INTERPLANETARY TRAJECTORY SIMULATION

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Abstract

The purpose of this paper is to demonstrate that an interplanetary trajectory simulation program which obtains velocities and positions of the astronomical bodies, as well as of the vehicle, by numerical integration yields a much smaller, simpler program than one that obtains planetary information by table look up; that no loss of accuracy results due to obtaining planetary position by numerical integration rather than by table look up; that the solution is accomplished in reasonable machine time; and that this approach allows launch to impact simulation of interplanetary flight as only a minor modification to existing ballistic missile programs.

Introduction

Developments in the field of rocket propulsion during recent years have generated the capability of launching rocket propelled vehicles from the surface of the earth at very high velocities. This capability was first employed to place artificial satellites in orbit about the Earth. This first step has been followed by probes which have impacted the Moon or escaped the Earth. The next logical step in this sequence will be to hurl an instrumented package to the neighborhood of another planet. Adequate planning for such interplanetary flight requires a study of the trajectory which such a vehicle would follow, including the problems of propulsion, guidance, aerodynamics, etc., of now familiar long-range missile simulations, and with the n-body problem added. For ballistic operation above the atmosphere, the n-body problem is the total problem. It is with this domain that this paper is concerned.

Much valuable qualitative and some quantitive information concerning interplanetary trajectories has been obtained by approximating the vehicle's orbit by the solution of three two-body problems, i.e., by assuming that the vehicle is affected first only by the Earth, then only by the Sun, and finally only by the target body. However, a numerical integration of the full n-body problem is still necessary for adequate design work (1). The actual formulation of this process for use with electronic computers can take several forms. The choice of form and the skill with which the formulation is accomplished greatly influence the capabilities and cost of operation of the resulting analysis tool. The Missiles and Space Systems Engineering Department of Douglas Aircraft Company has developed such a method. This has been programmed for the IBM 704 and 709 EDPM, and has been used for the simulation of both cislunar and interplanetary flight.

It is the intent of the present paper to show that the method used leads to a solution in a reasonable computing time, is conservative in use of machine memory, and may easily be adapted as the gravitational contribution in a total system simulation.

First, the form of Newton's law of gravitation which is used as the equation of motion will be explained. Second, methods of obtaining initial planetary positions and velocities from: (a) the elements of the orbit and Kepler's third law, and (b) by numerical differentiation of an ephemeris, will be indicated and compared for accuracy. An explanation of the integration scheme and why it was chosen will complete the discussion of the present model. Program extensions to include effects of thrust, aerodynamics and guidance will be indicated since simple extension is the main reason for this approach. Applications to lunar studies will be used to show the use of the automatic hunting procedure. The use of the program to check and extend inferential two-body methods of interplanetary trajectory studies will also be demonstrated.

DISCUSSION OF METHOD

Form of Gravitation Law

Programs for ballistic trajectory simulation in the solar system must use as the equation of motion of the vehicle some form of Newton's law for the gravitational attraction between two bodies of masses m_1 and m_2,

$$F = \frac{Gm_1m_2}{r^2} \qquad (1)$$

where
- F = force of attraction
- r = distance between two bodies
- G = gravitational constant

and Newton's second law,

$$F = ma \qquad (2)$$

where a = acceleration. For the n-body problem, equations (1) and (2) must be considered as vector equations. A common mechanization of these equations evaluates the components of these vectors in rectangular components. This may be written

$${}_m\ddot{X}_j = G \sum_{k=0}^{n} \frac{m_k\,({}_mX_j - {}_kX_j)}{d_{mk}^{\,3}} - \sum_{k=1}^{n} \frac{m_k\,({}_kX_j - {}_0X_j)}{d_{0k}^{\,3}}, \quad j=1,2,3 \qquad (3)$$

where n is the number of bodies whose effect on the vehicle is deemed significant, and

- m_k = mass of the kth body
- d_{mk} = the distance between the vehicle and kth body
- m_0 = mass of the Sun
- d_{0k} = the distance between the Sun and kth body
- X_1, X_2, X_3 are rectangular coordinates with origin at the center of the Sun and are rotationally fixed with respect to inertial space
- ${}_mX_j$ = j position coordinate of the vehicle
- ${}_kX_j$ = j position coordinate of kth body
- ${}_m\ddot{X}_j$ = jth component of vehicle acceleration.

The first term in the brackets of (3) is a solution of (1) and (2). The second term is the acceleration of the Sun due to the other bodies and is necessary because the origin of coordinates, traveling with the Sun, is not fixed in inertial space. It is assumed that the coordinates do not rotate.

Planetary Position and Velocity

There remains the major problem of specifying the position of the n-bodies as a function of time. Since the velocity of the missile with respect to selected bodies is usually desired as an output, the velocities of these bodies with respect to the sun must also be determined. The usual method is to store tables of planetary position and to interpolate and numerically differentiate at each point, or to store polynomials of planetary position and to evaluate these and their derivatives at each point.

The approach described herein is believed to lead to a simpler and more compact program. Equation (3) is rewritten

$$ {}_i\ddot{X}_j = G \sum_{\substack{k=0 \\ k\neq i}}^{10} \frac{m_k\left({}_iX_j - {}_kX_j\right)}{d_{ik}^{\;3}} - \sum_{\substack{k=0 \\ k\neq c}}^{10} \frac{m_k\left({}_kX_j - {}_cX_j\right)}{d_{ck}^{\;3}} \qquad (4) $$

where $j = 1,2,3$; and $i = 1,2,\ldots 11$; $i \neq c$.

The only changes from equation (3) have been the replacing of the subscript m with i and the subscript 0 with c.

Equations(4) is the total equation of motion of 12 bodies, 36 simultaneous 2nd order differential equations. If we associate

0 with the Sun (☉)
1 with the Earth (⊕)
2 with Mercury (☿)
3 with Venus (♀)
4 with Mars (♂)
5 with Jupiter (♃)
6 with Saturn (♄)
7 with Uranus (♅)
8 with Neptune (♆)
9 with Pluto (♇)
10 with the Moon (☾)

and

11 with the vehicle,

then equations (4) represent the equations of motion of the principal bodies of the solar system and the one other body, the vehicle, in which we are most interested. The subscript c replacing subscript 0 indicates that the coordinate system used has its origin at body c, the closest body to the vehicle. This was done to minimize round-off error in distances that would otherwise be prohibitive with the 704-709 floating point word length. Obviously, all three components of the acceleration of the body c are zero. Thus the system is a system of order 66 (33 second order equations). Given suitable initial conditions, the future relative positions and velocities of each body can be determined by numerical integration of equations (4).

The compactness of the program need hardly be discussed. Although equations(4) represents 33 equations, by simple looping in the coding, it can be made to occupy only little more space than the 3 equations (3) which all n-body space programs must carry; in addition no tables or polynomials are required. For a typical code, equations (3), for the missile alone, require 62 instructions, whereas equations (4), for the whole solar system, require 100 instructions.

To keep the form of simulation of the gravitational contribution to system performance compact is clearly attractive, for even though machines with 32,000 word memories are available,

the simulation of terrestrial missions, including guidance, controls, aerodynamics, thrust, etc., requires a very large percentage of this memory. Extra-terrestrial systems will include all of these elements plus the astronomical problems.

Since astronomical tables present only planetary positions, initial velocities must be calculated to commence the integration. These calculations are space consuming, but need not be in the memory during the simulation.

As to accuracy, note that the presently used ephemerides of the five outer planets have been obtained by numerical integration. Note also that the vehicle with its close approach to at least one, and probably two, other bodies will certainly have the most complex orbit of any of the n-bodies considered. If a trajectory program assumes, as is commonly the case, that the vehicle orbit may be accurately determined by a method of numerical integration, then the same integration method must certainly suffice for the simpler planetary orbits. Experimental verification of the accuracy will be indicated later.

The most important investigation for evaluating the feasibility of this method is that of determining the relative machine running time required for solving these many differential equations as compared to the more common practice of solving the three differential equations of motion of the vehicle and obtaining the positions and velocities of the planets by various other means.

First it will be mentioned that in no one mission is it expected that all 11 bodies will have a significant effect on the vehicle's orbit. The provisions for all these bodies were included to give the program the capability of simulating a variety of missions. In any particular case, their effect may be flagged out on the load sheets with a corresponding saving of computing time. For instance, in simulation of cislunar trajectories only the Earth and Moon, or Earth, Moon, and Sun, are normally considered; in a Mars vehicle design study, only the Earth, Mars, Sun, and Jupiter would normally be considered. On any particular run, almost no time is lost because the program was written "generally". The order of the system to be solved is then $6(r-1)$, where $r \leq 12$ is the number of bodies, including the vehicle, being considered.

Let us again examine the $3(r-1)$ equations(4). As noted previously, the second term is common to all equations and hence is evaluated only once. The calculation of d_{ik}^3, which involves a square root, is the largest time consumer in the evaluation. But each d_{ik} is used in 6 terms (since it is independent of j and $d_{ik} = d_{ki}$). Each Gkm/d_{ik}^3 appears in three terms. Thus the calculation of each term consists of one subtraction, one multiplication, one-third of a division, and one-sixth of the calculation of d_{ik}^3. The flow chart, figure 1, shows the steps used in an evaluation of these equations, so as

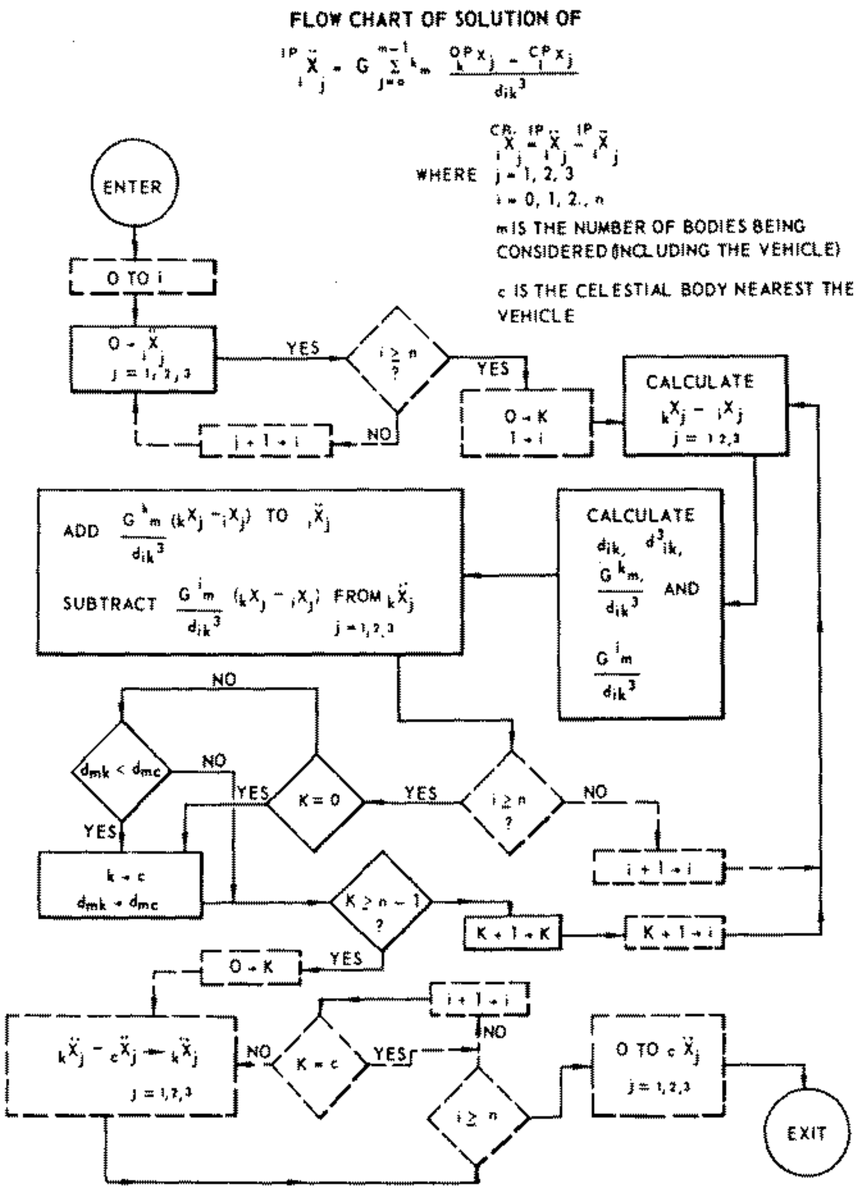


Figure 1.

to take advantage of the above facts while maintaining a compact code. (Figuratively speaking, those portions in solid boxes, or in solid portions of boxes, are necessary to solve only the vehicle's equation of motion. The dashed portions show what must be added to evaluate the equations of motion of the other bodies. Since an integration routine may be assumed to be present for integration of the vehicle's equations of motion, these dashed portions in essence replace all tables and table look up, and/or numerical differentiation, and/or polynominals and polynomial evaluation.) Care such as this has been important in attaining low running time with these equations of motion. With this technique the time for the evaluation of all equations of motion is about double that for the equations of motion of the missile alone. The time used for the numerical integration equations is almost directly proportional to the number of bodies considered, but the time to evaluate the integration formulae is only about 10% of the time required to evaluate the equations of motion.

Integration Technique

In integrating space trajectories, it is customary to use some "no-past-history" scheme of numerical integration such as Runge-Kutta to start the trajectory and then to shift to a scheme requiring less iterations, such as Adams-Moulton, as soon as the required number of points have been calculated. However, from a user's standpoint, the choice of print times should not be limited to required calculation times. If the principal integration scheme requires equally spaced points, it would generally need restarting by some Runge-Kutta type scheme at each special print point. Further, expansions of the program will most likely require discontinuous control programs. It was thus felt that schemes of integration other than Runge-Kutta would be of value only if they could represent a major gain in running time during that portion of the flight when they were being used. The adaption of the Runge-Kutta method outlined below, which was made by O. Senda, formerly of Douglas Aircraft Company, has proved so efficient that the inclusion of any other integration scheme has not to date shown any reduction in running time or improvement of accuracy.

Scarborough (2) gives the general Runge-Kutta equations for the numerical integration of a second order system as

$$k_1 = \Delta t\ f(t_n, x_n, \dot{x}_n)$$

$$k_2 = \Delta t\ f\left(t_n + \frac{\Delta t}{2}, x_n + \frac{\Delta t}{2}\dot{x}_n + \frac{\Delta t}{8}k_1, \dot{x}_n + \frac{k_1}{2}\right)$$

$$k_3 = \Delta t\ f\left(t_n + \frac{\Delta t}{2}, x_n + \frac{\Delta t}{2}\dot{x}_n + \frac{\Delta t}{8}k_1, \dot{x}_n + \frac{k_2}{2}\right)$$

$$k_4 = \Delta t\ f\left(t_n + \Delta t, x_n + \Delta t\ \dot{x}_n + \frac{\Delta t}{2}k_3, \dot{x}_n + k_3\right)$$

$$\Delta x = \Delta t\left[\dot{x}_n + \frac{1}{6}(k_1 + k_2 + k_3)\right]$$

$$\Delta\dot{x} = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

where f indicates any of the equations of motion (4) and Δt represents the computing interval. Scarborough also points out that if f is independent of velocity, then $k_2 = k_3$.

Because of the choice of rectangular components, which was done primarily for engineering convenience, the equations of motion are independent of velocity. For this special case, fourth order Runge-Kutta accuracy can be obtained from three, rather than the normal four, cycle computation. This leads to the interesting condition that the second order equations used are being integrated with less calculation than is generally required for first order equations. That is, the time for solution of the 6(r-1) order system is about three-fourths of what would normally be expected for a system with the same number of first order equations (a 2-1/4 (r-1) order system).

The method used to compute the time interval is a development from reference (3). In this approach a backward integration over the previous interval is made and the difference between the final and initial conditions is assumed to be twice the error in the forward integration. From the assumption that the error in fourth-order Runge-Kutta integration methods is proportional to the fifth power of the time interval and that the coefficient of this term varies slowly, the possible time interval for any allowable position error may then be calculated.

In this method, backward integration is used for only one component of one body. By methods more mesmeristic than mathematical, it has been decided that the position component having the biggest error and determining the time interval will be the one corresponding to the largest velocity component of the vehicle or the Moon, which ever has the greater acceleration at the moment.

When using this technique, very few integration steps are required for either lunar or Martian missions, as will be demonstrated in the examples below.

A self-computing time interval scheme is a necessity for efficient operation with the variations of time interval possible in an extra-terrestrial mission. This will be illustrated

by example. However, the requirement that the integration error be proportional to $(\Delta t)^5$ does require that the allowable integration error be one order of magnitude larger than round off error. Nevertheless, IBM 704 single precision floating point has sufficed. As has been mentioned above, this same limit on accuracy would exist if only the vehicle orbit were integrated and planetary positions and velocities were obtained by any other means. One indication of accuracy is provided by integrating the equations of motion of the solar system for 330 days and comparing the results to the American Ephemeris and Nautical Almanac (4). As an example, the error in the position of Jupiter was 1650 nautical miles (0.7" error in arc) when the integration was performed with a fixed 6 hour integration interval and 1400 nautical miles (0.6" error in arc) when the more rapid self-computing interval was used.

Further accuracy checks are given below in the examples of typical uses of the program.

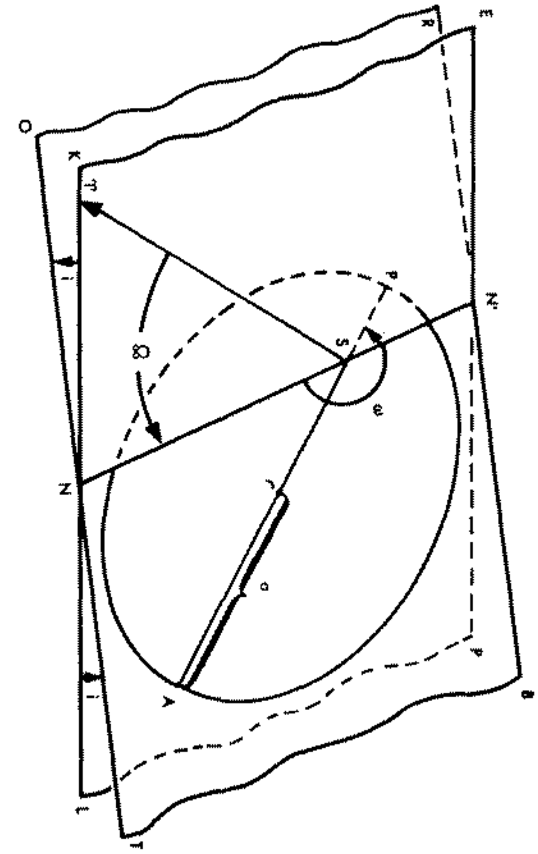
Position data of the principal objects of our solar system as a function of time are readily available, but velocities are not. Both are required as initial conditions for the numerical integration. Velocities may be derived from the elements of the orbits (i.e., inclination, semi-major axis, eccentricity, longitude of ascending node, longitude of perihelion, sidereal period, and mean longitude at epoch, see figure (2) and Kepler's law of equal areas (5). Mean elements for any future time may be calculated from simple formulas available in (6) and (7) and are shown in Table 1. For many astronautical purposes, the mean elements have been adequate and convenient. For more accurate calculations, osculating elements could be used. Unfortunately, these are not available for the whole period of astronautical interest.

Numerical differentiation of tables now available can provide extremely accurate velocities (error of less than 1 ft/sec in the case of the Earth), but is more awkward than obtaining velocities from the orbital elements.

A more detailed explanation of the mathematics involved in this program is presented in reference (5).

Analysis of Cislunar Trajectories

Studies of Earth to Moon trajectories conducted at the Douglas Aircraft Company have been based on the interplanetary trajectory simulation, (8) and (9). This has allowed the results to include the proper gravitational effects of Earth, Moon, and Sun. The effects of other planets on cislunar trajectories were examined and found to be negligible (the effect of including Jovian gravity in the calculations is a displacement of 0.01 miles at time of impact on the Moon).



NN' LINE OF NODES
i INCLINATION
a SEMI-MAJOR AXIS
☊ LONGITUDE OF THE ASCENDING NODE
☊ + ω LONGITUDE OF PERIHELION
EKLP PLANE OF ECLIPTIC
ORBT PLANE OF PLANET'S ORBIT
S SUN
A APHELION
P PERIHELION
♈ VERNAL EQUINOX
C GEOMETRIC CENTER OF ORBIT

Figure 2. Definition of Elements.

Table 1. Elements of the Planets' Orbits.

PLANET	MEAN DISTANCE	PERIOD	ECCENTRICITY, e	INCLINATION TO THE ECLIPTIC
1. EARTH	1.000000	1.00004	0.01675104−0.00004180T	
2. MERCURY	0.387099	0.24085	0.205615 +0.000020 T	7° 0' 10.6" + 6.3" T
3. VENUS	0.723332	0.61521	0.006818 −0.000050 T	3° 23' 37.1" + 4.5 T
4. MARS	1.523691	1.88089	0.093310 +0.000094 T	1° 51' 1.1" − 2.3" T
5. JUPITER	5.202803	11.86223	0.048335 +0.000164 T	1° 18' 31.4" − 20.5" T
6. SATURN	9.538843	29.45772	0.055892 −0.000345 T	2° 29' 33.1" − 14.0" T
7. URANUS	19.181945	84.01308	0.0470 +0.0002 T	0° 46' 20.9" + 2.3" T
8. NEPTUNE	30.057767	164.79405	0.0087 +0.00004 T	1° 46' 45.3" − 34.3" T
9. PLUTO	39.51774	248.4302	0.247	17° 8' 44.0" − 20.0" T

MEAN LONGITUDE OF ASCENDING NODE	PERIHELION	PLANET **
1. 0	101° 13' 15" + 6189" T	99° 41' 48.08" + 129,602,768.13" T + 1.089" T²
2. 47° 8' 43" + 4266" T	75° 53' 54" + 5596" T	178° 10' 44.68" + 538,106,654.80" T + 1.084" T²
3. 75° 47' 1" + 3260" T	130° 9' 8" + 5056" T	342° 46' 1.39" + 210,669,162.88" T + 1.1148" T²
4. 48° 47' 12" + 2786" T	334° 13' 6" + 6626" T	293° 44' 51.46" + 68,910,117.33" T + 1.1184" T²
5. 99° 26' 36" + 3638" T	12° 43' 15" + 5796" T	238° 2' 57.32" + 10,930,687.148" T + 1.20486" T² − 0.005936" T³
6. 112° 47' 25" + 3134" T	91° 5' 54" + 7050" T	266° 33' 51.76" + 4,404,635.581" T + 1.16835" T² − 0.021" T³
7. 73° 28' 38" + 1795" T	169° 3' 0" + 5800" T	244° 11' 50.89" + 1,547,508.265" T + 1.13774" T² − 0.002176" T³
8. 130° 40' 53" + 3956" T	43° 50' 0" + 2400" T	84° 27' 28.78" + 791,589.291" T + 1.15374" T² − 0.002176" T³
9. 108° 57' 17" + 4889" T	222° 48' 0" + 5000" T	137° 38' 0.00"

$$T = \frac{\text{JULIAN DAY NO.} - 2415020.0}{36525}$$

*ASTRONOMICAL QUANTITIES BY C. W. ALLEN
**FRENCH NAUTICAL ALMANAC, CONNAISSANCE DES TEMPS

If initial conditions for a trial trajectory are chosen so that the resulting path passes within several thousand miles of the moon, these initial conditions may then be corrected to reduce this miss distance by a system which is built into the program. This reduction amounts to about two-thirds of the miss distance per step, as shown in figure 3, for a typical case. The automatic system holds velocity fixed and computes correction to the flight path angle and azimuth at each step. The technique used was simply to resolve the miss distance into the planes of the initial azimuth angle and flight path angle and then to assure that the required change in input values of these angles would be the component of miss distance in their planes divided by the distance to the Moon (see figure 4). When this guess overcorrected to the other side of the center of the Moon, a linear interpolation was made for the next run. The original method was used again for the following run. The process was stopped when the miss distance was less than a specified tolerance or if any step failed to improve over the previous step by at least this same tolerance. The computer time required per trajectory is approximately 1.2 minutes. Because of the manner in which the integration interval is calculated by the program, this machine running time is essentially unaffected by moderate changes in initial velocity.

The number of computing steps required was approximately 88 including recalculation where the original estimate of the time interval proved too large. A typical calculating time interval versus flight time plot is shown in figure 5. The rapid change of time interval, shown here on a log plot, indicates the type of running time gains that are obtained by the self-computing intervals as compared to the method of using a step function based on the user's best guesses.

The final trajectory was rerun allowing double the error in each integration step. The impact position varied by one mile and the time of impact by 7.5 seconds. A check allowing 10 times the original error in the individual step indicated that total error is linear with step error, i.e., the error in the most accurate trajectory may be taken to be about 0.5 mile at impact with an impact time error of 4 seconds.

Application to Interplanetary Problems

For simulation of interplanetary trajectories, it is necessary to make a careful choice of initial conditions. A preliminary study using inferential techniques has proved useful as a basis for the selection of initial conditions. The choice should be made so that the vehicle trajectory will pass within a few million miles of the target planet. If this is accomplished, the miss distance can be reduced by a differential correction procedure. Each component of miss distance, ΔX_u, is

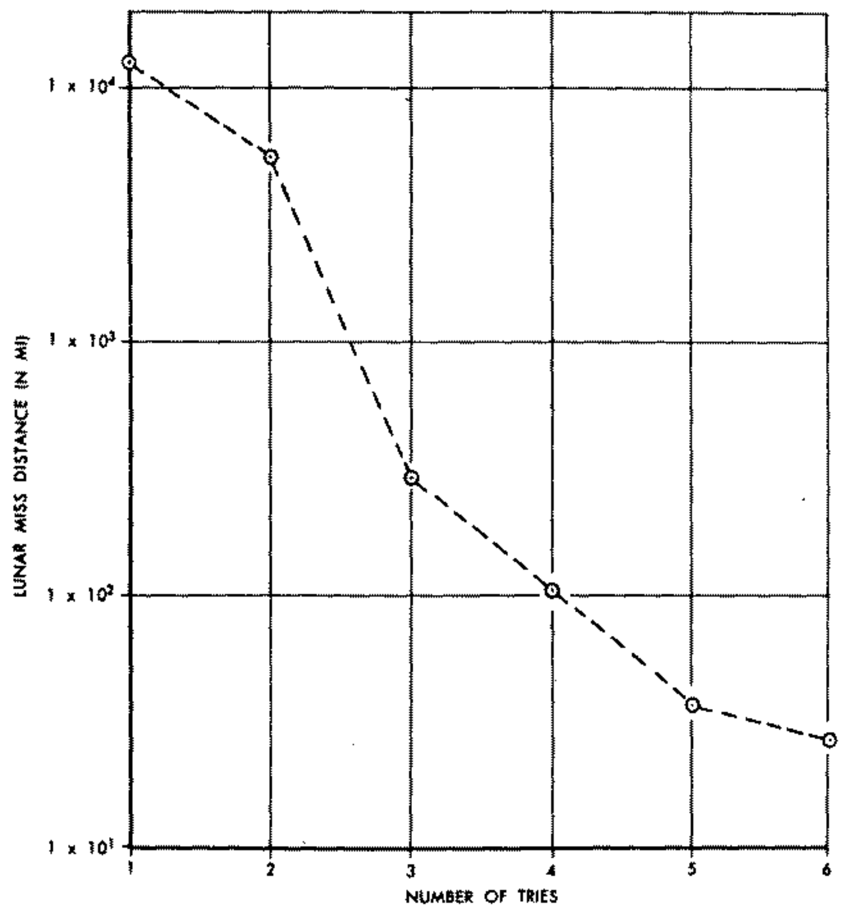


Figure 3. Convergence of the Lunar Hunting Procedure.

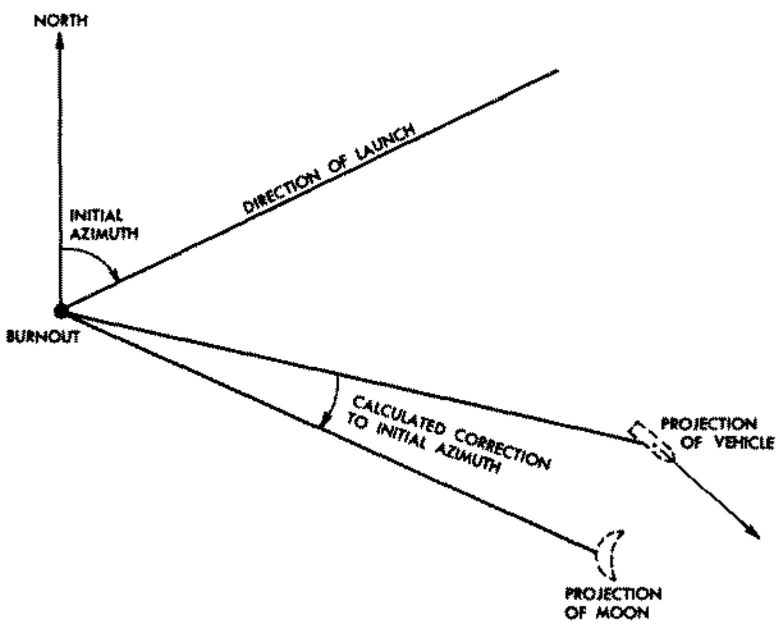


Figure 4. Projection of Miss Distance on Plan Tangent to the Earth at Point of Burnout.

FIFTH SYMPOSIUM ON BALLISTIC MISSILE AND SPACE TECHNOLOGY

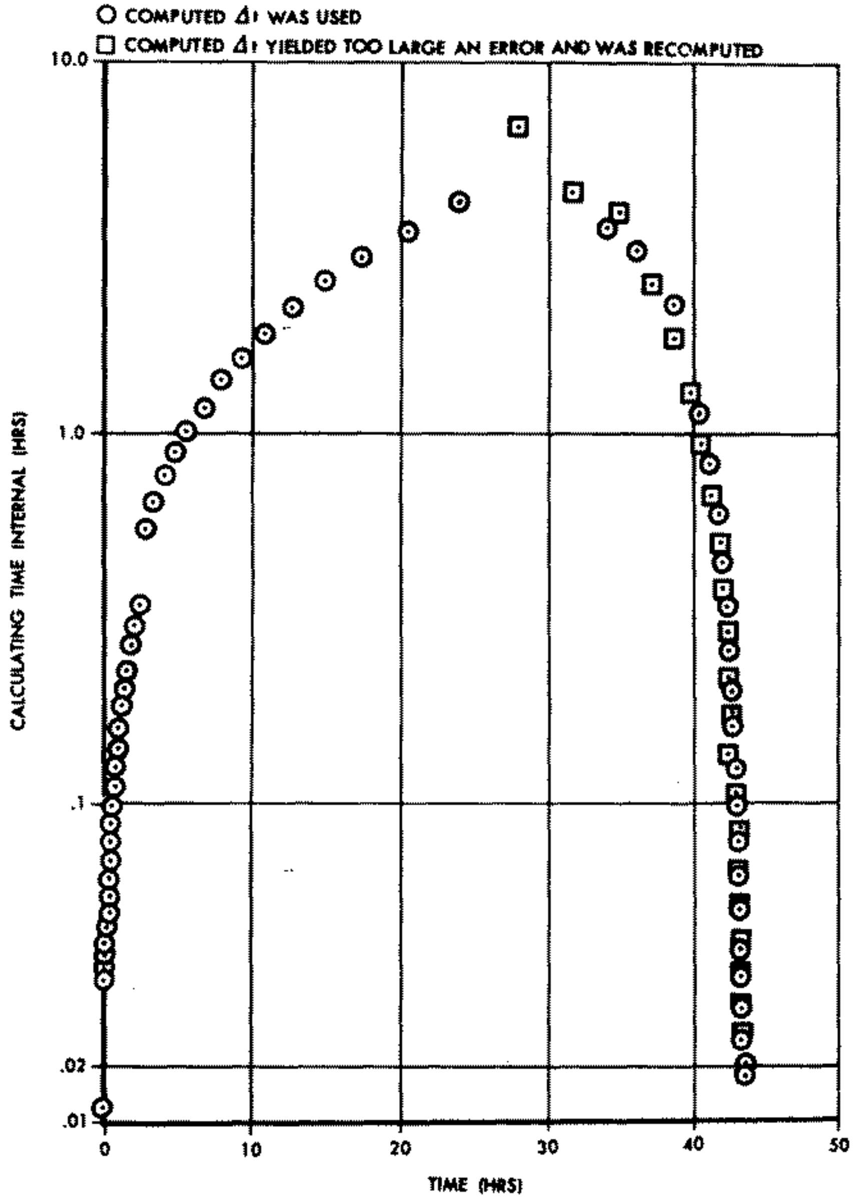


Figure 5. Computing Time Interval History for Typical Lunar Trajectory.

considered to be a function of initial velocity, V_0, flight path angle, γ_0, and azimuth of launch, A_{Z_0}. Fixed launch time and intercept time are assumed. Thus, Earth and Mars position and orientations are determined. From the initial trajectory, the partial derivative of each component with respect to one of the three input variables, e.g., velocity, V_0, is determined by running a trajectory varying only the initial velocity from the reference trajectory by a small increment ΔV and then making the approximation

$$\frac{\partial \Delta X_j}{\partial V} = \frac{\Delta X_j (\gamma_0, A_{Z_0}, V_0 + \Delta V) - \Delta X_j (\gamma_0, A_{Z_0}, V_0)}{\Delta V}, \quad j = 1,2,3$$

The partial derivatives with respect to flight path angle and azimuth angle are obtained from two more similar trajectories. If each component is expanded in Taylor Series about (γ_0, A_{Z_0}, V_0)

$$\Delta X_j (\gamma, A_Z, V) = \Delta X_j (\gamma_0, A_{Z_0}, V_0) + \frac{\partial \Delta X_j}{\partial \gamma} (\gamma - \gamma_0) + \frac{\partial \Delta X_j}{\partial A_Z} (A_Z - A_{Z_0}),$$

$$+ \frac{\partial \Delta X_j}{\partial V} (V - V_0) + \dots, \quad j = 1,2,3$$

where all terms above the first order have been ignored, then for impact the righthand terms must equal zero. The only unknowns are γ_0, A_{Z_0}, and V_0 and hence the three equations may be solved to yield new estimates of these quantities. The resulting miss distance, from the center of the target planet, can be reduced to a desired value (but not less than the error due to computational inaccuracy) by repeated application of this correction procedure. The amount of improvement available for this application has been slightly less than one order of magnitude on most of the trajectories considered to date. This reduction is illustrated in figure 6 for a typical Earth-to Mars flight. The final trajectory is illustrated schematically in figure 7. Additional information about the final trajectory of this series is contained in Table 2 and figure 8. The IBM 704 computing time required to obtain these results was approximately twenty minutes, with each trajectory requiring about 2.4 minutes of machine time to simulate the trajectory from launch to point of closest approach to target planet. Five and one-half bodies were considered: Sun, Earth, Venus, Mars, Jupiter, and the vehicle. The choice of velocity magnitude, flight path angle, and azimuth as independent variables is quite arbitrary; it is only necessary that the three independent

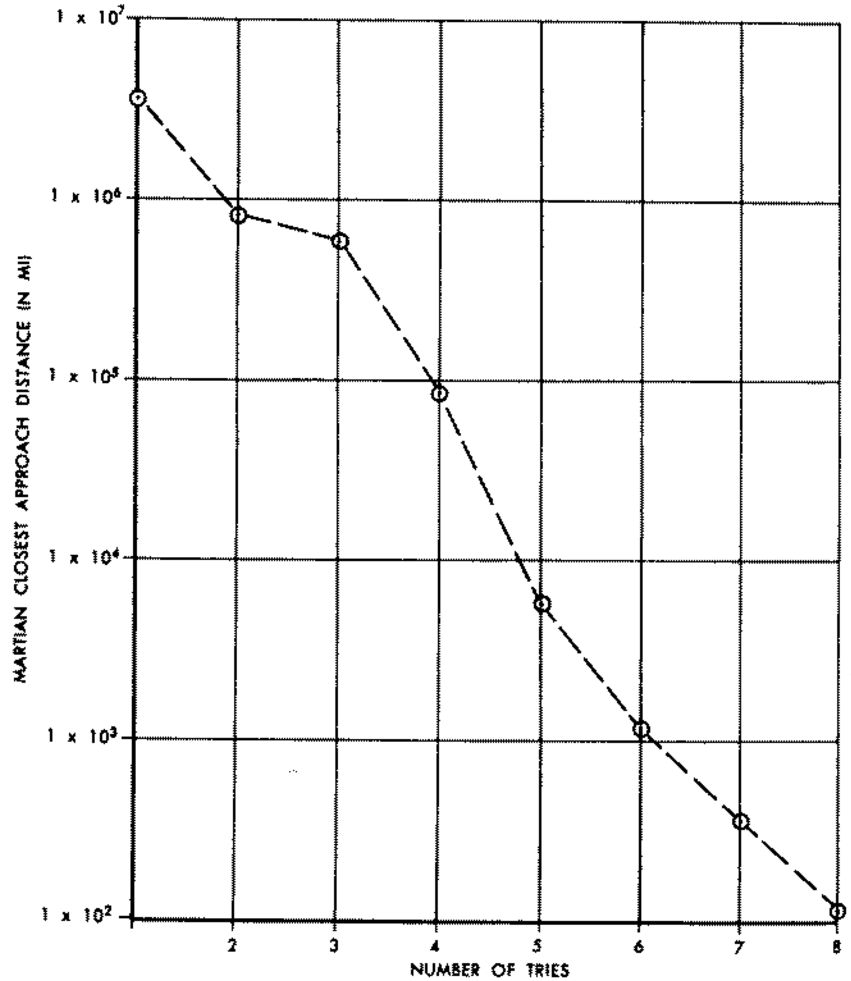


Figure 6. Convergence of the Interplanetary Hunting Procedure.

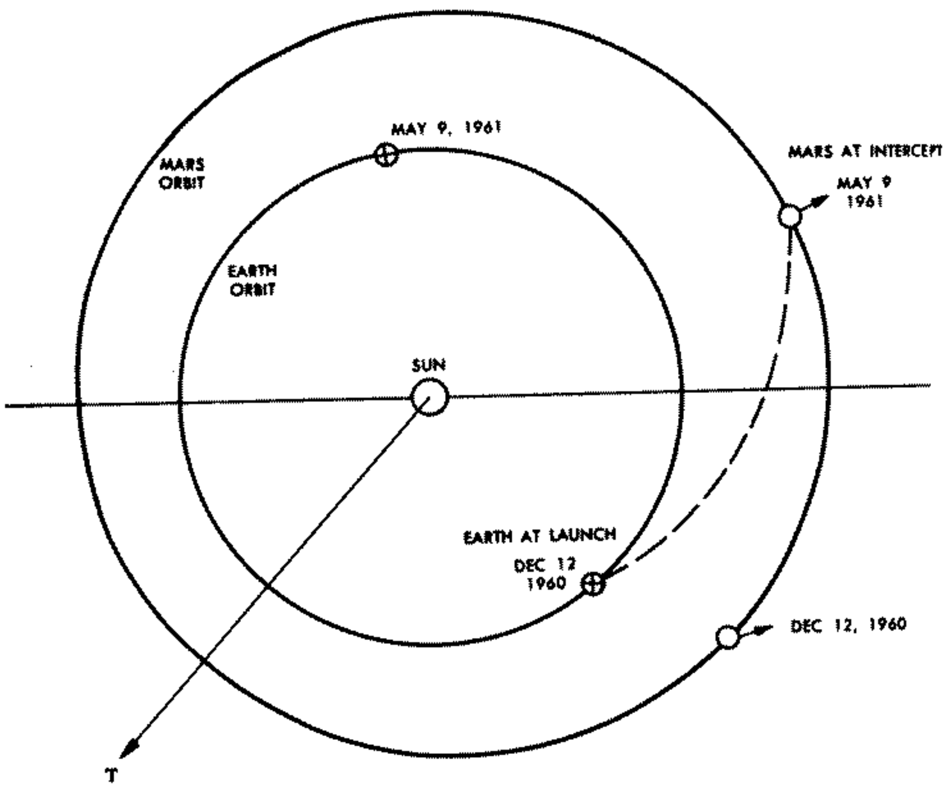


Figure 7. Typical Interplanetary Trajectory.

Table 2. Summary of Initial and Terminal Conditions and Permissible Deviations for Partian Impact

LAUNCH DATE	1960 DECEMBER 12
LAUNCH TIME	$4^h\ 59^m\ 27.89814^s$
TRANSFER ANGLE, $\bar{\beta}$, DEG.	81.17
NOMINAL LAUNCH CONDITIONS	
VELOCITY, V_L, FPS	56,314.02
FLIGHT PATH ANGLE, γ_L, DEG.	89.8
AZIMUTH ANGLE, A_Z, DEG.	89.943506
INCLINATION ANGLE, λ, DEG.	1.10
TERMINAL CONDITIONS	
VELOCITY AT IMPACT, V_i, FPS	23,344.63
IMPACT ANGLE, ψ, DEG.	6.4002
PERMISSIBLE ERRORS	
VELOCITY, ΔV_L, FPS	16.25
FLIGHT PATH ANGLE, $\Delta\gamma_L$, DEG.	0.00146
AZIMUTH ANGLE, ΔA_Z, DEG.	0.33
ASTRONOMICAL UNIT, $\frac{\Delta AU}{AU}$, %	0.0245
TRANSFER TIME, t, DAYS	146.5

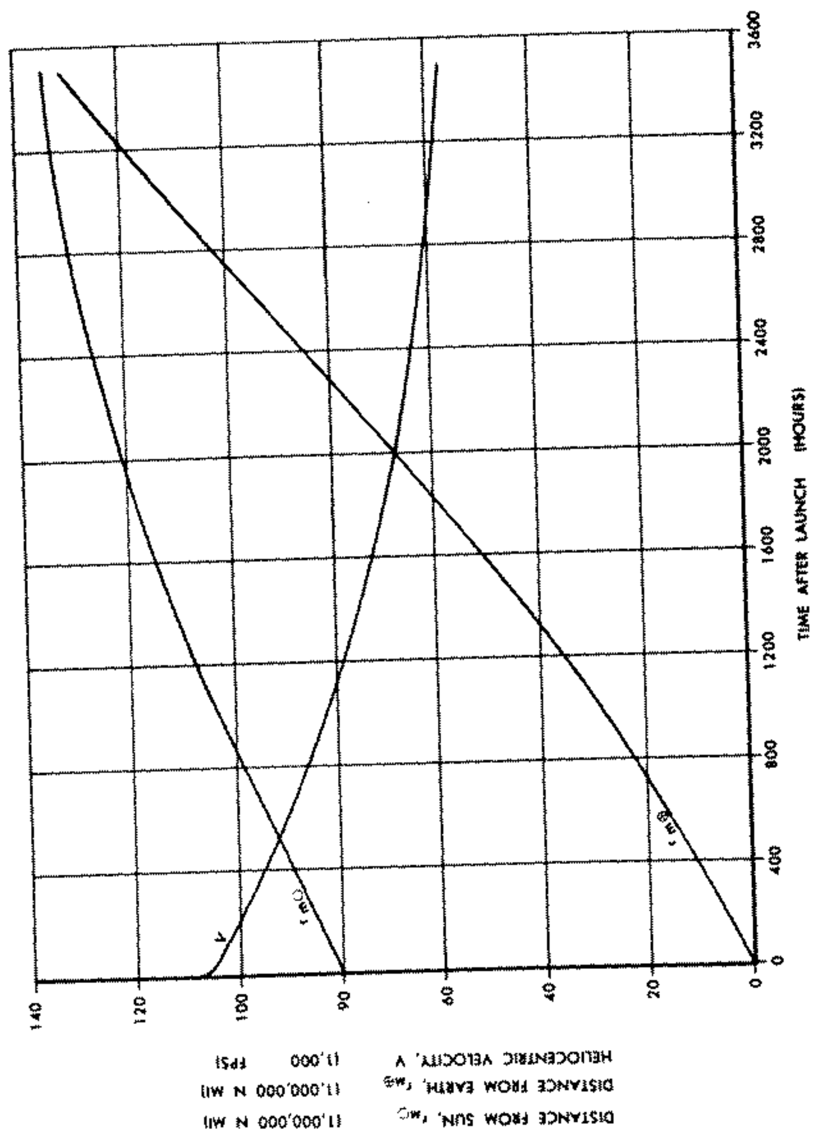


Figure 8. Velocity and Distance of Typical Martian Trajectory.

variables describe the initial velocity vector. Time interval versus time is indicated in figure 9. It is to be noted that the computing interval used is determined as an integral part of the program from an input accuracy requirement. This allows for the use of small increments in regions where gravitational forces are large (near planets) and large increments where gravitational forces are small (far from planets). Accordingly, much of the 2.4 minutes per trajectory is consumed in calculating the trajectory near the launch and target planet. The net result is that usage of the program for simulating trajectories of much greater duration does not require appreciably greater computation time. The total number of points required for the typical trajectory was 99.

As with the lunar case, trajectories were run with larger allowable errors. These indicated that the basic run's impact position on Mars had about 200 nautical miles of uncertainty due to integration error. The time of flight had an uncertainty of about six munutes due to the same cause. A similar check at a fixed time shortly before impact showed an uncertainty in missile position with respect to the Sun of 1070 miles, whereas the position of Mars checked within 13 miles. Since all programs must integrate the missile trajectory, it is felt that the much larger error in missile position supports the contention that no accuracy is lost by obtaining planetary positions by integration of the equations of motion.

Extensions for Total Mission Simulation

This mechanization of the n-body problem has been designed to be combined with existing ballistic missile trajectory methods to provide a means of simulating a total space mission. A typical simulation of the flight of a multi-stage long range missile, with guidance, on a rotating spheroidal Earth, proceeds as follows (see figure 10):

The variations of atmosphere and gravity with altitude and latitude are specified for each case. From these and the statement of initial missile position, the environment in which the missile is operating may be evaluated. Adding the missile specifications and initial velocity, the orders that would be issued by the guidance may be calculated and the accelerations due to aerodynamics and propulsion may be evaluated in missile coordinates. Then typically these may be rotated through the inertial platform coordinates to a set of coordinates located at the center of Earth (which are considered inertial). At this point the gravitational accelerations are easily added and the result rotated to coordinates at the surface of the Earth (with due allowance for Coriolis and Centrifugal accelerations) where they are integrated for new velocities and positions as would be seen by an observer on the Earth.

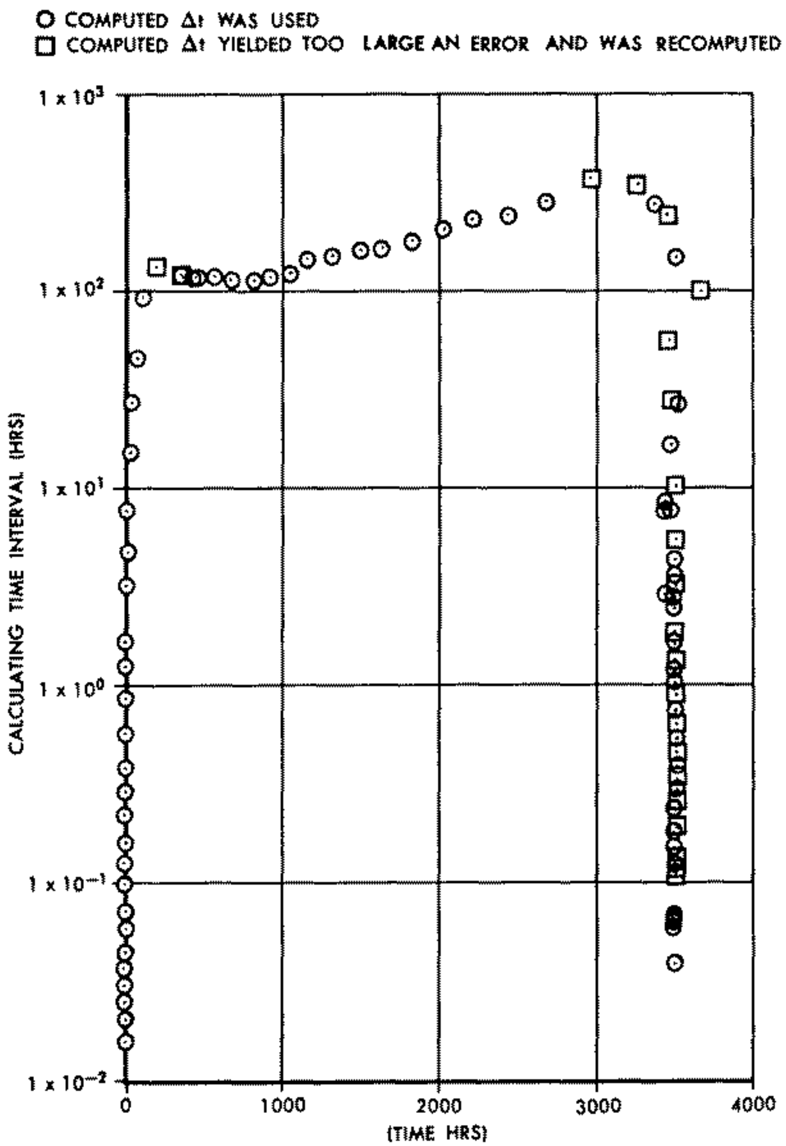


Figure 9. Computing Time Interval History for Typical Interplanetary Transfer.

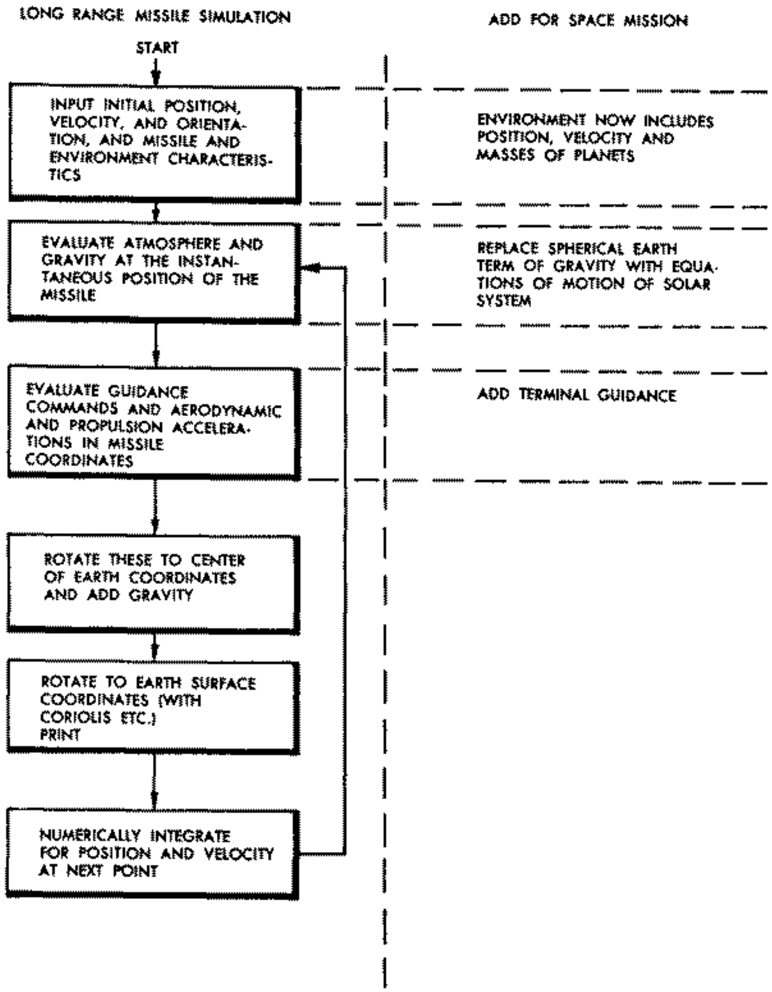


Figure 10. Relation Between Long Range Missile and Total Space Mission Simulation.

The changes to such a ballistic missile program to adapt it for a total simulation of a space mission would require:

(1) Specification of the atmosphere of each planet.
(2) Replacement of the spherical Earth term of the gravity calculation in the missile simulation by equations (4) in order to include the n-body effects and remove the assumption that the center of the Earth is inertially fixed.
(3) Additions to the guidance section for terminal and midcourse phases.

Obviously, forces that predominate in some portions of flight are insignificant in others. An efficient simulation will drop these out wherever possible, and will use four cycle integration where necessary and the simplified three cycle integration whenever possible.

Conclusions

It is concluded:

1. That integration of the equations of motion of the solar system is a compact way of obtaining planetary positions and velocities as a function of time for use in the vehicle equations of motion.
2. That the errors in planetary coordinates introduced by this method run one to two orders of magnitude less than the error in vehicle position, and hence, are insignificant in all applications.
3. That the machine time required to introduce the n-body's effect on the vehicle in this manner are comparable to those of table-look-up programs.
4. That when the n-body problem is handled in this way, expansion to a more general simulation of the entire mission is quite simple and does not lead to as large a program as other approaches do.

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