CHAOS

Making a New Science

JAMES GLEICK

VIKING

responded to a request from a North Vietnamese reporter by giving a press conference on the broad steps of Moscow University. He began by condemning the American intervention in Vietnam, and then, just as his hosts began to smile, added a condemnation of the Soviet invasion of Hungary and the absence of political freedom in the Soviet Union. When he was done, he was quickly hustled away in a car for questioning by Soviet officials. When he returned to California, the National Science Foundation canceled his grant.

Smale's Fields Medal honored a famous piece of work in topology, a branch of mathematics that flourished in the twentieth century and had a particular heyday in the fifties. Topology studies the properties that remain unchanged when shapes are deformed by twisting or stretching or squeezing. Whether a shape is square or round, large or small, is irrelevant in topology, because stretching can change those properties. Topologists ask whether a shape is connected, whether it has holes, whether it is knotted. They imagine surfaces not just in the one-, two-, and three-dimensional universes of Euclid, but in spaces of many dimensions, impossible to visualize. Topology is geometry on rubber sheets. It concerns the qualitative rather than the quantitative. It asks, if you don't know the measurements, what can you say about overall structure. Smale had solved one of the historic, outstanding problems of topology, the Poincaré conjecture, for spaces of five dimensions and higher, and in so doing established a secure standing as one of the great men of the field. In the 1960s, though, he left topology for untried territory. He began studying dynamical systems.

Both subjects, topology and dynamical systems, went back to Henri Poincaré, who saw them as two sides of one coin. Poincaré, at the turn of the century, had been the last great mathematician to bring a geometric imagination to bear on the laws of motion in the physical world. He was the first to understand the possibility of chaos; his writings hinted at a sort of unpredictability almost as severe as the sort Lorenz discovered. But after Poincaré's death, while topology flourished, dynamical systems atrophied. Even the name fell into disuse; the subject to which Smale nominally turned was differential equations. Differential equations describe the way systems change continuously over time. The tradition was to look at such things locally, meaning that engineers or physicists would

consider one set of possibilities at a time. Like Poincaré, Smale wanted to understand them globally, meaning that he wanted to understand the entire realm of possibilities at once.

Any set of equations describing a dynamical system—Lorenz's, for example—allows certain parameters to be set at the start. In the case of thermal convection, one parameter concerns the viscosity of the fluid. Large changes in parameters can make large differences in a system—for example, the difference between arriving at a steady state and oscillating periodically. But physicists assumed that very small changes would cause only very small differences in the numbers, not qualitative changes in behavior.

Linking topology and dynamical systems is the possibility of using a shape to help visualize the whole range of behaviors of a system. For a simple system, the shape might be some kind of curved surface; for a complicated system, a manifold of many dimensions. A single point on such a surface represents the state of a system at an instant frozen in time. As a system progresses through time, the point moves, tracing an orbit across this surface. Bending the shape a little corresponds to changing the system's parameters, making a fluid more viscous or driving a pendulum a little harder. Shapes that look roughly the same give roughly the same kinds of behavior. If you can visualize the shape, you can understand the system.

When Smale turned to dynamical systems, topology, like most pure mathematics, was carried out with an explicit disdain for real-world applications. Topology's origins had been close to physics, but for mathematicians the physical origins were forgotten and shapes were studied for their own sake. Smale fully believed in that ethos—he was the purest of the pure—yet he had an idea that the abstract, esoteric development of topology might now have something to contribute to physics, just as Poincaré had intended at the turn of the century.

One of Smale's first contributions, as it happened, was his faulty conjecture. In physical terms, he was proposing a law of nature something like this: A system can behave erratically, but the erratic behavior cannot be *stable*. Stability—"stability in the sense of Smale," as mathematicians would sometimes say—was a crucial property. Stable behavior in a system was behavior that would not disappear just because some number was changed a tiny bit. Any system could have both stable and unstable behaviors within it. The equations governing a pencil standing on its point have a good mathematical solution with the center of gravity directly above the point—but you cannot stand a pencil on its point because the solution is unstable. The slightest perturbation draws the system away from that solution. On the other hand, a marble lying at the bottom of a bowl stays there, because if the marble is perturbed slightly it rolls back. Physicists assumed that any behavior they could actually observe regularly would have to be stable, since in real systems tiny disturbances and uncertainties are unavoidable. You never know the parameters exactly. If you want a model that will be both physically realistic and robust in the face of small perturbations, physicists reasoned that you must surely want a stable model.

The bad news arrived in the mail soon after Christmas 1959, when Smale was living temporarily in an apartment in Rio de Janeiro with his wife, two infant children, and a mass of diapers. His conjecture had defined a class of differential equations, all structurally stable. Any chaotic system, he claimed, could be approximated as closely as you liked by a system in his class. It was not so. A letter from a colleague informed him that many systems were not so well-behaved as he had imagined, and it described a counterexample, a system with chaos and stability, together. This system was robust. If you perturbed it slightly, as any natural system is constantly perturbed by noise, the strangeness would not go away. Robust and strange—Smale studied the letter with a disbelief that melted away slowly.

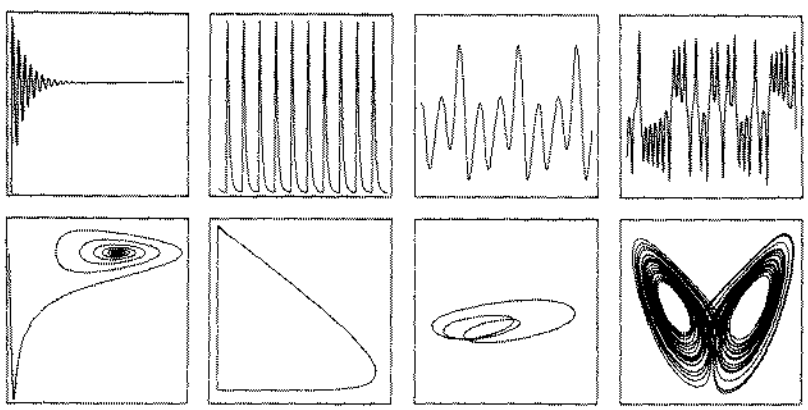
Chaos and instability, concepts only beginning to acquire formal definitions, were not the same at all. A chaotic system could be stable if its particular brand of irregularity persisted in the face of small disturbances. Lorenz's system was an example, although years would pass before Smale heard about Lorenz. The chaos Lorenz discovered, with all its unpredictability, was as stable as a marble in a bowl. You could add noise to this system, jiggle it, stir it up, interfere with its motion, and then when everything settled down, the transients dying away like echoes in a canyon, the system would return to the same peculiar pattern of irregularity as before. It was locally unpredictable, globally stable. Real dynamical systems played by a more complicated set of rules than anyone had imagined. The example described in the letter from Smale's colleague was another simple system, discovered more than a generation earlier and all but forgotten. As it happened, it was a pendulum in disguise: an oscillating electronic circuit. It was nonlinear and it was periodically forced, just like a child on a swing.

It was just a vacuum tube, really, investigated in the twenties by a Dutch electrical engineer named Balthasar van der Pol. A modern physics student would explore the behavior of such an oscillator by looking at the line traced on the screen of an oscilloscope. Van der Pol did not have an oscilloscope, so he had to monitor his circuit by listening to changing tones in a telephone handset. He was pleased to discover regularities in the behavior as he changed the current that fed it. The tone would leap from frequency to frequency as if climbing a staircase, leaving one frequency and then locking solidly onto the next. Yet once in a while van der Pol noted something strange. The behavior sounded irregular, in a way that he could not explain. Under the circumstances he was not worried. "Often an irregular noise is heard in the telephone receivers before the frequency jumps to the next lower value," he wrote in a letter to *Nature*. "However, this is a subsidiary phenomenon." He was one of many scientists who got a glimpse of chaos but had no language to understand it. For people trying to build vacuum tubes, the frequency-locking was important. But for people trying to understand the nature of complexity, the truly interesting behavior would turn out to be the "irregular noise" created by the conflicting pulls of a higher and lower frequency.

Wrong though it was, Smale's conjecture put him directly on the track of a new way of conceiving the full complexity of dynamical systems. Several mathematicians had taken another look at the possibilities of the van der Pol oscillator, and Smale now took their work into a new realm. His only oscilloscope screen was his mind, but it was a mind shaped by his years of exploring the topological universe. Smale conceived of the entire range of possibilities in the oscillator, the entire phase space, as physicists called it. Any state of the system at a moment frozen in time was represented as a point in phase space; all the information about its position or velocity was contained in the coordinates of that point. As the system changed in some way, the point would move to a new position in phase space. As the system changed continuously, the point would trace a trajectory.

For a simple system like a pendulum, the phase space might just be a rectangle: the pendulum's angle at a given instant would determine the east-west position of a point and the pendulum's speed would determine the north-south position. For a pendulum swinging regularly back and forth, the trajectory through phase space would be a loop, around and around as the system lived through the same sequence of positions over and over again.

Smale, instead of looking at any one trajectory, concentrated on the behavior of the entire space as the system changed—as more driving energy was added, for example. His intuition leapt from the physical essence of the system to a new kind of geometrical essence. His tools were topological transformations of shapes in phase space—transformations like stretching and squeezing. Sometimes these transformations had clear physical meaning. Dissipation in a system, the loss of energy to friction, meant that the system's shape in phase space would contract like

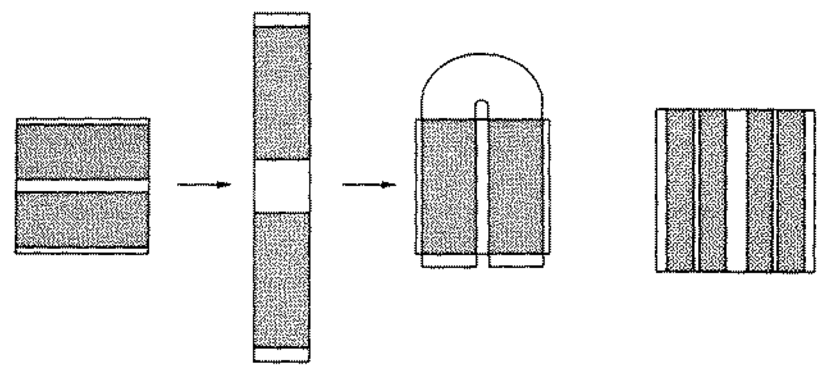


MAKING PORTRAITS IN PHASE SPACE. Traditional time series (*above*) and trajectories in phase space (*below*) are two ways of displaying the same data and gaining a picture of a system's long-term behavior. The first system (*left*) converges on a steady state—a point in phase space. The second repeats itself periodically, forming a cyclical orbit. The third repeats itself in a more complex waltz rhythm, a cycle with "period three." The fourth is chaotic.

a balloon losing air—finally shrinking to a point at the moment the system comes to a complete halt. To represent the full complexity of the van der Pol oscillator, he realized that the phase space would have to suffer a complex new kind of combination of transformations. He quickly turned his idea about visualizing global behavior into a new kind of model. His innovation—an enduring image of chaos in the years that followed—was a structure that became known as the horseshoe.

To make a simple version of Smale's horseshoe, you take a rectangle and squeeze it top and bottom into a horizontal bar. Take one end of the bar and fold it and stretch it around the other, making a C-shape, like a horseshoe. Then imagine the horseshoe embedded in a new rectangle and repeat the same transformation, shrinking and folding and stretching.

The process mimics the work of a mechanical taffy-maker, with rotating arms that stretch the taffy, double it up, stretch it again, and so on until the taffy's surface has become very long, very thin, and intricately self-embedded. Smale put his horseshoe through an assortment of topological paces, and, the mathematics aside, the horseshoe provided a neat visual analogue of the sen-



SMALE'S HORSESHOE. This topological transformation provided a basis for understanding the chaotic properties of dynamical systems. The basics are simple: A space is stretched in one direction, squeezed in another, and then folded. When the process is repeated, it produces a kind of structured mixing familiar to anyone who has rolled many-layered pastry dough. A pair of points that end up close together may have begun far apart.

sitive dependence on initial conditions that Lorenz would discover in the atmosphere a few years later. Pick two nearby points in the original space, and you cannot guess where they will end up. They will be driven arbitrarily far apart by all the folding and stretching. Afterward, two points that happen to lie nearby will have begun arbitrarily far apart.

Originally, Smale had hoped to explain all dynamical systems in terms of stretching and squeezing—with no folding, at least no folding that would drastically undermine a system's stability. But folding turned out to be necessary, and folding allowed sharp changes in dynamical behavior. Smale's horseshoe stood as the first of many new geometrical shapes that gave mathematicians and physicists a new intuition about the possibilities of motion. In some ways it was too artificial to be useful, still too much a creature of mathematical topology to appeal to physicists. But it served as a starting point. As the sixties went on, Smale assembled around him at Berkeley a group of young mathematicians who shared his excitement about this new work in dynamical systems. Another decade would pass before their work fully engaged the attention of less pure sciences, but when it did, physicists would realize that Smale had turned a whole branch of mathematics back toward the real world. It was a golden age, they said.

"It's the paradigm shift of paradigm shifts," said Ralph Abraham, a Smale colleague who became a professor of mathematics at the University of California at Santa Cruz.

"When I started my professional work in mathematics in 1960, which is not so long ago, modern mathematics in its entirety—in its *entirety*—was rejected by physicists, including the most avant-garde mathematical physicists. So differentiable dynamics, global analysis, manifolds of mappings, differential geometry—everything just a year or two beyond what Einstein had used—was all rejected. The romance between mathematicians and physicists had ended in divorce in the 1930s. These people were no longer speaking. They simply despised each other. Mathematical physicists refused their graduate students permission to take math courses from mathematicians: *Take mathematics from us. We will teach you what you need to know. The mathematicians are on some kind of terrible ego trip and they will destroy your mind.* That was 1960. By 1968 this had completely turned around."

Eventually physicists, astronomers, and biologists all knew they had to have the news.

A MODEST COSMIC MYSTERY: the Great Red Spot of Jupiter, a vast, swirling oval, like a giant storm that never moves and never runs down. Anyone who saw the pictures beamed across space from Voyager 2 in 1978 recognized the familiar look of turbulence on a hugely unfamiliar scale. It was one of the solar system's most venerable landmarks—"the red spot roaring like an anguished eye/ amid a turbulence of boiling eyebrows," as John Updike described it. But what *was* it? Twenty years after Lorenz, Smale, and other scientists set in motion a new way of understanding nature's flows, the other-worldly weather of Jupiter proved to be one of the many problems awaiting the altered sense of nature's possibilities that came with the science of chaos.

For three centuries it had been a case of the more you know, the less you know. Astronomers noticed a blemish on the great planet not long after Galileo first pointed his telescopes at Jupiter. Robert Hooke saw it in the 1600s. Donati Creti painted it in the Vatican's picture gallery. As a piece of coloration, the spot called for little explaining. But telescopes got better, and knowledge bred ignorance. The last century produced a steady march of theories, one on the heels of another. For example:

The Lava Flow Theory. Scientists in the late nineteenth century imagined a huge oval lake of molten lava flowing out of a volcano. Or perhaps the lava had flowed out of a hole created by a planetoid striking a thin solid crust.

The New Moon Theory. A German scientist suggested, by contrast, that the spot was a new moon on the point of emerging from the planet's surface.

The Egg Theory. An awkward new fact: the spot was seen to be drifting slightly against the planet's background. So a notion put forward in 1939 viewed the spot as a more or less solid body floating in the atmosphere the way an egg floats in water. Variations of this theory—including the notion of a drifting bubble of hydrogen or helium—remained current for decades.

The Column-of-Gas Theory. Another new fact: even though the spot drifted, somehow it never drifted far. So scientists proposed in the sixties that the spot was the top of a rising column of gas, possibly coming through a crater.

Then came Voyager. Most astronomers thought the mystery would give way as soon as they could look closely enough, and indeed, the Voyager fly-by provided a splendid album of new data, but the data, in the end, was not enough. The spacecraft pictures in 1978 revealed powerful winds and colorful eddies. In spectacular detail, astronomers saw the spot itself as a hurricane-like system of swirling flow, shoving aside the clouds, embedded in zones of east-west wind that made horizontal stripes around the planet. *Hurricane* was the best description anyone could think of, but for several reasons it was inadequate. Earthly hurricanes are powered by the heat released when moisture condenses to rain; no moist processes drive the Red Spot. Hurricanes rotate in a cyclonic direction, counterclockwise above the Equator and clockwise below, like all earthly storms; the Red Spot's rotation is anticyclonic. And most important, hurricanes die out within days.

Also, as astronomers studied the Voyager pictures, they realized that the planet was virtually all fluid in motion. They had been conditioned to look for a solid planet surrounded by a paper-thin atmosphere like earth's, but if Jupiter had a solid core anywhere, it was far from the surface. The planet suddenly looked like one big fluid dynamics experiment, and there sat the Red Spot, turning steadily around and around, thoroughly unperturbed by the chaos around it.

The spot became a gestalt test. Scientists saw what their intuitions allowed them to see. A fluid dynamicist who thought of turbulence as random and noisy had no context for understanding an island of stability in its midst. Voyager had made the mystery doubly maddening by showing small-scale features of the flow, too small to be seen by the most powerful earthbound telescopes. The small scales displayed rapid disorganization, eddies appearing and disappearing within a day or less. Yet the spot was immune. What kept it going? What kept it in place?

The National Aeronautics and Space Administration keeps its pictures in archives, a half-dozen or so around the country. One archive is at Cornell University. Nearby, in the early 1980s, Philip Marcus, a young astronomer and applied mathematician, had an office. After Voyager, Marcus was one of a half-dozen scientists in the United States and Britain who looked for ways to model the Red Spot. Freed from the ersatz hurricane theory, they found more appropriate analogues elsewhere. The Gulf Stream, for example, winding through the western Atlantic Ocean, twists and branches in subtly reminiscent ways. It develops little waves, which turn into kinks, which turn into rings and spin off from the main current—forming slow, long-lasting, anticyclonic vortices. Another parallel came from a peculiar phenomenon in meteorology known as blocking. Sometimes a system of high pressure sits offshore, slowly turning, for weeks or months, in defiance of the usual east-west flow. Blocking disrupted the global forecasting models, but it also gave the forecasters some hope, since it produced orderly features with unusual longevity.

Marcus studied those NASA pictures for hours, the gorgeous Hasselblad pictures of men on the moon and the pictures of Jupiter's turbulence. Since Newton's laws apply everywhere, Marcus programmed a computer with a system of fluid equations. To capture Jovian weather meant writing rules for a mass of dense hydrogen and helium, resembling an unlit star. The planet spins fast, each day flashing by in ten earth hours. The spin produces a strong Coriolis force, the sidelong force that shoves against a person walking across a merry-go-round, and the Coriolis force drives the spot.

Where Lorenz used his tiny model of the earth's weather to print crude lines on rolled paper, Marcus used far greater computer power to assemble striking color images. First he made contour plots. He could barely see what was going on. Then he made slides, and then he assembled the images into an animated movie. It was a revelation. In brilliant blues, reds, and yellows, a checkerboard pattern of rotating vortices coalesces into an oval with an uncanny resemblance to the Great Red Spot in NASA's animated film of the real thing. "You see this large-scale spot, happy as a clam amid the small-scale chaotic flow, and the chaotic flow is soaking up energy like a sponge," he said. "You see these little tiny filamentary structures in a background sea of chaos."

The spot is a self-organizing system, created and regulated by the same nonlinear twists that create the unpredictable turmoil around it. It is stable chaos.

As a graduate student, Marcus had learned standard physics, solving linear equations, performing experiments designed to match linear analysis. It was a sheltered existence, but after all, nonlinear equations defy solution, so why waste a graduate student's time? Gratification was programmed into his training. As long as he kept the experiments within certain bounds, the linear approximations would suffice and he would be rewarded with the expected answer. Once in a while, inevitably, the real world would intrude, and Marcus would see what he realized years later had been the signs of chaos. He would stop and say, "Gee, what about this little fluff here." And he would be told, "Oh, it's experimental error, don't worry about it."

But unlike most physicists, Marcus eventually learned Lorenz's lesson, that a deterministic system can produce much more than just periodic behavior. He knew to look for wild disorder, and he knew that islands of structure could appear within the disorder. So he brought to the problem of the Great Red Spot an understanding that a complex system can give rise to turbulence and coherence at the same time. He could work within an emerging discipline that was creating its own tradition of using the computer as an experimental tool. And he was willing to think of himself as a new kind of scientist: not primarily an astronomer, not a fluid dynamicist, not an applied mathematician, but a specialist in chaos.

Life's Ups and Downs

The result of a mathematical development should be continuously checked against one's own intuition about what constitutes reasonable biological behavior. When such a check reveals disagreement, then the following possibilities must be considered:

a. A mistake has been made in the formal mathematical development;

b. The starting assumptions are incorrect and/or constitute a too drastic oversimplification;

c. One's own intuition about the biological field is inadequately developed;

d. A penetrating new principle has been discovered.

—HARVEY J. GOLD,
Mathematical Modeling of Biological Systems