analytical theory, i.e., approximate analytical integration of differential equations of motion, consists usually of several successive stages, the first of which is the selection of an undisturbed orbit; the following stages consist of the detection of disturbances of different orders.

An undisturbed orbit or an undisturbed motion are selected in such a way as to take into account to a first approximation the most influential factors, so that the character of motion will be determined in the main and that, in future, only small corrections need be introduced, i.e., the deviations of actual motion from selected undisturbed motion (disturbances) will be small and easily determined.

In investigations of translational motion of an artificial celestial body, following traditions and customs of astronomers for such an undisturbed motion, undisturbed Keplerian motion is usually taken with an elliptic (or hyperbolic) orbit, all formulas of which are simple for calculation and graphic in use.

The analytical theory consists in the construction of the following approximations, i.e., in the determination of disturbances of the Keplerian motion caused by the effect of different disturbing forces or factors (for instance, the influence of the Earth's shape, drag of the atmosphere, radiant pressure, attraction of other celestial bodies, etc.).

Because of this, the creation of an analytical theory depends on the consideration of different disturbances and on the selection of the main constants that characterize undisturbed motion. Such variables may be rectangular or spherical polar coordinates of the object or arbitrary constants of undisturbed motion: some orbital elements (Keplerian or some other elements) or some other variables which may be selected certainly by a variety of methods.

Then the methods of the perturbation theory give us formulas that represent deviations of selected variables from their values in Keplerian motion in the form of a sum of an usually large number of terms, each of which is some simple function of time, and some parameters. Each of these terms, called inequalities in celestial mechanics, represents a deviation in some coordinate from the value that it has in undisturbed motion in an elliptic (or hyperbolic) orbit caused by the effect of one of the perturbation causes, if perturbations of the first order are considered.

Really, in celestial mechanics it is demonstrated that, to a first approximation which determines disturbances of the first order (i.e., the most large and essential), different inequalities correspond to different disturbing causes, but the same disturbing cause produces many inequalities of the first order.

However, in next approximations, i.e., at consideration of disturbances of the second and higher orders, the same inequality can by produced by simultaneous action of several disturbing forces, which leads to the effect called reimposition of disturbances and which can cause very essential changes in the undisturbed magnitude of the value under consideration.

We should mention also the essential circumstance, namely, that different inequalities of the same order have very different numerical values, depending on different characteristic peculiarities of the disturbing effect and, of course, on initial conditions.

Thus, the so-called secular inequalities are the most essential, each of which contains time (or, in general, an independent variable) as a multiple (in the first or higher degree) and also resonance inequalities which, because of ratios of commensurability, contain very small quantities in the denominator of the coefficient of the trigonometric term.

The forementioned refers to such problems of astrodynamics in which, as in the majority of the problems of celestial mechanics, disturbances imposed on Keplerian motion are, in general, sufficiently small or remain small during some not very large time interval.

As pointed out previously, in some problems of classical celestial mechanics there are cases when the described procedure of the perturbation method turns out to be poorly applicable. In astrodynamics, such cases are more frequent.

In such cases, it is practically inconvenient and inefficient to take a Keplerian (elliptic or hyperbolic) orbit for the basis of calculations, and some other methods should be searched for. The search for an intermediate

orbit with the subsequent determination of disturbances of this intermediate motion elements may serve as an example of such methods.

By an intermediate orbit in celestial mechanics an orbit is meant which differs from a Keplerian one and which is closer to the actual orbit that the celestial body describes. In accordance with this, by an intermediate motion a motion is meant which differs from motion according to Kepler's laws and which is closer to actual motion.

We try to select an intermediate orbit in such a way as, first, to insure formulas of intermediate motion sufficiently simple and easy to use and, secondly, to take into account (maybe partly) some of the main perturbing forces in differential equations of intermediate motion.

In classical celestial mechanics, periodic orbits of the restricted or general three-body problem are known best of all among intermediate orbits. A great deal of different methods have been suggested for finding such orbits (POINCARÉ, LYAPUNOV, SCHWARZSCHILD, etc.).

Periodic intermediate orbits can be used and really are already used in different problems of astrodynamics relating to the theory of motion of artificial satellites which can perform a sufficiently great number of revolutions about the Earth.

Another source of the origin of intermediate orbits is a classical two-fixed-center problem which is of great interest, since differential equations of this problem are integrated in quadratures that permit us to obtain sufficiently simple formulas representing this intermediate motion and to carry out a total analysis of the properties of motion.

In recent years in the Soviet Union, this technique has been used for investigations of motion of the Earth's artificial satellite in the Earth's gravitational field, since it turned out that the Earth's gravitational potential can be represented with sufficient accuracy by a force function of two fixed masses which are at an imaginary distance from each other. Certainly some other selection of intermediate motion is possible.

At last an analysis of the stability of the investigated motion of an artificial celestial body is of great theoretical and practical importance in celestial mechanics. Let us discuss briefly what this problem is about. This problem is often referred to the field of theoretical mechanics, but for the first time it appeared in celestial mechanics in analysis of the solar system stability problem.

Let us discuss a problem on motion of some artificial object under the influence of quite definite forces whose laws of actions are known. In this case motion depends solely on the initial conditions, and, provided their definite selection is made, motion and its properties will be also quite definite. But it is well known that the initial conditions cannot be realized with absolute accuracy and are complicated by errors of different origin.

Therefore, a question naturally arises as to how inaccuracy in the initial conditions influences subsequent motion of an object and whether small changes in the initial conditions will lead to considerable changes of the properties of motion and values of coordinates and velocity of the moving body. On the other hand, it is very important to be in a position to estimate these subsequent variations knowing the corresponding initial conditions variations and, on the contrary, to determine increases in the initial values corresponding to the known (for instance, from observations) fluctuations in values of interest to us at some instant of time.

All these problems can be solved by methods of the general motion stability theory created by the famous Russian scientist A. M. LYAPUNOV and developed subsequently by contributions of many scientists.

It is especially important that LYAPUNOV's theory not only gives the possibility of solving the stability problem, i.e., of establishing whether motion under investigation will be stable or unstable, but also permits us to establish a correlation between initial and subsequent deviations (disturbances) without integration of motion either analyticaly or by numerical methods.

The other significant problem of the motion stability theory is investigation of the influence of minor disturbing forces constantly acting during motion, but not taken into account at the raising of the problem because they are relatively small or not known to us beforehand.

Among such forces are, for example, forces of interactions between an artificial object and casually flying meteorites or asteroids, or some space forces not known to us, etc.

This problem is similar to one just described but differs from it by the following. Here disturbances are not instantaneous and act constantly during motion. That is why, in new differential equations of motion, one can say that random terms appear. They are small but can produce considerable changes in motion calculated preliminarily.

It is clear that stability of such a kind should play a very important role in astrodynamics, and, therefore, methods of the solution of the stability problem at constantly acting disturbances should be worked out in detail, so that by means of such methods necessary numerical estimates can be obtained.

4. Orbit Projecting

The development of space flights has made it possible to work out different methods of orbit projecting. By orbit projecting the choosing of space flight orbits is meant with the aim of the most complete solution of the basic flight problem at the most economical use of technical means.

In preliminary analysis, the whole totality of orbits is essential, which insures, at least in principle, the solution of the chief objective of flight. For instance, at projecting of flight to the Moon it is important to imagine all the trajectories along which such travel is feasible in order to be able to determine the initial conditions necessary for the realization of different trajectories and to determine kinematic and dynamic characteristics of orbits, such as the time of flight, the velocity of encounter with the Moon, the value of required initial energy, conditions of observation from predetermined points at the Earth's surface, etc.

Analyzing the totality of possible orbits, one can select those among them which satisfy best the necessary requirements both from the point of view of the efficiency of the solution of the main objective of flight and from the point of view of simplicity and economical efficiency of the realization. These requirements are mostly contradictory, and final solution is usually the result of a compromise and consideration of available actual engineering means.

One of the most important requirements is economical efficiency of the launching from the viewpoint of the power necessary for injecting the object into orbit. Practically, two main methods of the placing of vehicles in space orbits are used: the continuous powered stage and the launching from a satellite orbit. The first method is technically more simple. However, in some cases it is connected with difficulties.

The matter is that the selection of the place of launching is necessarily limited, and for acceleration of a space vehicle with the prescribed mission it may become necessary to use the trajectories steeply inclined to the local horizon. This leads to the increase of losses due to the necessity of overcoming the gravitational force and reducing available final velocity, or, at the prescribed conditions of the placing in orbit, of reducing the possible weight of the space vehicle. Therefore, the continuous acceleration method limits the range of velocity directions at the beginning of motion along the orbit, making not only orbits with maximally low initial velocity more desirable but also orbits with maximally low inclination of the velocity vector to the local horizon at the end of the powered flight.

The method of the launching from a satellite orbit is free from energy limitations imposed on the acceleration direction. Any direction of the velocity vector is obtained by the proper choice of the two time instants: the time of launching into a parking satellite orbit which gives the aiming azimuth at the expense of the turn of the parking orbit, together with the Earth in its 24-hr motion, and the time of the launching from a satellite orbit which gives an aiming elevation angle because the transfer from a satellite orbit occurs in the place where motion in a satellite orbit has the necessary direction. Acceleration of a space vehicle during its injection to a satellite orbit and during its transfer from it takes place at minimum inclination angles to the local horizon and insures the maximum use of the booster's energy potentialities. The method of the launching of space vehicles from a satellite orbit used by Soviet scientists and engineers is a distinguished engineering achievement. The use of such a method of launching restricts only the value of initial velocity of motion along the orbit, permitting us to place heavy space vehicles in orbit, sharply extending the range of possible orbits and facilitating the conditions of their expedient selection.

Another complex of problems associated with the orbit projecting is investigation of required accuracies of the realization of the selected nominal orbit and the selection of the correction method. In cases when flight is carried out without correction of the trajectory in its path, the problem consist in the search for such a region of parameter deviations at the end of the acceleration stage that the main objective of flight can be achieved if deviations do not go beyond the indicated region. For instance, if the objective of flight is to reach the Moon, then such launching parameter deviations are searched for at which the orbits pass through the Moon, and hence the Moon is reached. It is natural that the less rigorous are limitations on the region of the launching parameter spread, the simpler is the realization of flight, the less are requirements on the accuracy of launching devices, the lower is the weight of these devices, and the higher is their reliability. Therefore, the choosing of space flight orbits which permit maximum deviations of launching parameters is desirable. This requirement can contradict and usually does contradict the energy optimalness of the orbit, and such situation is typical in orbit projecting.

It often turns out that the permissible region of initial deviations is exceedingly small and cannot be realized by existing technical means. Besides, knowledge of celestial mechanics constants, such as the solar parallax or planetary orbital elements, can turn out to be insufficient, so that even ideal realization of launching conditions does not guarantee the accomplishment of the objective of spaceflight. In these cases, in-flight orbit correction should be used—correction of motion parameters which can be carried out by imparting impulses of the appropriate value and direction in some places of the orbit. During the flight, such orbit correction can take place once or several times.

Orbit correction requires the onboard correcting of propulsive installation and fuel store. The value of additional weight that should be taken aboard the space vehicle in connection with orbit correction depends on the value of the correcting impulse or the magnitude of the total impulse in the case of correction performed several times. The value of the correcting impulse depends on the motion parameter spread at the end of the acceleration stage, and will be the greater, the greater is the spread region. Besides, the value of the impulse necessary for orbit correction depends on the place in the orbit where this correction is carried out. For instance, if correction is carried out too close to the target, a very large change of velocity of motion and too large correcting impulse, and, therefore, considerable additional weight aboard the space vehicle can be required.

While choosing orbits, we should prefer those orbits which allow the most simple and economical correction. At the same time, the correction optimization problem arises, i.e., such selection of the orbit and such selection of correction points in it that the execution of correction will require the minimum total impulse and minimum additional weight aboard the space vehicle.

The solution of the correction problem requires exact determination of the actual motion parameters during the flight, the calculation of deviations of the motion parameters from the nominal ones, and the calculation of the necessary correction parameters. The determination of the orbital elements is a classical problem of celestial mechanics.

However, its solution in connection with space vehicle flights is associated with a number of specific requirements. For instance, it is often necessary to determine the orbital elements with maximum speed. Therefore, the calculation algorithm, which usually contains an iterative process, should be very economical and insure the small number of iterations and small time of the performance of each iteration. The calculation algorithm should be very reliable and faultless, guaranteeing the process convergence even in the case of not sufficiently adequate selection of an initial approximation.

The accuracy of the determination of the actual orbital elements depends on the combination and accuracy of measured parameters and also on the location and extent of the measured interval in the orbit. At the prescribed scope and accuracy of measurements, the larger is the orbit section at which measurements are carried out, the more accurate, as a rule, can be the determination of the actual motion parameters, and the more reliable and accurate will be orbit correction. However, excessive delay is unrational, since it may lead to extremely late correction and extremely large value of the correcting impulse. Early correction may turn out to be more economical. However, not sufficient accuracy in the determination of orbital elements by the correction moment may lead to its insufficient accuracy and to the necessity of its repetition.

The forementioned considerations illustrate the complexity and the contradictory character of the problem of the projecting of the system of measurements and orbit correction, i.e., projecting of the flight control system of a space vehicle. The optimum solution of the flight control problem consists in the creation of a system insuring the solution of the main flight objective most simply and reliably and most economically as far as onboard weights are concerned. Therefore, in orbit selection we should prefer those orbits for which it is possible to carry out flight control closer to the optimum one.

Orbit projecting consists in the detection and consideration of a number of contradictory requirements on the orbit, some of which are briefly mentioned in the foregoing, in the performance of the complex analysis, and in the selection of such an orbit which maximally satisfies all the necessary requirements. The performance of an all-round analysis of flight orbits requires calculation of a great deal of variants. At the same time, the requirements on the accuracy of calculations at the initial stage of projection are usually not very high. Therefore, the rational solution is the development and use of various approximate methods that permit us simply, economically, and graphically, though with limited accuracy, to analyze orbits from the point of view of the degree of satisfaction of requirements on the orbit and to search for compromise variants giving the optimum solution to the problem on the whole.

The next projecting stage is associated with the performance of the more accurate calculations for the selected variants in which all of the necessary factors affecting the space vehicle flight are taken into account. Such calculations are usually carried out by numerical integration methods with the use of the most precise constants and are aimed at obtaining exact values of flight parameters and parameters determining the placing in a space orbit. Since more exact calculations often are very labor-consuming, the development of effective calculation methods is not less urgent than the development of calculation methods for the preliminary projecting stage. The effective more accurate calculation technique should combine the accuracy and speed of operations. The maximum use of our knowledge of orbits is a rational path to creation of such methods. For instance, motion of a space vehicle with respect to the Earth inside its sphere of action is close to motion along a conic section with the focus in the Earth's center. Motion beyond the sphere of the Earth's action is close to heliocentric motion along an undisturbed orbit, etc. The consideration of these circumstances introduces a new moment into the technique of more exact calculations and opens up the path to their improvement. Certainly, other paths are possible.

Methods of orbit investigations are essentially determined by the character of flight. Multirotational orbits and orbits with small angular distance can be sharply differentiated. Orbits of the Earth's, Moon's, and planetary satellites which perform during the period of their existence a large number of circuits belong to the orbits of the first type. Investigation and projecting of such orbits is associated with the use of methods permitting us to reveal the picture of the evolution of the osculating orbit parameters with time under the influence of disturbing factors, such as noncentrality of the gravitational field, the effect of the atmosphere, disturbances from other celestial bodies, the effect of the pressure of light, etc. The evolution process calculation problem can be considered as a problem of a nonlinear theory of oscillations. A wide use of different averaging methods and the technique of the construction of asymptomic solutions can insure the creation of simple and efficient methods for preliminary and more accurate calculations.

Orbits with small angular distance are, for instance, orbits of travels from the Earth to the Moon and from the Earth to Mars, Venus, and other planets. Orbits of such travels are to a first approximation arcs of conic sections, and evolution problems do not arise here. It is rational to create approximate methods, either completely ignoring disturbances or taking them roughly into account.

The orbit of travel to Mars may be regarded as consisting of pieces of conic-sections—undisturbed geocentric motion in the sphere of the Earth's action, undisturbed heliocentric motion beyond the sphere of action of the Earth and Mars, and undisturbed conic section with the focus in the center of Mars when motion takes place inside the sphere of its action.

Such are some of considerations on orbit projecting. This new field of investigations is at the juncture of celestial mechanics and space engineering, and, because of the progress in space conquest, it is being intensively developed. The development of astrodynamics promotes the more successful investigation and conquest of interplanetary space and planets of the solar system by man, and the penetration of inquisitive human thought into the most remote corners of the universe.