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Astrodynamics

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The engineering and scientific aspects of space exploration rely heavily on astrodynamics, and for this reason it has become of considerable importance in the past few years, under various names—space dynamics, applied celestial mechanics, orbit mechanics, etc., depending on the interest, orientation, and aim of the project. It is a powerful engineering tool on one hand and a science with far-reaching potentiality on the other. Accordingly, its scope includes application of celestial mechanics, astronomy, analytical dynamics, and optimization techniques to missile and spacecraft motion; orbit prediction (perturbation techniques), orbit determination (use of observational data), trajectory modification (optimization of powered flight), and trajectory selection (system and mission studies). In addition, the aim of this discipline is to obtain new knowledge regarding the geophysical, lunar, and planetary environments through interpretation of the orbital data of artificial earth satellites and lunar and interplanetary probes.

To appreciate its problems and accomplishments, it is essential to understand that this newborn discipline consists of continuous interplays, between modern and classical problems, as well as between new and old techniques. First, for the researcher to make significant contributions, he requires, as a prerequisite, familiarity with the background sciences of celestial mechanics, with the fast-widening circle of applications, and with modern computer and mathematical techniques. Several of the present-day problems were unknown to the high priests of classical celestial mechanics, and these problems require the development of new techniques. Other problems of astrodynamics can be handled with the well-known classical approaches. It is significant that a powerful feedback exists in the field, not only because space explorations helped to rediscover and rejuvenate celestial mechanics, but also because high-speed electronic computers and modern (for example, topological) mathematical methods are today capable of handling several of the classical problems far more efficiently than ever before.

In what follows, the status of the major areas of astrodynamics will be described to emphasize problems and accomplishments, rather than to review individual papers. The interdisciplinary aspects, the impact of space projects on astrodynamics, the influence of astrodynamics on space exploration, and the roads to new discoveries will be stressed, since for our purposes here familiarity with the whole picture is more important than mosaic credits to the many whose brilliant contributions made the advances.

A classification of the main areas in astrodynamics according to mission requirements, while useful in planning future space explorations, turns out to be repetitious and overlapping and prevents the presentation of the basic problems. Considering the technical areas involved, four major items emerge:

1. Orbit prediction—the precomputation of trajectories.
2. Orbit determination—the establishment of trajectories from observational information.
3. Orbit modification—the changing of trajectories by applying thrust.
4. Orbit selection—the choosing of trajectories applicable to given missions.

These major areas show strong interdependence. Orbit determination aims at establishing ephemerides for natural and artificial celestial bodies, and it starts with obtaining and analyzing observational data from which a preliminary orbit is established. To improve this first approximation, one modifies the orbit mathematically. So orbit modification and determination are closely related. On the other hand, once the initial conditions and the forces acting on the vehicle are known, its future is determined. To compute an orbit from given initial conditions is the subject of orbit prediction (item 1), so if orbit determination (item 2) gives the initial conditions, the problem is reduced to item 1. Furthermore, selecting the "best" orbit (item 4) requires a knowledge of the totality of possible trajectories, an undertaking belonging to item 1. Whenever the mission requirements cannot be satisfied with ballistic or free-fall trajectories, orbit selection (item 4) will lead to thrust requirements (item 3). Many other examples of the interdependences exist, showing that the whole field of astrodynamics benefits from progress along any avenue.

The question of the largest sensitivity emerges naturally: What are the *central problems*, solutions of which will result in the greatest advancement of the whole field?

Such a global question by its nature is controversial, and a great variety of answers is available. The significant fact, however, is that a common element persists from whatever orientation or specialization the answers come. To arrive at this critical problem the following two considerations are offered.

Firstly, we recall that the special and general perturbation methods used for orbit prediction require the knowledge of a reference orbit, that is, an analytical expression of an approximate solution. Orbit determination is based on such a reference orbit, called the preliminary orbit. Orbit modification studies the influence of thrust on the standard trajectory, and therefore it needs the results of orbit prediction, that is, an analytical expression for an approximate or exact solution. Orbit selection is based on the representation in a useful form of many possible orbits. If the accuracy of an approximate solution is known, the systems engineer can proceed with his work.

Secondly, when we go further into the question of environment or physical constants, the same basic problem seems to emerge. The engineering questions are: How accurately do we *have to know* the gravitational field of the earth, of the moon, and of the planets; and how much error can be allowed in density determinations and in the value of the astronomical unit? The scientific question is slightly more basic: How accurately do we *know* these values? The fact that recent radar and older determinations of the solar parallax are associated with *smaller* standard deviations than the difference between the means of these results cannot be discussed here in detail. Nevertheless, it indicates that neither the engineering nor the scientific question is closed. Exact or approximate analytical solutions of trajectory problems allow the evaluation of the effects of uncertainties in the physical constants on orbits of natural and artificial bodies and, in an admittedly roundabout way, allow finding the answers to the preceding questions.

Returning, then, to the subject of "critical problems," and allowing a properly broad interpretation of the discussion just given, we may formulate the common underlying "conditio sine qua non" problem to significant progress in astrodynamics as follows: *determination of approximate solutions with well-defined accuracy in analytical form.* Used as the basis of evaluating the progress made in astrodynamics in the few years of its existence, this statement permits an orderly review of the significant advances.

IN the field of *orbit predictions*, significant advances were made along the lines of special as well as general perturbations. Individual orbit computations, performed mostly by high-speed electronic computers, are generally referred to as special perturbations, since their validity is limited to single orbits, that is, to certain initial conditions. Computer programs of great variety exist today with applicability ranging from satellite orbits (including higher order gravitational harmonics) to lunar and interplanetary trajectories. Some of these programs include capability to establish initial conditions required for a certain mission, this way solving the so-called two point boundary value problem. Systematic trajectory computations have been performed, resulting in a large number of trajectories available for the systems engineer to perform his orbit-selection activities. Error coefficients (sometimes called guidance, influence, or differential correction coefficients) also have been computed on a systematic basis to offer assistance to designers of orbit-modification equipment. Such systematic numerical studies often solve the immediate trajectory problem and might give insight to the nature of the solution, and it is not inconceivable that a deeper understanding of the totality of solutions might be forthcoming.

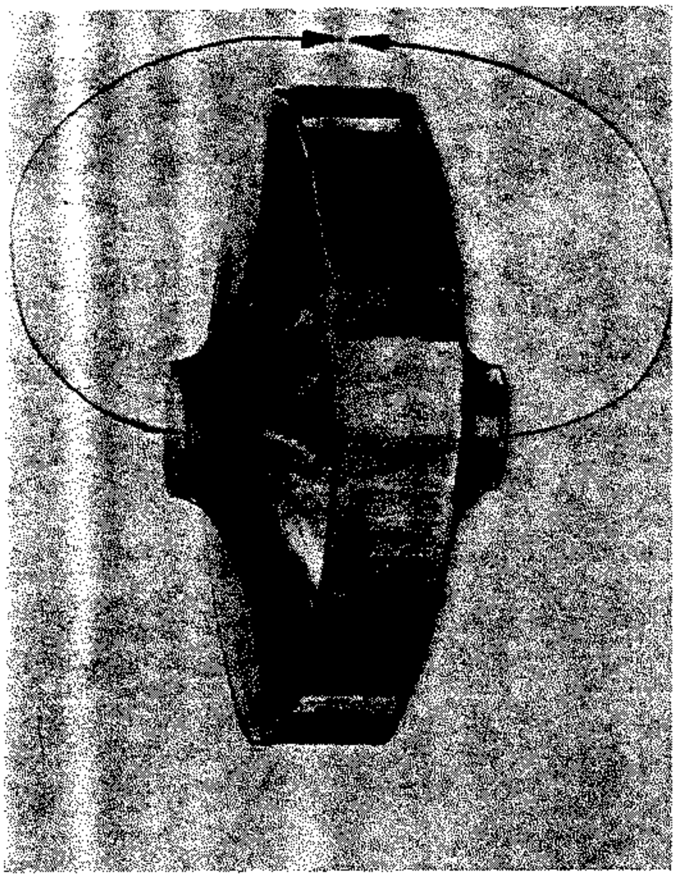
The two fundamental techniques involved in special perturbation work are the Cowell and the Encke methods. The first consists of integrating the actual differential equations of motion using rectangular coordinates, while the second scheme integrates the differences between a conic-section approximation and the actual orbit. Considering the latter, it is clear that a reference orbit that approximates lunar and interplanetary trajectories better than a conic section will reduce the numerical work significantly. Improved reference orbits have been proposed recently, such as the two fixed force center solution, and conic-section approximations have been studied in great detail, yielding analytically expressed guidance coefficients.

While the aim of general perturbations is more ambitious, the methods of general and special perturbations often merge and are not necessarily clearly distinguishable. The search for new variables, another favorite undertaking of astrodynamicists, is of course along the lines of establishing new reference orbits which can be used to obtain the final solution. If the final solution is obtained by numerical integration, we refer to the technique as special perturbation, while the process of improving the reference orbit analytically can be regarded as general perturbation. The power of the general-perturbation approach became apparent in the past few years, when analytical solutions became available for the artificial earth-satellite problem (without drag).

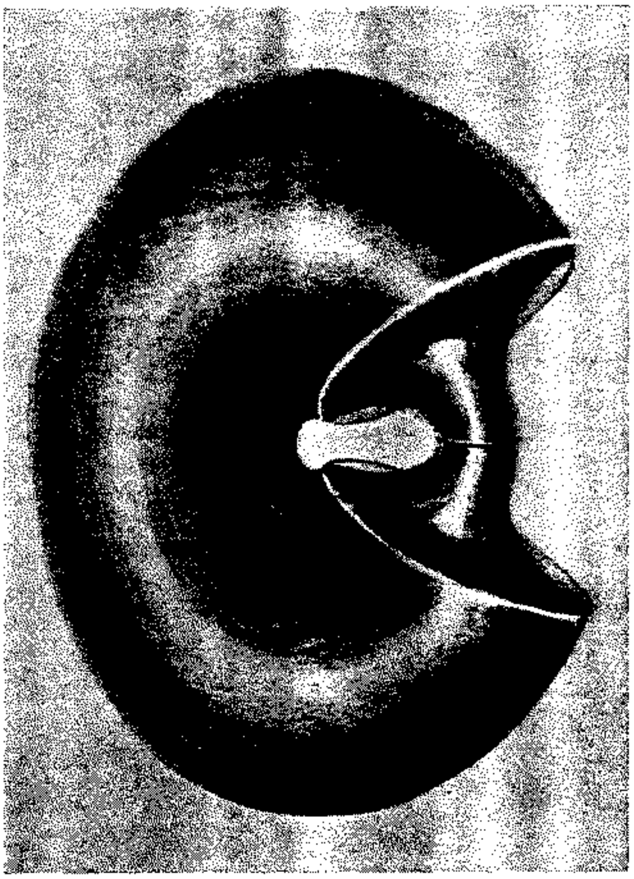
Another powerful approach is to invert the problem at hand and search for an approximate description of the force field. In other words, we can approximate either the actual force field or the actual orbit. Both of these approaches have been presented in the literature with not negligible success.

Orbit determination problems are treated today in a sophisticated and highly automatized manner which, less than a decade ago, would not have been thought possible. The important progress made a large amount of various types of observational data digestible, and realtime orbit determinations became feasible. Further, comprehensive studies of various types of tracking systems on the one hand and more sophisticated statistical analyses on the other are indicated.

Precise knowledge of the forces acting on the vehicle is mandatory for long-time-orbit determination. This problem takes us back to the question of physical constants. Accurate earth-satellite ephemerides require better



Orbit selection produces flywheel-like objects such as this which show totality of orbits in restricted three-body problem.



Joining axle ends produces anatomic form of this type, with outer shell going to infinity, representing hyperbolic-type orbits.

information of the atmospheric density, its time dependence, and the gravitational field than is available today. High-density satellites (that is, low area/mass ratio) or satellites placed in critical (resonance-type) orbits will furnish better values of the gravitational harmonics of the earth, while careful monitoring of the solar radiation and establishing its correlation with atmospheric-drag effects will result in improvements of our knowledge of the properties of the density of the atmosphere. Low-altitude satellites, orbits near the critical angle of inclination, and 24-hr satellites are of principal interest to the astrodynamicist. To obtain new knowledge regarding the gravitational field of the moon and planets is another area which will contribute to the engineering and scientific significance of astrodynamics. The theory needed to obtain such information from tracking data has been developed; the hardware and the satellites are lagging.

Orbit modification by high- or low-thrust devices of continuous or intermittent operation is one of the new problems with which the predecessors of present-day astrodynamicists were not faced. The two major advances were made in guidance analysis and by the establishment of optimization techniques. The first area relies on astrodynamics to furnish trajectory information which can then be used for guidance analysis, that is, which is amenable to analytical treatment. This requirement emphasizes once again the importance of establishing approximate or exact solutions in analytical form.

The field of optimization is judged by many as one of the most important areas in astrodynamics. It has been shown several times during recent years that properly formulated general optimization procedures can decide the feasibility question of certain missions. Such approaches as the calculus of variations, dynamic programming, and the method of steepest descent have gained in their applicability to space dynamics in the past few years. Necessary and sufficient conditions were established and discontinuous solutions were admitted. Nevertheless, not all of the theoretical considerations have been implemented on computers.

Orbit modifications by impulse have been studied in considerable detail with highly valuable practical results.

The problem of rendezvous might be mentioned in this field as one of principal interest to manned lunar missions. A classical problem of celestial mechanics from an analytical point of view is closely related to this most-recent problem. The equations describing the motion of Trojan asteroids under certain conditions show remarkable similarity to the rendezvous problem of a vehicle meeting another one, the latter being in an elliptic orbit.

The field of *orbit selection* is closely related to orbit modification, its main problems being optimization and the establishment of the totality of possible orbits. Progress with the second avenue has taken place along two lines—the mission-oriented numerical computation of a large number of orbits and topologically oriented qualitative studies. It is interesting to note that the field of orbit selection, which is almost completely systems-engineering oriented, might benefit greatly from the results obtained by the most sophisticated mathematical techniques.

This fact is not surprising when the standard systems-engineering question is recalled regarding the search for trajectories satisfying certain mission requirements. The desired trajectories—if they exist—can be computed by a digital machine, but their existence and their sensitivity to firing errors and delays are questions of higher order difficulty. Anyone who participated in establishing trajectories applicable to the basic or modified Apollo missions can testify regarding the difficulties involved. And is there anybody who did not participate? Round-trip interplanetary missions are on an even higher level of difficulty.

Orbit prediction, furnishing analytical results or a set of possible orbits; *orbit determination,* giving accurate physical constants and, during the flight, realtime information on the orbit; *orbit modification,* offering optimum trajectories and, during the flight, the guidance information; and finally, *orbit selection,* establishing the whole mission, constitute the four-masted ship of astrodynamics.

This discussion would not be complete without a few words concerning the *literature.* The fast-growing field of astrodynamics has a rich and time-honored literature since its foundations rest in celestial mechanics. The classics—Charlier, Encke, Euler, Gauss, Hamilton, Hill, Jacobi, Lagrange, Laplace, Poincaré, Tisserand, von Zeipel, to mention a few—are available in any self-respecting center of advanced work. The recently published volume, "Methods of Celestial Mechanics," by Brouwer and Clemence (Academic Press, 1961) treats some of the work of the classics, emphasizing orbit prediction. In the field of orbit determination, the lack of a modern text discussing radar and Doppler techniques is hurting the profession. Regarding orbit modification and selection, reference is made to the almost incomprehensibly fertile literature in the form of government, industrial, and university reports and journal articles.

The leading periodicals are the *Astronomical Journal* of the American Astronomical Society (with a large number of pure *and* applied papers), the translation of the *Astronomical Journal of the Academy of Sciences of the USSR* (large number of analytical papers), the *Journal of the American Rocket Society,* the *Astronautica Acta* of the International Astronautical Federation, the *Journal of the Astronautical Sciences* of the American Astronautical Society, the *Journal of the British Interplanetary Society,* and the *Space Sciences Reviews.*

Recently published books for the astrodynamicist's library are by Ehricke ("Space Flight," Van Nostrand, 1960), Baker ("An Introduction to Astrodynamics," Academic Press, 1960), Thomson ("Introduction to Space Dynamics," J. Wiley, 1961), Miele ("Flight Mechanics," Addison-Wesley, 1962), Danby ("Fundamentals of Celestial Mechanics," MacMillan, 1962), and, in handbook style, Jensen, et al. ("Design Guide to Orbital Flight," McGraw-Hill, 1962). The "bridge-between" book connecting celestial mechanics with astrodynamics, impatiently awaited by the profession, is being prepared by S. Herrick.

Excellent sources of up-to-date information are the volumes of papers offered at national and international meetings. In addition to the American Astronautical Society's yearly volume, called "Advances in the Astronautical Sciences," publishing the papers given at their national meeting, the Committee on Space Research of the International Council of Scientific Unions (COSPAR) publishes "Space Research," and the International Astronautical Federation publishes its volume of presented papers. Other volumes of collected papers are edited by the American Mathematical Society ("Orbit Theory," 1959), by Seifert ("Space Technology," J. Wiley, 1959), by Ordway ("Advances in Space Science," Academic Press, 1959), by Kurnosova ("Artificial Earth Satellites," Plenum Press, 1960), by Berkner and Odishaw ("Science in Space," McGraw-Hill, 1961), and by Koelle ("Handbook of Astronautical Engineering," McGraw-Hill, 1961).

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