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# MINIMUM ENERGY ENTRY INTO ORBITS AROUND MERCURY*

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ABSTRACT

The velocity requirements for entry into circular orbits around Mercury are determined, assuming transfer along a substantially cotangential semi-ellipse from Earth, and making allowance for the eccentricity and inclination of the orbit of Mercury. It is shown that the lowest velocity requirement is obtained when the transfer orbit is in the plane of the orbit of Mercury with arrival at perihelion of Mercury's orbit. The same transfer technique is preferred when entry is to be made into a parabolic orbit close to the planet's surface.

It has been indicated in an earlier note by the author[1] that for each planet there is a circular satellite orbit, entry into which calls for the minimum total velocity requirement. In these earlier studies, it was assumed that the orbits of the planets were circular, concentric and coplanar, and transfer was effected along a minimum energy semi-ellipse.

The planet for which it appeared to be of greatest advantage to take account of this effect was Mercury. With the single exception of Pluto, for which these studies are of only academic interest, Mercury is the planet with an orbit having the greatest eccentricity (0·206) and inclination (7°). Further studies were therefore carried out to investigate the influence of these factors on the results obtained for this planet.

From the viewpoint of simple computation, it is a convenience that perihelion and aphelion of the orbit of Mercury lie close to the ecliptic. For these studies, the assumption is made that these points lie in the ecliptic. The various alternative flight paths open to us are, of course, infinite in number, but only the four bounding possibilities listed below were examined, transfer in each case being effected along a substantially cotangential semi-ellipse:

(1) Correct for inclination of orbit of Mercury in departing from Earth, and so enter a transfer orbit in the plane of the orbit of Mercury. Arrive at Mercury's perihelion.

(2) As (1), but arrive at Mercury's aphelion.

(3) Enter a transfer orbit in the ecliptic, and correct for inclination of orbit of Mercury when reaching Mercury. Arrive at Mercury's perihelion.

(4) As (3), but arrive at Mercury's aphelion.

The main results of these studies are presented in Table I, alongside corresponding data for the simplified study based on circular, concentric coplanar orbits for the planets. The circular orbit considered around Mercury is that closest to the surface, since this gives the minimum total velocity requirement in each case.

As expected, transfers to Mercury's perihelion involve a more eccentric transfer orbit than those to the planet's aphelion, and the velocity requirements for departure

TABLE I.—*Breakdown of velocity requirements (in miles/sec.) for flights from Earth to orbit of Mercury and to close circular orbits around Mercury respectively, with transfer in the orbital planes of Earth and Mercury respectively, and with arrival at perihelion and aphelion respectively of the orbit of Mercury.*

| | Data based on simplified model of circular, concentric, coplanar orbits | Data allowing for eccentricity and inclination of orbit of Mercury | | | |
|---|---|---|---|---|---|
| | | Transfer orbit in Earth orbit plane | | Transfer orbit in Mercury orbit plane | |
| | | Arrival at perihelion | Arrival at aphelion | Arrival at perihelion | Arrival at aphelion |
| Escape from Earth and enter transfer orbit .. .. | 8·34 | 9·02 | 7·84 | 9·22 | 8·10 |
| Enter orbit of Mercury distant from planet .. .. | 6·00 | 6·60 | 8·21 | 4·60 | 7·50 |
| Enter circular orbit around Mercury close to surface | 4·67 | 5·23 | 6·74 | 3·42 | 6·08 |
| Total for entry into orbit of Mercury distant from planet .. .. .. .. .. .. | 14·34 | 15·62 | 16·05 | 13·82 | 15·60 |
| Total for entry into circular orbit around Mercury close to surface .. .. .. .. .. | 13·01 | 14·25 | 14·58 | 12·64 | 14·18 |
| Saving .. .. .. .. .. .. .. | 1·33 | 1·37 | 1·47 | 1·18 | 1·42 |
| Saving, % of total .. .. .. .. .. | 9·3 | 8·2 | 9·2 | 8·5 | 9·1 |

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from Earth and entry into these transfer orbits are correspondingly higher. The requirements at perihelion are seen to be lower than at aphelion. The total requirements for entry into the close circular orbit around Mercury are roughly the same for all cases except that for transfer in the orbit plane of Mercury and arrival at the planet's perihelion. For this case the velocity requirement is significantly lower, and is usefully lower than that indicated in the simplified study that ignored the inclination of the planetary orbits.

The preferred transfer orbit is thus in the plane of the orbit of Mercury with arrival at the planet's perihelion. The percentage saving possible through entry into a close circular orbit around the planet is 8·5, a little less than that previously indicated in the simplified study, but still very useful.

The situation for arrival at Mercury's perihelion is illustrated in Fig. 1. The transfer takes 96·5 days, just

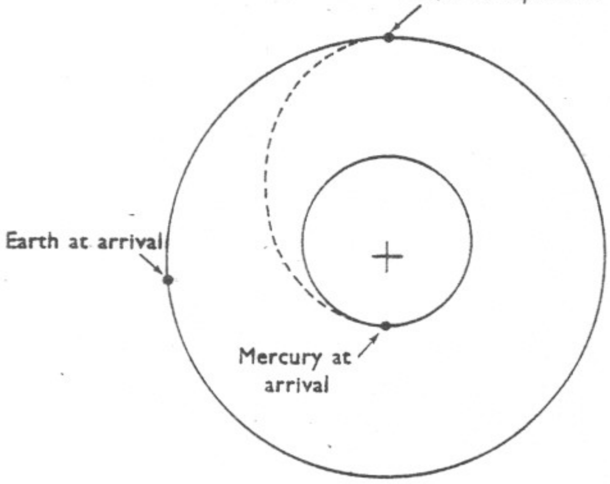


FIG. 1. Earth-to-Mercury transfer orbit. The situation illustrated is for arrival at Mercury's perihelion.

over three months, and the relative position of the two planets and the Sun on completion of the transfer is shown. This should favour communication with the minimum interference from solar noise.

For the most suitable case, the velocity requirements for entry into circular orbits around Mercury at different distances from the planet are given in Fig. 2.

Similar studies to the above were carried out for entry into parabolic orbits around Mercury following transfer along a substantially cotangential semi-ellipse, for direct comparison with the simplified studies of an earlier note

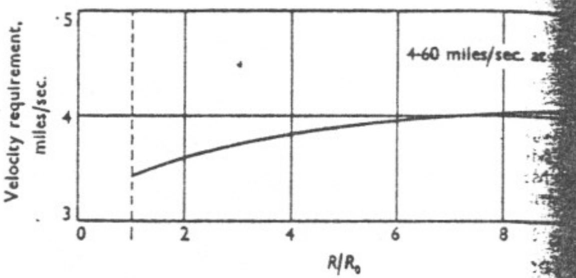


FIG. 2. Velocity requirement for entry into circular around Mercury from transfer orbit. The transfer orb the Mercury orbit plane, and arrival is at Mercury's peri

by the author.[2] Again, it was found that the mini total velocity requirement is given by transfer in the plane of Mercury with arrival at perihelion. The results are compared in Table II.

TABLE II.—*Comparison of simplified and real ve requirements (in miles/sec.) for flights from Ea orbit of Mercury and to close parabolic orbit a Mercury respectively. The real case shown is that the lowest total velocity requirement.*

| | Data based on simplified model of circular, concentric, coplanar orbits | Data allo for eccent and inclin of orb of Merc Transfer in Merc orbit pl Arrival perihel |
|---|---|---|
| Total for entry into orbit of Mercury distant from planet | 14·34 | 13·8 |
| Total for entry into parabolic orbit around Mercury close to surface .. .. .. | 12·22 | 11·8 |
| Saving .. .. .. .. | 2·12 | 1·9 |
| Saving, % of Total .. .. | 14·8 | 14·2 |

A conclusion of both sets of studies is that the assump tion of circular, concentric, coplanar planetary orbits satisfactory for introductory consideration of the velocity requirements for entry into orbits around planets, though more realistic assumptions will ob be required for more advanced studies.


REFERENCES

1. S. W. Greenwood, *J.B.I.S.*, 1961–62, 11, 73.
2. S. W. Greenwood, *J.B.I.S.*, 1961–62, 11, 108.